A System of Equations of Conservation

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Abstract

In classical mechanics, this paper presents a system of equations of conservation.

Equations of Conservation

If we consider a system of N particles then the equations of conservation are as follows:

(1) $\sum_{i=1}^{N} \left(m_i \, \mathbf{a}_i - \mathbf{F}_i \right) = 0$

(2) $\sum_{i=1}^{N} \left(m_i \, \overline{\mathbf{a}}_i \cdot \overline{\mathbf{r}}_i - \mathbf{F}_i \cdot \overline{\mathbf{r}}_i \right) = 0$

(3) $\sum_{i=1}^{N} \left(m_i \, \overline{\mathbf{a}}_i \cdot \overline{\mathbf{v}}_i - \mathbf{F}_i \cdot \overline{\mathbf{v}}_i \right) = 0$

(4) $\sum_{i=1}^{N} \left(m_i \, \overline{\mathbf{a}}_i \cdot \overline{\mathbf{a}}_i - \mathbf{F}_i \cdot \overline{\mathbf{a}}_i \right) = 0$

(5) $\sum_{i=1}^{N} \left(m_i \, \overline{\mathbf{a}}_i \times \overline{\mathbf{r}}_i - \mathbf{F}_i \times \overline{\mathbf{r}}_i \right) = 0$

(6) $\sum_{i=1}^{N} \left(m_i \, \overline{\mathbf{a}}_i \times \overline{\mathbf{v}}_i - \mathbf{F}_i \times \overline{\mathbf{v}}_i \right) = 0$

(7) $\sum_{i=1}^{N} \left(\Delta \tfrac{1}{2} \, m_i \, \overline{\mathbf{v}}_i \cdot \overline{\mathbf{v}}_i - \int_1^2 \mathbf{F}_i \cdot d\overline{\mathbf{r}}_i \right) = 0$

(8) $\sum_{i=1}^{N} \left(\Delta \tfrac{1}{2} \, m_i \, \overline{\mathbf{v}}_i \cdot \overline{\mathbf{v}}_i + \Delta \tfrac{1}{2} \, m_i \, \overline{\mathbf{a}}_i \cdot \overline{\mathbf{r}}_i - \int_1^2 \mathbf{F}_i \cdot d\overline{\mathbf{r}}_i - \Delta \tfrac{1}{2} \, \mathbf{F}_i \cdot \overline{\mathbf{r}}_i \right) = 0$

where $\overline{\mathbf{r}}_i = \mathbf{r}_i - \mathbf{r}_{cm}$, $\overline{\mathbf{v}}_i = \mathbf{v}_i - \mathbf{v}_{cm}$, $\overline{\mathbf{a}}_i = \mathbf{a}_i - \mathbf{a}_{cm}$, $\mathbf{r}_i$, $\mathbf{v}_i$ and $\mathbf{a}_i$ are the position, the velocity and the acceleration of the i-th particle, $\mathbf{r}_{cm}$, $\mathbf{v}_{cm}$ and $\mathbf{a}_{cm}$ are the position, the velocity and the acceleration of the center of mass of the system of particles, m_i is the mass of the i-th particle, and $\mathbf{F}_i$ is the net force acting on the i-th particle.

Now, if the system of particles is isolated and if Newton's third law of motion is valid then the equations of conservation are as follows:

(1) $\sum_{i=1}^{N} \; (m_i \; \mathbf{a}_i) = 0$

(2) $\sum_{i=1}^{N} \; (m_i \; \overline{\mathbf{a}}_i \cdot \overline{\mathbf{r}}_i - \mathbf{F}_i \cdot \mathbf{r}_i) = 0$

(3) $\sum_{i=1}^{N} \; (m_i \; \overline{\mathbf{a}}_i \cdot \overline{\mathbf{v}}_i - \mathbf{F}_i \cdot \mathbf{v}_i) = 0$

(4) $\sum_{i=1}^{N} \; (m_i \; \overline{\mathbf{a}}_i \cdot \overline{\mathbf{a}}_i - \mathbf{F}_i \cdot \mathbf{a}_i) = 0$

(5) $\sum_{i=1}^{N} \; (m_i \; \overline{\mathbf{a}}_i \times \overline{\mathbf{r}}_i - \mathbf{F}_i \times \mathbf{r}_i) = 0$

(6) $\sum_{i=1}^{N} \; (m_i \; \overline{\mathbf{a}}_i \times \overline{\mathbf{v}}_i - \mathbf{F}_i \times \mathbf{v}_i) = 0$

(7) $\sum_{i=1}^{N} \; \left(\Delta \tfrac{1}{2} \, m_i \; \overline{\mathbf{v}}_i \cdot \overline{\mathbf{v}}_i - \int_1^2 \mathbf{F}_i \cdot d\mathbf{r}_i\right) = 0$

(8) $\sum_{i=1}^{N} \; \left(\Delta \tfrac{1}{2} \, m_i \; \overline{\mathbf{v}}_i \cdot \overline{\mathbf{v}}_i + \Delta \tfrac{1}{2} \, m_i \; \overline{\mathbf{a}}_i \cdot \overline{\mathbf{r}}_i - \int_1^2 \mathbf{F}_i \cdot d\mathbf{r}_i - \Delta \tfrac{1}{2} \, \mathbf{F}_i \cdot \mathbf{r}_i\right) = 0$

Observations

Equation (1) can be applied in any inertial reference frame without the necessity of introducing fictitious forces, and it is invariant under transformations between inertial reference frames.

Equations (2), (3), (4), (5), (6) and (7) can be applied in any non-rotating reference frame (inertial or non-inertial) without the necessity of introducing fictitious forces, and they are invariant under transformations between non-rotating reference frames.

Equation (8) can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces, and it is invariant under transformations between reference frames.

Bibliography

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