

# Cantor Dust as Dark Matter: A Brief Introduction

Ervin Goldfain

Global Institute for Research, Education and Scholarship (GIRES), USA

E-mail [ervingoldfain@gmail.com](mailto:ervingoldfain@gmail.com)

## Abstract

The goal of this brief report is to summarize the main points of the Cantor Dust (CD) model of Dark Matter (DM). The report is formatted as a user-friendly narrative, tailored primarily for undergraduate physics students and high-school enthusiasts.

**Key words:** Dark Matter, Cantor Dust, multifractals, dimensional condensate, flat rotation curves, Tully-Fisher relationship, modified Newtonian dynamics (MOND), core-cusp problem, Dark Matter models.

## 1. Introduction

In the context of early Universe phenomena, the Cantor Dust concept (CD) has a **dual interpretation**. It can be thought of either,

- a) a **multifractal distribution of mass** emerging from sustained dimensional fluctuations of primordial spacetime or,
- b) a **large-scale condensate** of continuous spacetime dimensions, emerging far above the electroweak scale ( $v = 246\text{GeV}$ ).

What makes CD particularly appealing is how it deploys the mathematics of **Cantor Sets** – infinitely fragmented and self-similar geometric structures – to solve cosmological puzzles that traditional particle or modified gravity DM theories struggle with [1 – 6, 8 – 10].

In line with both a) and b), CD describes a **cosmological phase of continuous spacetime dimensions**, naturally forming **topological defects** and **carrying intrinsic gravitational properties**. Specifically, the CD model assumes that, in the high-energy environment of primordial Universe, spacetime unfolded in **evolving non-integer dimensions**. As the Universe cooled, these dimensions "condensed" into the 3+1 dimensions of our familiar spacetime.

In this picture, the relic/leftover fragments of CD seed a multifractal network throughout the Universe, seen today as the **cosmic web of voids, walls and filaments**.

## 2. CD Replaces "Particles" with "Geometry"

In standard particle-based models, DM represents a "fluid" or a cloud of "dust" consisting of elusive ultralight or massive particles. By contrast, in the CD framework, DM emerges from a **singular multifractal measure**, characterized by the following attributes:

- It acts as a **random fractal scaffolding** with built-in gravitational effects.
- It has a spatial Hausdorff dimension  $D_H < 3$ , meaning that it **does not fill** space the way normal matter does.
- The Hausdorff dimension  $D_H$  *is not* a fixed parameter but flows with the observation/redshift scale. This flow is neither **monotonic** nor **deterministic**.

### **3. Flat Rotation Curves**

One of the deepest mysteries in astrophysics and cosmology is why stars at the edges of galaxies rotate just as fast as stars near the center. The CD solution works as follows: for a statistically isotropic filamentary network, the enclosed mass grows linearly with the radius ( $M(r) \propto r$ ). According to the divergence theorem, this geometry automatically produces a gravitational field that stays constant or drops off slowly ( $g(r) \propto r^{-1}$ ), creating flat rotation curves without requiring fine-tuning the density and/or distribution of particles in DM halos [1 - 2]

### **4. Baryonic Tully-Fisher Relationship**

Fractal geometry of CD defines a **universal acceleration scale** ( $a_0$ ) that regulates how baryonic matter (present in normal stars and gas) collapses, leading to the Tully-Fisher relationship. This relationship connects the galaxy mass to its orbital speed [1 -2].

## 5. Core-cusp Problem

Some standard DM simulations predict the formation of high-density "cusps" at the center of galaxies, but observations usually show flat "cores." CD addresses the "core-cusp problem" by showing that **gravitational diffusion and transport** on a fractal support naturally creates smoother, finite-density cores. Stated differently, since CD is a non-smooth structure, endowed with memory and long-range correlations, it creates effective diffusion-like effects. This naturally smears the center of galaxies into cores, matching what astrophysical observations show.

## 6. CD as Unified Theory of DM Models

The most compelling aspect is that CD acts as an "**ultraviolet completion**" or a master framework. Different ways of evaluating CD by coarse graining make it appear as unrelated manifestations of DM:

- **Self-Interacting Dark Matter (SIDM)**: Emerges as a result of gravitational diffusion in fractal spacetime. Our results indicate that

CD provides a unified, scale-dependent description of halo formation, relaxation, and the genesis of the cosmic-web, grounded in the behavior of the spacetime dimension [13].

- **Fuzzy/Axion Dark Matter**: The phenomenology commonly attributed to Fuzzy Dark Matter —including suppression of small-scale power, solitonic core formation, and wave-like interference effects—emerges naturally from gravitational dynamics defined on a multifractal spacetime.
- **MOND (Modified Newtonian Dynamics)**: The CD hypothesis provides a geometric rationale for why a scale-dependent range of accelerations exist in MOND-based models. Modified dynamics is thus deeply tied to the intrinsic gravitation carried by the CD structures.

## **7. Sigma 8 Anomaly**

Sigma 8 Anomaly refers to discrepancies in the amplitude of matter fluctuations, where low redshift observations favor weaker matter clustering

than predicted by standard cosmology. Elaborating from the hypothesis of **continuous spacetime dimensions**, [11 - 12] offer an alternative explanation of the anomaly, based on scale dependent corrections to the evolution of density fluctuations. In this interpretation, the anomaly can be viewed as observational evidence for primordial effects of CD at cosmological scales.

### **8. Stochastic Gravitational Wave Background**

Recently, several collaborations reported evidence for continuous low frequency gravitational waves (GW). Unlike the high-frequency GW's seen by ground-based instruments like the Laser Interferometer Gravitational-wave Observatory (LIGO), this low-frequency signal forms a *stochastic gravitational wave background (SGWB)*, which can only be detected over long times using Pulsar Timing Arrays. The current belief is that there are several possible astrophysical sources of SGWB, besides supermassive Black Holes binaries that were abundant in the early Universe. As detailed in [7], CD offers a fundamentally new explanation of SWGB.

## 9. From Cosmology to Information Theory

The CD framework suggests that cosmological entropy is **non-extensive** (meaning the total is not just the sum of its parts). It deploys the so-called **Tsallis and Rényi entropies** to describe how clustering of cosmic structures encodes the vast **fractal memory** of the primordial Universe [1, 14].

### References

1. <https://www.researchgate.net/publication/399127390>
2. <https://www.researchgate.net/publication/398638254>
3. <https://www.researchgate.net/publication/399914498>
4. <https://www.researchgate.net/publication/399806314>
5. <https://www.researchgate.net/publication/399763752>
6. <https://www.researchgate.net/publication/391156207>
7. <https://www.researchgate.net/publication/390213024>

8. <https://www.researchgate.net/publication/385000604>
9. <https://www.researchgate.net/publication/384922557>
10. <https://www.researchgate.net/publication/380376059>
11. <https://www.researchgate.net/publication/402676458>
12. <https://www.researchgate.net/publication/395241668>
13. <https://www.researchgate.net/publication/401435868>
14. <https://www.researchgate.net/publication/393946834>