From fractality of quantum mechanics to Bohr-Sommerfeld's quantization of planetary orbit distance

Victor Christianto, sciprint.org, email: victorchristianto@gmail.com

Abstract

In the present paper, we use periodic orbit quantization as suggested by Bohr-Sommerfeld in order to analyze quantization in astrophysical phenomena, i.e. planetary orbit distances. It is known that one can deduce Bohr-Sommerfeld quantization rules from Burger's turbulence [4], and recently such an approach leads to a subfield in physics known as quantum turbulence [5]. Further recommendation for generalizing Bohr-Sommerfeld quantization rules is also mentioned.

Introduction

It is known that quantum mechanics exhibits fractality at $d_F=2$, and an extensive report has been written on this subject and its related issues [1]. Moreover, a fractal solution of time-dependent Schrodinger equation has been suggested some time ago by Datta [2]. On the other side, if one takes a look at planetesimals in the case of planetary system formation, interstellar gas and dust in the case of star formation, the description of the trajectories of these bodies is in the shape of non-differentiable curves, and we obtain fractal curves with fractal dimension 2 [3]. This coincidence between fractality of quantum mechanics and fractal dimension of astrophysical phenomena seems to suggest that we can expect to use quantum mechanical methods such as wave mechanics and periodic orbit quantization to analyze astrophysical phenomena. Such an analysis has been carried out for example by Nottale and Celerier [3] in order to describe these phenomena from the viewpoint of macroscopic Schrodinger equation.

In the present paper, we use periodic orbit quantization as suggested by Bohr-Sommerfeld in order to analyze quantization in astrophysical phenomena, i.e. planetary orbit distances. It is known that one can deduce Bohr-Sommerfeld quantization rules from Burger's turbulence [4], and recently such an approach leads to a subfield in physics known as quantum turbulence [5]. Therefore, turbulence phenomena can also yield quantization, which also seems to suggest that turbulence and quantized vortice is a fractal phenomenon.

We will present Bohr-Sommerfeld quantization rules for planetary orbit distances, which will obtain the same result with a formula based on macroscopic Schrodinger equation.

Further recommendation for generalizing Bohr-Sommerfeld quantization rules is also mentioned.

Bohr-Sommerfeld quantization rules and planetary orbit distances

It was suggested in [6] and [7] that Bohr-Sommerfeld quantization rules can yield an explanation of planetary orbit distances of the solar system and exoplanets. Here, we begin with Bohr-Sommerfeld's conjecture of quantization of angular momentum. As we know, for the wavefunction to be well defined and unique, the momenta must satisfy Bohr-Sommerfeld's quantization condition:

$$\oint_{\Gamma} p.dx = 2\pi.n\hbar, \tag{1}$$

for any closed classical orbit Γ. For the free particle of unit mass on the unit sphere the left-hand side is:

$$\int_{0}^{T} v^2.d\tau = \omega^2.T = 2\pi.\omega, \tag{2}$$

Where $T = \frac{2\pi}{\omega}$ is the period of the orbit. Hence the quantization rule amounts to quantization of the rotation frequency (the angular momentum): $\omega = n\hbar$. Then we can write the force balance relation of Newton's equation of motion:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}. \tag{3}$$

Using Bohr-Sommerfeld's hypothesis of quantization of angular momentum (2), a new constant g was introduced:

$$mvr = \frac{ng}{2\pi}. \tag{4}$$

Just like in the elementary Bohr theory (just before Schrodinger), this pair of equations yields a known simple solution for the orbit radius for any quantum number of the form:

$$r = \frac{n^2.g^2}{4\pi^2.GMm^2}, \tag{5}$$

or

$$r = \frac{n^2.GM}{v_o^2}, \tag{6}$$

Where r, n, G, M, v_0 represents orbit radii (semimajor axes), quantum number (n=1,2,3,...), Newton gravitation constant, and mass of the nucleus of orbit, and specific velocity, respectively. In equation (6), we denote:

$$v_0 = \frac{2\pi}{g} GMm. \tag{7}$$

The value of m and g in equation (7) are adjustable parameters.

Interestingly, we can remark here that equation (6) is exactly the same with what is obtained by Nottale using his Schrodinger-Newton formula [8]. Therefore here we can verify that the result is the same, either one uses Bohr-Sommerfeld quantization rules or Schrodinger-Newton equation. The applicability of equation (6) includes that one can predict new exoplanets (extrasolar planets) with remarkable result.
Furthermore, one can find a neat correspondence between Bohr-Sommerfeld quantization rules and motion of quantized vortice in condensed-matter systems, especially in superfluid helium [9]. In this regards, a fractional Schrodinger equation has been used to derive two-fluid hydrodynamical equations for describing the motion of superfluid helium in the fractal dimension space [10]. Therefore, it appears that fractional Schrodinger equation corresponds to superfluid helium in fractal dimension space.

Discussion and results

With the help of equation (6) one can describe planetary orbit distances of both the inner planets and Jovian planets in the solar system [7]. See Table 1. Moreover, we were able to predict three new planets in the outer-side of Pluto. This new prediction of three planets beyond the orbit distance of Pluto is made based on our method called CSV (Cantorian Superfluid Vortex) [7].

Table 1: Comparison of prediction and observed orbit distance of planets in Solar system (in 0.1AU unit)

Object	No.	Titius-Bode	Nottale [8]	CSV [7]	Observed	Δ (%)
	1		0.4	0.43		
	2		1.7	1.71		
Mercury	3	4	3.9	3.85	3.87	0.52
Venus	4	7	6.8	6.84	7.32	6.50
Earth	5	10	10.7	10.70	10.0	-6.95

Object	No.	Titius-Bode	Nottale [8]	CSV [7]	Observed	Δ (%)
Mars	6	16	15.4	15.4	15.24	-1.05
Hungarias	7		21.0	20.96	20.99	0.14
Asteroid	8		27.4	27.38	27.0	1.40
Camilla	9		34.7	34.6	31.5	-10.00
Jupiter	2	52		45.52	52.03	12.51
Saturn	3	100		102.4	95.39	-7.38
Uranus	4	196		182.1	191.9	5.11
Neptune	5			284.5	301	5.48
Pluto	6	388		409.7	395	-3.72
2003EL61	7			557.7	520	-7.24
(Sedna)	8	722		728.4	(760)	(4.16)
2003UB31	9			921.8	970	4.96
Unobserv.	10			1138.1		
Unobserv.	11			1377.1		

For inner planets, our prediction values are very similar to Nottale's (1996) values, starting from n = 3 for Mercury; for n = 7 Nottale reported minor object called Hungarias. It is worth noting here, we don't have to invoke several *ad hoc* quantum numbers to predict orbits of Venus and Earth as Neto *et al*. (2002) did [7]. We also note here that the proposed method results in prediction of orbit values, which are within a 7% error range compared to observed values, except for Jupiter which is within a 12.51% error range.

The departure of our predicted values compared to Nottale's predicted values (1996, 1997, 2001) appear in outer planet orbits starting from n = 7. We proposed some new predictions of the possible presence of three outer planets beyond Pluto (for n = 7, n = 8, n = 9) [7]. It is very interesting to remark here, that this prediction is in good agreement with Brown-Trujillo's finding (March 2004, July 2005) of planetoids in the Kuiper belt [13][14][15]. Although we are not sure yet of the orbit of Sedna, the discovery of 2003EL61 and 2003UB31 are apparently in quite good agreement with our prediction of planetary orbit distances based on CSV model.

Therefore, we can conclude that while our method as described herein may be interpreted as an oversimplification of the real planetary migration process which took place sometime in the past, at least it could provide us with useful tool for prediction [6b]. Now we also provide new prediction of other planetoids which are likely to be observed in the near future (around 113.8AU and 137.7 AU). It is recommended to use this prediction as guide to finding new objects (in the inner Oort Cloud).

What we would like to emphasize here is that the quantization method does not have to be the *true* description of reality with regards to celestial phenomena. As always this method could explain some phenomena, while perhaps lacks explanation for other phenomena. But

at least it can be used to predict something quantitatively, i.e. measurable (exoplanets, and new planetoids in the outer solar system etc.).
In the mean time, a correspondence between Bohr-Sommerfeld quantization rules and Gutzwiller trace formula has been shown in [11], indicating that the Bohr-Sommerfeld quantization rules may be used also for complex systems. Moreover, a recent theory extends Bohr-Sommerfeld rules to a full quantum theory [12].

Concluding remarks

In the present paper, we use periodic orbit quantization as suggested by Bohr-Sommerfeld in order to analyze quantization in astrophysical phenomena, i.e. planetary orbit distances. It is known that one can deduce Bohr-Sommerfeld quantization rules from Burger's turbulence [4], and recently such an approach leads to a subfield in physics known as quantum turbulence [5].

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