First Draft

Conceptual Limitations of Structure Formation in Primordial Cosmology

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Abstract

Conventional wisdom says that the formation of large-scale structures in cosmology follows from the evolution of density perturbations under the combined effect of gravitation and cosmic expansion. We survey here the reasons why this framework fails to hold in primordial cosmology, due to the severe limitations placed on the Friedmann-Robertson-Walker (FRW) metric, the continuity hypothesis and the standard equations of fluid flows. We insist that understanding primordial cosmology must rely instead on a complex dynamics model of *evolving dimensional fluctuations,* conjectured to come into play far above the electroweak scale. A key outcome of this model is the universal generation of *topological defects (vortices) and condensates* emerging from the complex Ginzburg-Landau equation.

Key words: scalar density perturbations, primordial Universe, FRW cosmology, structure formation, complex dynamics, dimensional fluctuations.

1. Introduction

Relativistic cosmology models the Universe as a nearly homogeneous and isotropic expanding spacetime, as described by the FRW metric. In this framework, the observed large-scale structure of the Universe (galaxies, clusters, cosmic web including voids, filaments, halos etc.) develops from small perturbations of the smooth overall background. Perturbations are studied through *cosmological perturbation theory (CPT)*, which tracks how primordial inhomogeneities evolve into large-scale structures from fluctuations in energy density, pressure, and spacetime curvature. Despite its undisputed successes, CPT remains however inadequate for the study of early Universe formation, on spacetime-scales approaching the Planck regime.

The goal of this report is to survey the reasons why CPT fails to hold in primordial cosmology, due to the severe limitations placed on the FRW metric, the continuity hypothesis and the standard equations of fluid flows. In our view, understanding of primordial cosmology must rely instead on a complex dynamics model of *evolving dimensional fluctuations,* conjectured to come into play far above the electroweak scale. A key outcome of this model is the universal generation of *topological defects and topological condensates* emerging from the complex Ginzburg-Landau equation.

The report is structured as follows: next section is a condensed review of how scalar density perturbations are dealt with in standard cosmology. Section 3 goes over the limitations of the FRW model and CPT in explaining structure formation in primordial cosmology. Section 4 details our perspective on the *complex dynamics* of primordial dimensional fluctuations and their impact on structure formation via the complex Ginzburg Landau equation (CGLE) and self-organized criticality (SOC). The Appendix section elaborates on the role of Kolmogorov entropy in the evolution of dimensional fluctuations and their connection to primordial gravity.

For the sake of clarity and accessibility, the report is designed in a "user-friendly" format, with emphasis on pedagogical exposition rather than formal derivations.

2. Scalar Density Perturbations in Standard Cosmology

We start by reviewing the evolution equation of scalar density perturbations in the standard model of cosmology (the Lambda-CDM model) [ ]. Let $\rho(\vec{x},t)$ denote the local matter or energy density. The main parameter of interest in CPT is the *density perturbation*, which quantifies the local density deviation normalized to the average density of the Universe,

$$\delta(\vec{x},t) = \frac{\Delta\rho(\vec{x},t)}{\overline{\rho}} = \frac{\rho(\vec{x},t) - \overline{\rho}}{\overline{\rho}} \qquad (1)$$

The density perturbation (1) can be studied in Fourier space using the decomposition in modes of momenta $\vec{k}$,

$$\delta(\vec{x},t)=\int d^3k\,\delta_k(t)\exp(i\vec{k}\cdot\vec{x}) \tag{2}$$

Statistical distribution of modes is described using the concept of *power spectrum* $P(k)$. Observations indicate that the amplitude of density perturbations is *nearly scale invariant*, meaning that it stays roughly the same across all length scales. As a result, primordial fluctuations carry about the same strength and large- and small-scale structures start off with nearly similar density perturbations. In terms of the power spectrum, one has

$$P(k)\propto k^{n_s-1} \tag{3}$$

in which n_s denotes the *scalar spectral index*. While $n_s=1$ corresponds to exact scale invariance, observations hint that $n_s\approx 0.965$, which shows an actual slight red tilt (meaning more power allocated to large scales).

The linear scalar perturbations to a flat FRW metric takes the form

$$ds^2=a^2(\eta)[-(1+2\Psi)d\eta^2+(1-2\Phi)\delta_{ij}dx^i dx^j] \tag{4}$$

where η is the conformal time and Ψ,Φ are scalar potentials (with $\Psi=\Phi$, if there are no anisotropic stresses). Relation (2) defines the FRW metric in the so-called *conformal Newtonian gauge*. There are two key equations of CPT in Fourier space, namely,

a) The *continuity equation* reflecting energy conservation and given by,

$$\dot{\delta}+(1+w)(\theta-3\dot{\Phi})+3H(c_s^2-w)\delta=0 \tag{5}$$

b) *Euler equation* reflecting momentum conservation and written as,

$$\dot{\theta}+H(1-3w)\theta+\frac{\dot{w}}{1+w}\theta-\frac{k^2c_s^2}{1+w}\delta-k^2\Psi=0 \tag{6}$$

Here, $H=a'/a$ is the conformal Hubble parameter, $a'=da/d\eta$, $w=\overline{p}/\overline{\rho}$, $c_s^2=\delta p/\delta\rho$ is the square of the sound speed and $\theta=ik^j v_j=i\vec{k}\cdot\vec{v}$ stands for the velocity potential with wavevector $\vec{k}$.

Using (5) – (6), the equation for the evolution of scalar perturbations in the Lambda-CDM Universe assumes the form,

$$\boxed{\ddot{\delta}+2H\dot{\delta}-4\pi G\overline{\rho}\,\delta=0} \quad \text{(Newtonian limit)} \tag{7}$$

The full relativistic equation beyond the Newtonian limit (7) reads

$$\boxed{\ddot{\delta}+A(H,w)\dot{\delta}+B(c_s^2,k^2,a^2,w,\overline{\rho})\delta=S(\Phi)} \tag{8}$$

in which $A(...)$, $B(...)$ are functions of variables indicated in (8) and $S(\Phi)$ is the source term induced by Φ.

3. Limitations of FRW Cosmology in the Primordial Universe

While being adequate in describing the late evolution of the Universe, FRW cosmology faces several key limitations when applied to the evolution of the primordial Universe:

1) Along with General Relativity (GR), FRW breaks down near the Big Bang singularity where large and highly unstable curvature fluctuations ruin the *smooth topology of ordinary four-dimensional spacetime*. It is also conceivable that, in line with the Dimensional Reduction conjecture [ ], spacetime dimensionality drops down in some continuous but unpredictable fashion.

2) The cosmological principle posits a smooth and uniform Universe, yet these assumptions are bound to be violated near the Big-Bang singularity, as hinted by the Sakharov conditions for baryogenesis and the far-from-equilibrium properties of Planck physics. In the same context, strong nonlinearities associated with (1), that is, $\delta = O(1)$ are prone to ruin the basis of the entire CPT model.

3) FRW does not naturally explain the horizon and flatness puzzles of cosmology without fine-tuned initial conditions—issues typically resolved by *inflation*, which is still hypothetical and external to the Lambda-CDM paradigm.

4) FRW lacks a mechanism of structure formation from primordial fluctuations as it serves only as a background for inflationary cosmology.

5) The relevance of quantum gravity effects near the Planck scale remains highly controversial [ ].

6) The continuity equation (5) fails when mass/energy/momenta are not conserved due to the presence of sources or sinks. Likewise, Bianchi identities of GR break down under strong curvature fluctuations (spacetime manifold becomes non-smooth and singular) or when the effects of deterministic or stochastic chaos *cannot be discarded* [ ].

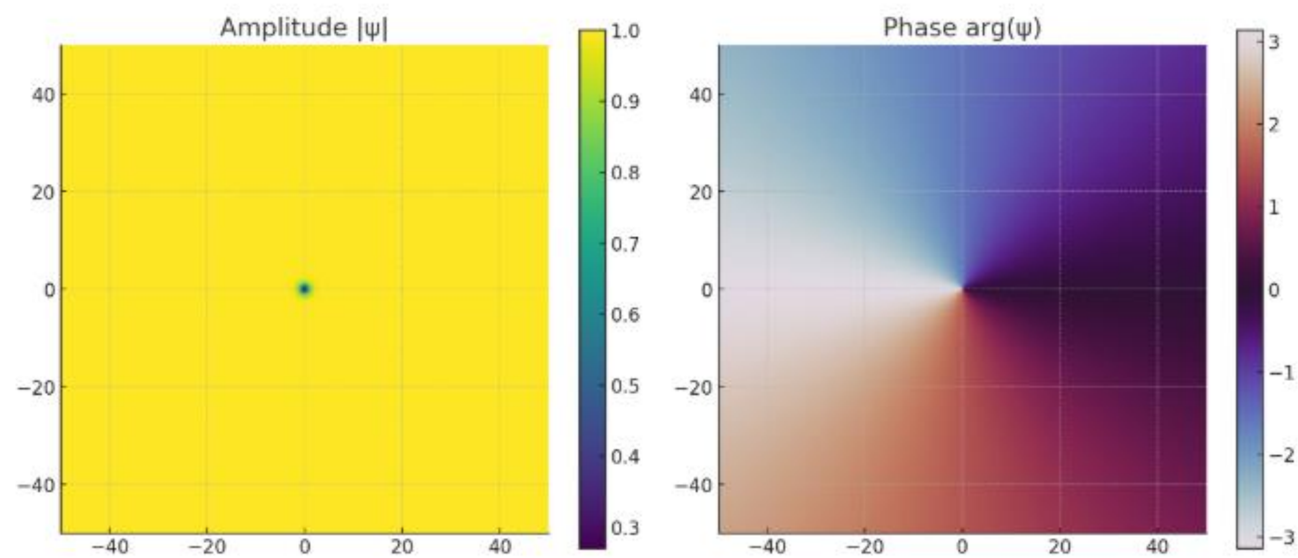
7) Fluid equations of the type (6) become invalid in the presence of boundary layers, unpredictable dissipation or turbulence, shocks, non-Newtonian stress-strain relationship or rarefied flow conditions.

5. Beyond CPT with Complex Dynamics of Dimensional Fluctuations.

It is generally believed that, if the ongoing cosmological observations continue to diverge from predictions, our current formulation of Lambda-CDM may require a fundamental paradigm shift. Parallel with the current situation in cosmology, independent research indicates that nonlinear dynamics of far-from-equilibrium systems has *universal features* linking the complex Ginzburg-Landau equation (CGLE) with the tenets of Quantum

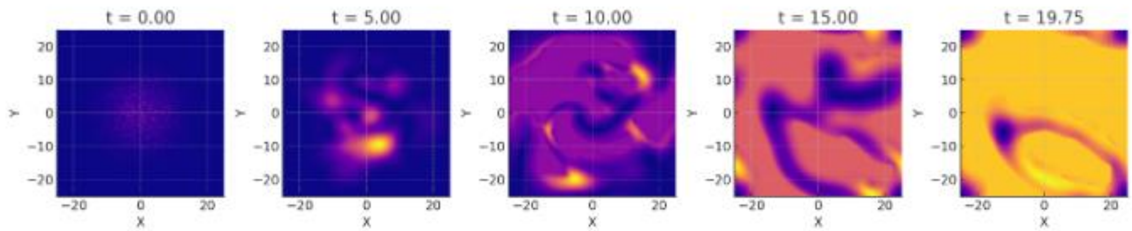
Field Theory (QFT), Standard Model of Particle Physics (SM) and the high-energy regime of primordial gravitation. Here we consolidate this view by arguing that CGLE lies behind structure formation in cosmology and the emergence of *topological defects and condensates* in high-energy physics. From this standpoint, *complex dynamics* appears to provide the most sensible way out of the current crisis in cosmology.

…To be continued…

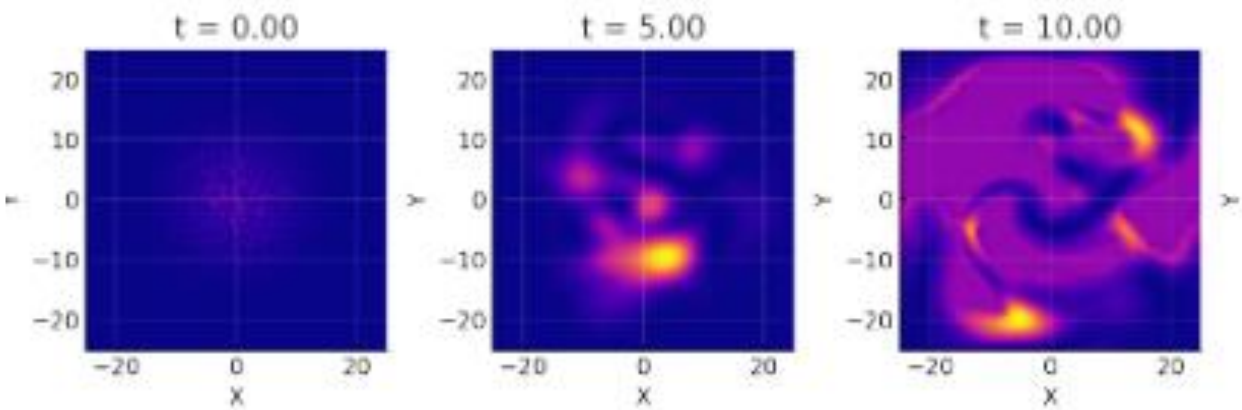


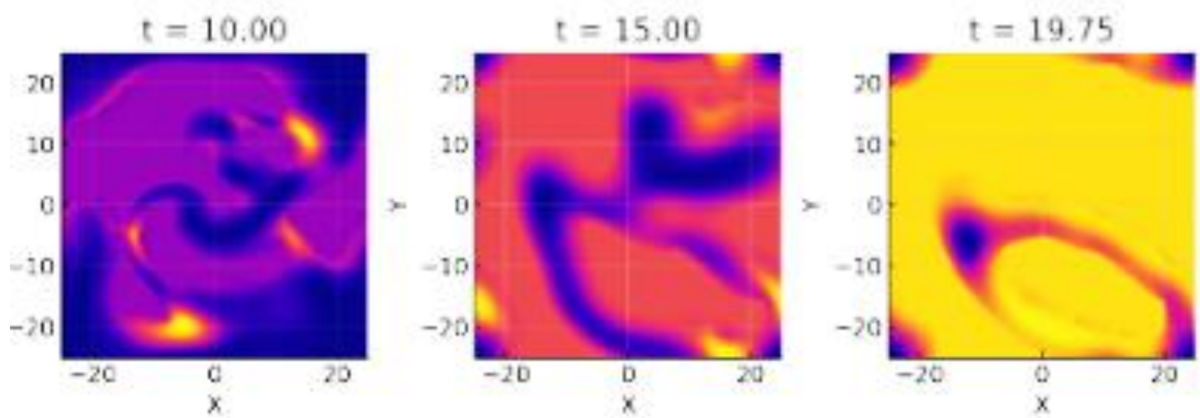
Here's a visualization of a **topological vortex** from the complex Ginzburg-Landau framework:

- The **left panel** shows the amplitude $|\psi|$, which vanishes at the center (the vortex core).
- The **right panel** shows the phase $\arg(\psi)$, which winds by 2π around the core—defining the vortex's topological charge.

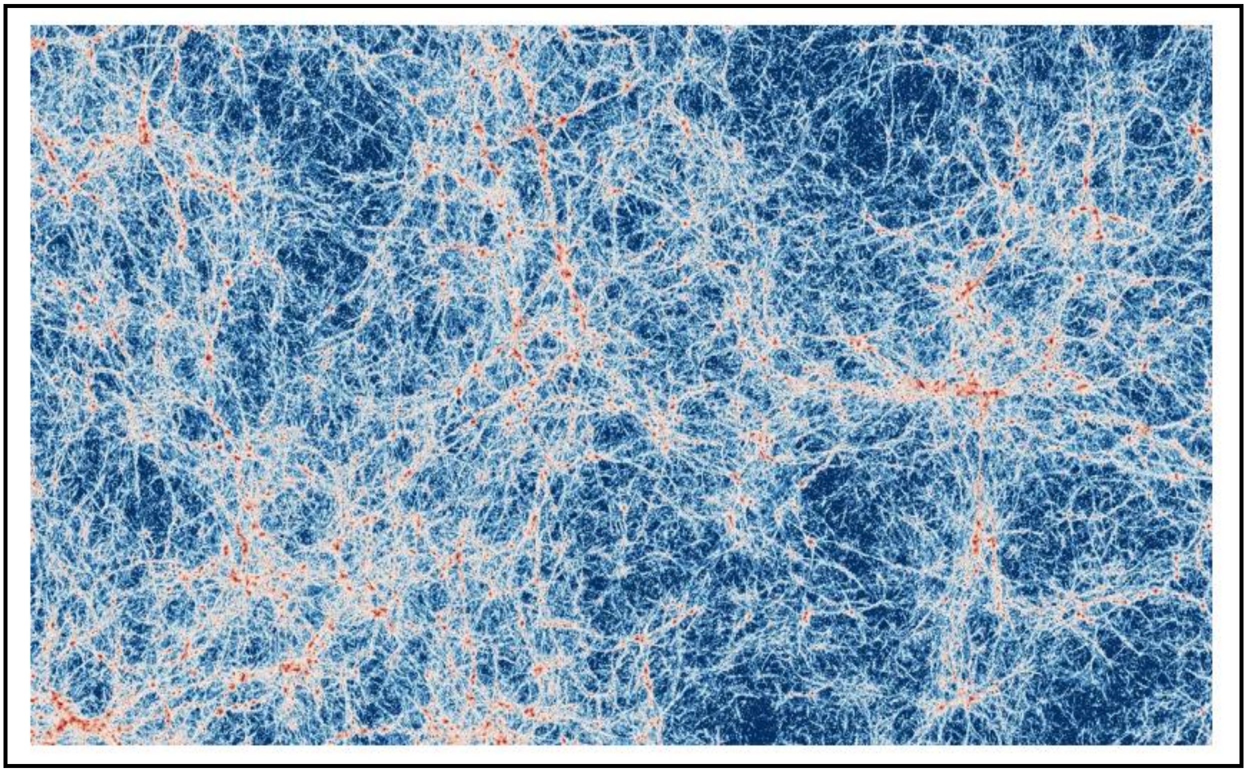


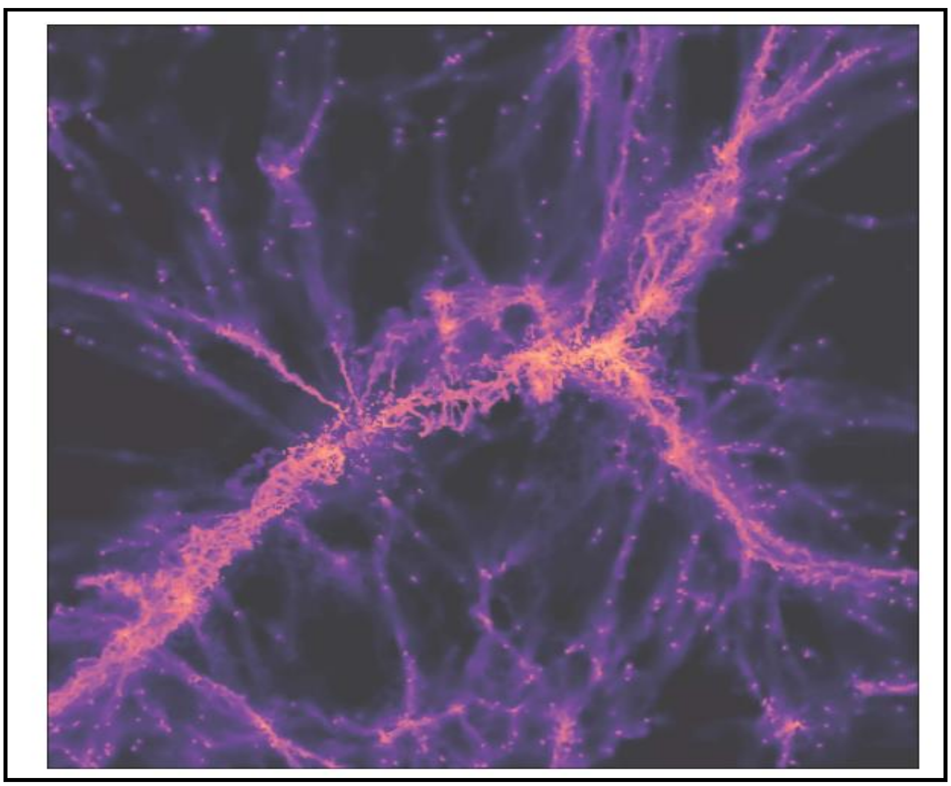
Here are snapshots of the density contrast field $|A(x, y, t)|^2$ at different time intervals. These images reveal how structures evolve dynamically over time, exhibiting features reminiscent of vortex lattices or cosmic voids. Let me know if you'd like further analysis or specific modifications!





Recent images of cosmic filaments and the cosmic web:





Vortex lines in the three-dimensional complex Ginzburg-Landau equation: