Two-Stream Instability as a Mechanism for Toroidal Magnetic Field Generation in the Magnetosphere of Crab Pulsar

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Abstract. New plasma mechanism for the generation of toroidal magnetic field in the magnetosphere of Crab pulsar is presented. It is based on the development of two-stream instability in the relativistic electron-positron plasma of the pulsar magnetosphere. In particular, pulsar magnetosphere relativistic plasma is penetrated by ultrarelativistic electron beam and two-stream instability develops, as a result of which toroidal magnetic field is generated.

1. Introduction

Nowadays it is widely accepted that pulsars are rapidly rotating neutron stars [1-4] with strong magnetic field of the order $10^{12} \div 10^{13}$ G. Neutron star is surrounded with magnetosphere, which is filled with relativistic electron-positron plasma (see example [5]). Pulsar radiation is generated in its magnetosphere most probably as a result of the development of different plasma processes in the region above the pulsar magnetic poles ([6]). In order to find the processes which are responsible for the generation of pulsar radiation it is essential to know in detail the structure of the magnetosphere where these processes develop.

In the pulsar magnetosphere, close to its surface, magnetic field has a dominant role - its energy exceeds the energy of the magnetospheric relativistic plasma by many orders of the magnitude. Magnetic field of pulsar has the dipole structure. It is frozen in magnetospheric plasma and in pulsar too. Therefore, solid body type rotation - corotation of pulsar, its magnetic field and magnetospheric relativistic plasma takes place. In the region of the magnetosphere where the magnetic field lines are closed magnetospheric plasma is confined by the magnetic field and it can not leave the magnetosphere. In this region we have "quiet corotation" if one can say so. Plasma can leave the magnetosphere only from the conical region (with small angle of opening) above the pulsar magnetic poles. In this region magnetic field lines are "opened" and since plasma particles follow these lines they leave pulsar magnetosphere and form relativistic pulsar wind. In case of Crab pulsar opened magnetic field lines practically lie in the equatorial plane of rotation because pulsar magnetic axis is nearly perpendicular to its rotation axis [7,8]. In general, opened magnetic field lines of pulsar are considered as almost straight radial lines in the region close to its surface, because in this region their curvature is small. Besides, in

this region we have rigid corotation - plasma particles rotate together with the magnetic field lines and also move along them. It is evident that this can not take place on large radial distance. In particular, corotation is strictly impossible beyond the light cylinder (cylindrical surface on which the corotation velocity equals to the speed of light). At the same time in this region, which is called wind zone, we have not the magnetospheric relativistic plasma but the relativistic pulsar wind. In the wind zone magnetic field is practically purely toroidal, it is still frozen in plasma, but its energy is smaller then the energy of the relativistic pulsar wind. From all the above mentioned it follows that somewhere in the pulsar magnetosphere - inside the light cylinder - toroidal magnetic field must be generated and corotation must be violated.

In the present paper one possible plasma mechanism for the generation of toroidal magnetic field in the magnetosphere of Crab pulsar is suggested. In the forthcoming section pulsar magnetosphere structure before the generation of toroidal magnetic field is discussed. In third section the mechanism of toroidal magnetic field generation in the pulsar magnetosphere is presented. The mechanism is based on the development of two-stream instability in the magnetospheric relativistic electron-positron plasma.

2. The Structure of the pulsar magnetosphere before the generation of toroidal magnetic field

As it has been already mentioned above pulsar magnetosphere is filled with relativistic electron-positron plasma. This plasma appears there as a result of cascade process which develops in the following way. Since matter inside pulsar is in superconductive state magnetic field is frozen in pulsar and rotates together with it. As a result of this rotation electric field is generated which extracts charged particles from pulsar surface [9]. Depending of the direction of generated electric field the particles extracted from the pulsar surface may be electrons [10], or positrons [11] and ions [12]. Here it will be assumed that charged particles extracted from the pulsar surface by electric field are electrons. In the pulsar magnetosphere electrons are accelerated by electric field and acquire ultrarelativistic velocities. Electrons follow the magnetic field lines and have only the longitudinal (with respect to the magnetic field line) component of velocity, because perpendicular component is lost in the strong magnetic field of pulsar after the rapid radiation with synchrotron mechanism. Since magnetic field lines are curved, the electrons moving along them with ultrarelativistic velocities radiate curvature radiation γ-quanta. Then, in the strong magnetic field of pulsar γ-quanta decay into electron-positron pairs. These electrons and positrons are also accelerated by electric field and emit curvature radiation γ-quanta, which again decay into electron-positron pairs etc. This cascade process leads to the formation of dense relativistic electron-positron plasma in the pulsar magnetosphere [5] . This plasma is penetrated by primary ultrarelativistic electron beam.

Now about the simplified geometric model of Crab pulsar magnetic field which will be used below. As it has been already mentioned above rotation axis and magnetic axis of Crab pulsar are nearly perpendicular. So, pulsar magnetic field lines are considered as radial straight lines located in the equatorial plane of rotation (see the Fig. 1). This assumption is justified and at the same time this is not an approximation of monopolar
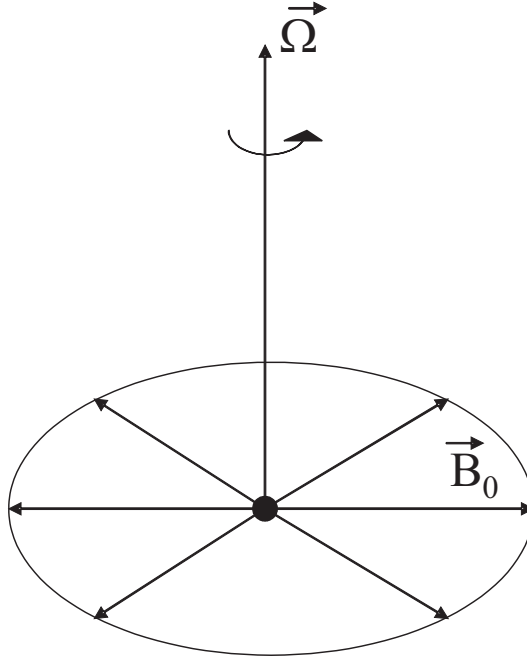
Figure 1: Geometric model of crab pulsar magnetic field.

magnetic field for the following reasons: first of all only the open magnetic field lines which come out from one magnetic pole of pulsar are discussed (all the results obtained below will be the same for the open magnetic field lines which come out from another magnetic pole of pulsar, just the direction of these magnetic field lines will be the opposite). Besides, these magnetic field lines are discussed in the thin layer of the magnetosphere close to pulsar surface. The thickness of this layer ($\approx 10^7$ cm) is much less than the curvature radius of Crab pulsar magnetic field lines ($\approx 10^8$ cm), therefore in this layer magnetic field lines can be considered straight. The reason why one gets for the magnetic field the picture seen on Fig. 1 is the rotation of those group of magnetic field lines which have been just defined above.

3. Two-stream instability development and toroidal magnetic field generation in the pulsar magnetosphere

As we have seen above relativistic electron-positron plasma in the pulsar magneto-sphere is penetrated by ultrarelativistic electron beam. As a result of the interaction of ultrarelativistic electron beam with relativistic plasma of the pulsar magnetosphere two-stream instability may develop. The possibility of the generation of pulsar radiation as a result of two-stream instability development has been studied in the papers [11-19]. In the present paper the possibility of toroidal magnetic field generation in the magneto-sphere of Crab pulsar as a result of two-stream instability development is investigated. For this purpose the standard set of equations describing the dynamics of cold relativistic

magnetized plasma is used:

$$\frac{\partial \vec{p}_{(\alpha)}}{\partial t} + \left(\vec{V}_{(\alpha)}\vec{\nabla}\right)\vec{p}_{(\alpha)} = \frac{e_{(\alpha)}}{m}\left(\vec{E} + \left[\vec{V}_{(\alpha)}\times\vec{B}\right]\right), \tag{1}$$

$$\frac{\partial n_{(\alpha)}}{\partial t} + div\left(n_{(\alpha)}\vec{V}_{(\alpha)}\right) = 0, \tag{2}$$

$$rot\vec{E} = -\frac{\partial\vec{B}}{\partial t}, \tag{3}$$

$$rot\vec{B} = 4\pi\vec{j} + \frac{\partial\vec{E}}{\partial t}, \tag{4}$$

where $e_{(\alpha)}$ and m are particle electric charge and mass respectively, $\vec{V}_{(\alpha)}$ and $\vec{p}_{(\alpha)} = \gamma_\alpha m\vec{V}_{(\alpha)}$ (γ_α being particle Lorentz-factor) are particle tree-velocity and momentum, $n_{(\alpha)}$ is the particle density, $\vec{j}$ is the current density and $\vec{E}$ and $\vec{B}$ are electric and magnetic fields. Subscript (α) denotes the group of particles (we have two groups - plasma ($\alpha = 1$) and beam ($\alpha = 2$)). In the equations (1-4) so-called "geometric" unites - $c = G = 1$ are used and momentum is changed by normalized momentum $\vec{p}\rightarrow\vec{p}/m$.

The dynamics of electromagnetic perturbations in the system plasma-beam is studied. These studies are performed in the reference frame of a rotating magnetic field line (the geometry of the magnetic field lines being discussed in the previous section). At the same time the reference frame in which the investigations are performed is moving radially outwards along the magnetic field line with such a constant velocity that in this frame the velocities of plasma and beam are equal ($|\vec{V}_{(1)}| = |\vec{V}_{(2)}| = V$) and directed in opposite direction (beam velocity $\vec{V}_{(2)}$ being directed radially outwards and plasma velocity $\vec{V}_{(1)}$ - radially inwards). Here we discuss the dynamics of the perturbations wave vector of which is directed along the x-axis (x-axis is parallel to the pulsar rotation axis) - $\vec{k}(k_x, 0, 0)$, perturbed electric field is directed opposite to the z-axis (z-axis being directed radially outwards from pulsar) - $\vec{E}_1(0, 0, -E_{1z})$, unperturbed magnetic field B_0 (which is pulsar magnetic field) is also directed along the z-axis and perturbed magnetic field has only the toroidal - y-component - $\vec{B}_1(0, B_{1y}, 0)$, see the Fig. 2. For these kind of perturbations from the set of equations (1-4) one can obtain the following dispersion relation:

$$\omega^2 - k^2 = \frac{\omega_p^2}{\gamma_0^3}\left(1 - \frac{k^2 V_0^2}{\omega_c^2/\gamma_0^6 - \omega^2}\right). \tag{5}$$

Here V_0 and γ_0 are unperturbed velocity and Lorentz-factor of particles, $\omega_c = eB_0/m$ is the cyclotron frequency (B_0 being the unperturbed magnetic field of pulsar) and $\omega_p = \sqrt{4\pi(n_{0p} + n_{0b})e^2/m}$ is the plasma frequency (n_{0p} and n_{0b} being the unperturbed particle density of pulsar magnetosphere relativistic electron-positron plasma and ultrarelativistic electron beam respectively).

Equation (5) can be rewritten in the following form:
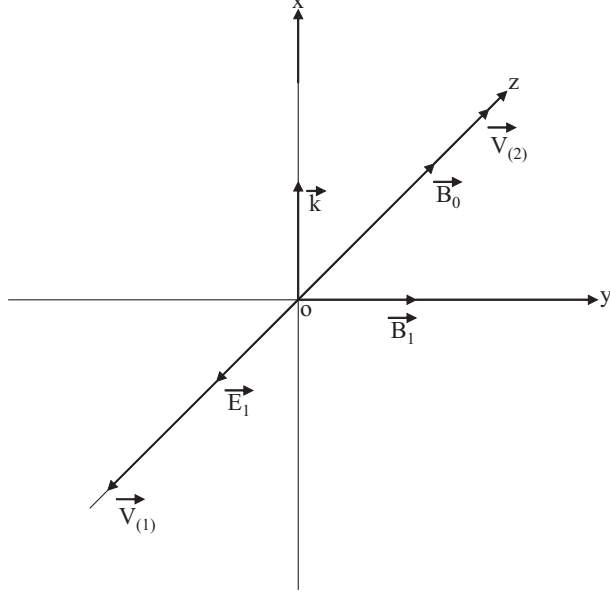
$$a\omega^4 + b\omega^2 + c = 0, \tag{6}$$

Figure 2: Orientation of the perturbations under study.

where

$$a = \gamma_0^6, \tag{7}$$

$$b = -\left(\omega_c^2 + \omega_p^2 \gamma_0^3 + k^2 \gamma_0^6\right), \tag{8}$$

$$c = k^2 \omega_c^2 + \frac{\omega_p^2 \omega_c^2}{\gamma_0^3} - \omega_p^2 \gamma_0^3 k^2 V_0^2. \tag{9}$$

From the expressions (6)-(9) one can conclude that if the condition

$$k^2 V_0^2 > \frac{k^2 \omega_c^2}{\omega_p^2 \gamma_0^3} + \frac{\omega_c^2}{\gamma_0^6}, \tag{10}$$

is fulfilled, than $\omega^2 < 0$ and ω is entirely imaginary. The time dependence of perturbed quantities and namely perturbed toroidal magnetic field is exponential $B_1 \sim exp\left(-i\omega t\right)$. Therefore, when the condition (10) is fulfilled, then two-stream instability develops in the magnetosphere of Crab Pulsar and exponentially growing toroidal magnetic field is generated.

Substituting the parameters appropriate to Crab pulsar and its magnetosphere one can easily find that the condition (10) is really fulfilled.

As a result of the development of two-stream instability exponentially growing toroidal magnetic field is generated in the magnetosphere of Crab pulsar (for other possible mechanisms of toroidal magnetic field generation in the magnetosphere of Crab Pulsar see [20] and [21]. The source of energy for the generation of this field is the pulsar rotation

slowing down. Really, as it has been already mentioned above, as a result of pulsar rotation together with frozen-in magnetic field, electric field is generated. This electric field gets energy from pulsar rotation slowing down. The electric field extracts electrons from pulsar surface, accelerates them to ultrarelativistic velocities and thus the ultrarelativistic electron beam is formed. In the strong magnetic field of pulsar electron-positron pairs appear from the beam particles and dense relativistic plasma of the pulsar magnetosphere is formed. This plasma is penetrated by ultrarelativistic electron beam. The system plasma-beam is unstable and two-stream instability develops in it, as a result of which toroidal magnetic field is generated in the pulsar magnetosphere. The energy source for the generation of the toroidal magnetic field is beam kinetic energy. The beam itself acquires its kinetic energy from the electric field. As we have been just mentioning the electric field is generated during pulsar rotation together with frozen-in magnetic field and gets energy from its rotation slowing down. Thus, the energy of the generated toroidal magnetic field comes from pulsar rotation slowing down.

Superposition of generated toroidal magnetic field and pulsar magnetic field will give the spiral configuration magnetic field. Since plasma particles follow the magnetic field lines corotation will be violated in the pulsar magnetosphere and instead of it we will have differential rotation or shear flow of magnetospheric plasma.

On large radial distances the step of the magnetic field spiral should decrease and beyond the light cylinder magnetic field will become practically purely toroidal. On larger radial distance from pulsar - around 10^{17} cm this magnetic field is reconnected with the magnetic field of Crab Nebula, which has also toroidal structure (about one possible mechanism for the generation of this field see [22]).

4. Conclusions

Thus, in the magnetosphere of Crab pulsar relativistic electron-positron plasma is penetrated by ultrarelativistic electron beam. The system plasma-beam is unstable and two-stream instability develops in it. As a result exponentially growing toroidal magnetic field is generated in the magnetosphere of Crab pulsar and after this magnetic field structure changes to spiral. Since plasma particles follow the magnetic field lines corotation will be violated. On large radial distances step of the magnetic field spiral decreases and we get practically purely toroidal magnetic field which is finally reconnected with the magnetic field of Crab nebula. Toroidal magnetic field is generated in the pulsar magnetosphere at the expense of energy released during pulsar rotation slowing down.

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