Effects of dimers on cooperation in the spatial prisoner's dilemma game[*]

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Abstract

We investigate the evolutionary prisoner's dilemma game in structured populations by introducing dimers, which are defined as that two players in each dimer always hold a same strategy. We find that influences of dimers on cooperation depend on the type of dimers and the population structure. For those dimers in which players interact with each other, the cooperation level increases with the number of dimers though the cooperation improvement level depends on the type of network structures. On the other hand, the dimers, in which there are not mutual interactions, will not do any good to the cooperation level in a single community, but interestingly, will improve the cooperation level in a population with two communities. We explore the relationship between dimers and self-interactions and find that the effects of dimers are similar to that of self-interactions. Also, we find that the dimers, which are established over two communities in a multi-community network, act as one type of interaction through which information between communities is communicated by the requirement that two players in a dimer hold a same strategy.

Keywords: Prisoner's dilemma game, Cooperation frequency, Networks

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I. INTRODUCTION

The spontaneous emergence of cooperation in groups of selfish individuals is ubiquitous in human society and biological systems and the evolutionary game theory has been considered as an important approach to investigate the cooperative behavior in those systems. As one of the most intriguing games, the evolutionary prisoner's dilemma game (PDG) has attracted much attention over the last few decades [1] for gaining understanding the emergence of cooperation. In a PDG, each individual chooses cooperation (C) or defection (D) as her competing strategy. When the population is well-mixed, a PDG fails to sustain cooperation, which is often at odds with reality where mutual cooperation may also be the final outcome of the game [2–4]. In Nowak and May's seminal works [5, 6], the two-dimensional (2D) square lattice and the interaction between nearest neighbors enable cooperators to protect themselves against exploitation of defectors by forming compact clusters on the lattice. From then on, the evolutionary PDG on a structured population has been a hot spot. The influences on cooperation by different factors of models have been studied intensively. For example, a large set of strategies were used [7–9], different evolutionary rules were introduced [10, 11], different types of randomness were considered [12–16], and so on. Also, the influences of population structure on cooperation have attracted much attention, for example, types of network structure [5, 17–26] and the topological properties of structure such as average degree [27], degree-mixing patterns [28] and clustering coefficient [29, 30]. Generally, the opinion that the diversity in personalities of individuals or in population structures could enhance cooperation in an evolutionary PDG [10, 31–37] has been widely accepted.

Now, the investigation on an evolutionary PDG has been extended to structured populations with communities. Lozano *et al.* studied the PDG in two practical networks in which inter-community structure and intra-community structure were designed and found that cooperation depends strongly on both intra-community heterogeneity and inter-community connectivity [38, 39]. In [40], the authors considered the population with two communities and studied the effects of inter-connection on cooperation. They found that cooperation may display a resonance-like behavior with the variation of the number of inter-connections.

Consider that, in human society, there always exist many small coalitions such as family members, colleagues, friends, collaborators, and so on. The members within one coalition

always hold the same belief and behave in a consistent way. It is an interesting question that how cooperation varies when the small coalitions are introduced into structured populations. In this work, we consider the influences of the factors on cooperation by introducing dimers to an evolutionary PDG in structured populations. In each dimer, two players always hold a same strategy. The paper is organized as follows. In section 2, the model incorporating dimers and the categories of dimers are introduced. In section 3, we firstly discuss the effects of different categories of dimers on cooperation in square lattices and ER networks [41]. And then, we compare the effects of dimers and self-interaction. Finally, we expand the study to populations with two communities [40] and more rich phenomena are found. In the final section, we give some discussions and conclusions.

II.   MODEL

In a standard evolutionary PDG, there are two steps in one generation. In the first step, each player follows cooperation $s_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ or defection $s_x = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. The payoff of a player x accumulating by playing PDGs with her neighbors can be expressed as

$$P_x = \sum_{y \in \Omega_x} s_x^+ Q s_y, \tag{1}$$

where s_x^+ denotes the transpose of the state vector s_x. Ω_x includes all of the neighbors of the player x. For simplicity, but without loss of generality, we follow the previous work [12] and adopt the re-scaled payoff matrix Q depending on one single parameter r for PDG

$$Q = \begin{pmatrix} 1 & -r \\ 1+r & 0 \end{pmatrix}, 1 < 1+r < 2. \tag{2}$$

In this notation, $1 < 1+r < 2$ measures a defector's temptation to exploit the neighboring cooperators and $-r$ denotes the sucker's payoff for a cooperator encountering a defector. Here, r denotes the ratio of the costs of cooperation to the net benefits of cooperation. In the second step, the player x will adopt the strategy s_y of a randomly chosen neighbor y with a probability which is determined by the payoff difference between them [15]:

$$W(s_x \leftarrow s_y) = 1/[1 + exp[(P_y - P_x)/K]], \tag{3}$$

where the parameter K, which is analogous to the temperature in Fermi-Dirac distribution in statistical physics, characterizes the stochastic uncertainties in making decisions for the player x [14, 15]. Throughout the work, we set $K = 0.1$.

In this work, the players in the population are divided into two groups: one with ordinary players who follow the standard evolutionary PDG, and the other with dimers. The players in dimers behave differently from the ordinary players only in the step of strategy updating: in each dimer, the one with higher payoff updates her strategy ordinarily and the other just follows her partner. Based on whether the two players in a dimer play game with each other or not, dimers can be classified into two categories: interacting ones (I-Dimer) and non-interacting ones (N-Dimer). Each category of dimers can be subdivided into local dimers (L-Dimer) and distant dimers (D-Dimer) depending on whether the players in a dimer are neighbors or not on the given network. To be noted, for ID-Dimer, the given network structure is modified by extra connections between players in dimers and, for NL-Dimer, the given network structure is modified by cutting the connections between players in dimers. In this work, we do not consider NL-Dimer and just focus on the effects of the other three categories of dimers on cooperation in different types of population structures such as square lattices with periodic boundary conditions and degree $z = 4$, Erdös-Rényi (ER) [41] networks with mean degree $z = 8$ and structural populations with two communities. In a two-community-structure population where one community is a square lattice with $z = 4$ and the other a square lattice with $z = 8$, dimers are established over these two communities, that is, two players in a dimer locate on different communities.

Throughout this work, we set the number of players in the population to be $N = 10,000$. Initially, in the Monte Carlo simulations, m dimers are randomly assigned through the population and players take the strategy of C or D with equal probability. To measure the cooperation level, the cooperator frequency ρ_c will be monitored when the evolution of strategy pattern reaches its steady state. All the following data are obtained with synchronous strategy updating and each point is gained by averaging 1000 generations after a transient time of 6000 generations and by averaging over 100 independent realizations.
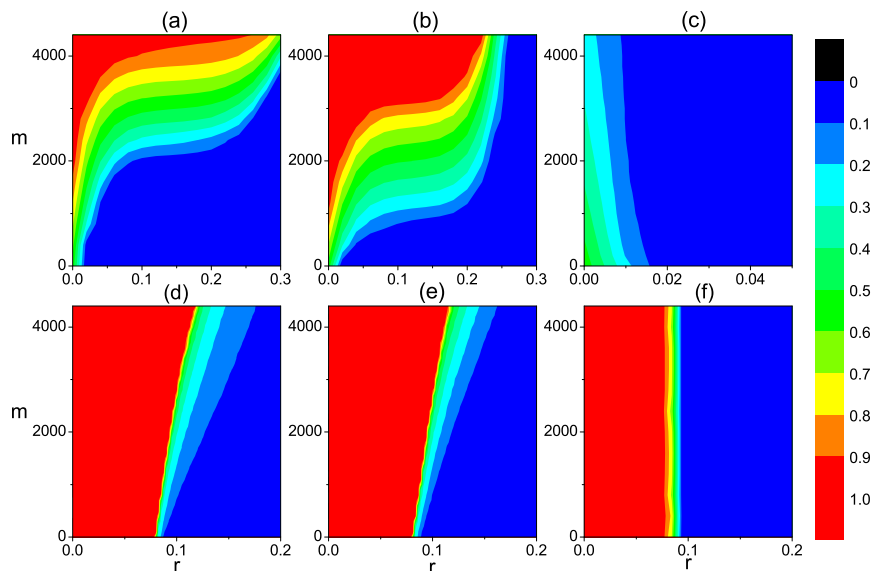
FIG. 1: (color online) The contour graphs of the cooperation level ρ_c for evolutionary PDGs, as functions of r and the number of dimers m on square lattices with $z = 4$ (top panel) and ER networks with mean degree $z = 8$ (bottom panel). m increases from 0 to 4400 in all cases. (a, d) For IL-Dimer. (b, e) For ID-Dimer. (c, f) For ND-Dimer.

III.   SIMULATION RESULTS AND ANALYSIS

Generally, cooperation is maintained in an evolutionary PDG on networks by forming cooperator clusters (C-clusters). The interactions between cooperators at the boundaries of C-clusters and those inside C-clusters enable cooperators at the boundaries to have high payoffs to compete with surrounding defectors. Intuitively, the same strategy held by the players in a dimer make it possible that cooperator dimers (C-dimer) serve as seeds for C-clusters and tend to accelerate the expansion of C-clusters. Therefore, it seems that the presence of dimers in an evolutionary PDG would improve cooperation. However, as we show below, the effects of dimers on cooperation are not self-evident and whether cooperation is improved or not depends on the type of dimers and the structure of the underlying networks.

Firstly, we investigate the effects of dimers on cooperation in populations without multi-community. The contour graphs of cooperator frequency ρ_c as functions of r and the number of dimers m are presented in figure 1. The top panel shows the results for IL-Dimer, ID-Dimer

and ND-Dimer on square lattices, respectively. For the range of r that is not covered here, almost all of ρ_c reaches 0 or will descend to 0 and no more raise of ρ_c appears. Interestingly, the presence of interaction between players in a dimer plays a decisive role on cooperation. Dimers enhance cooperation strongly for both IL-Dimer and ID-Dimer. Especially, in these two situations, the state that all players become cooperators (All-C state) may be reached for a large range of r where cooperators die off in the absence of dimers. Furthermore, ID-Dimer shows a faster growth of ρ_c with the number of dimers than IL-Dimer in the range of $r < 0.2$, which indicates a stronger cooperation enhancement for ID-Dimer than IL-Dimer. The stronger cooperation enhancement for ID-Dimer results from the presence of shortcuts established by the two players in each dimer. Together with a same strategy held by players in a dimer, these shortcuts shorten the average distance between any two players, which speeds up the expansion of C-clusters. On the other hand, the presence of ND-Dimer deteriorates cooperation, i.e., ρ_c decreases with the number of dimers. The deterioration of cooperation for ND-Dimer could be explained by the behavior of C-dimer. Since there are no interactions between players in these C-dimers, the advantage in payoff for cooperators at the boundaries of C-clusters over defectors may be weakened. Additionally, due to the absence of interaction between the players in these C-dimers, the speeding up of the expansion of C-clusters is lost and the defectors will benefit from these C-dimers provided that they are the neighbors of these C-dimers. Both of these suppress cooperation in ND-Dimer and the suppression of cooperation increases with the number of dimers.

The bottom panel in figure 1 shows the results for IL-Dimer, ID-Dimer and ND-Dimer on ER networks, respectively. In comparison with the top panel in figure 1, the improvement of cooperation by IL-Dimer and ID-Dimer is observed, though the improvement is not prominent. However, the influence of ND-Dimer on cooperation is quite different from that on square lattices: ρ_c is insensitive to the presence of dimers in this situation. Consider that, in ER networks where the mean distance between any two players is short, the players in C-clusters are always exposed to defectors and defectors always benefit from cooperators in C-clusters in an ordinary PDG. When ND-Dimer is introduced, defectors cannot get more benefits from C-dimer than those in the absence of ND-Dimer. Therefore, the deterioration of cooperation by ND-Dimer on square lattices is missed on ER networks and cooperation is independent of the number of dimers.

It is well known that the inclusion of self-interaction in an evolutionary PDG on networks
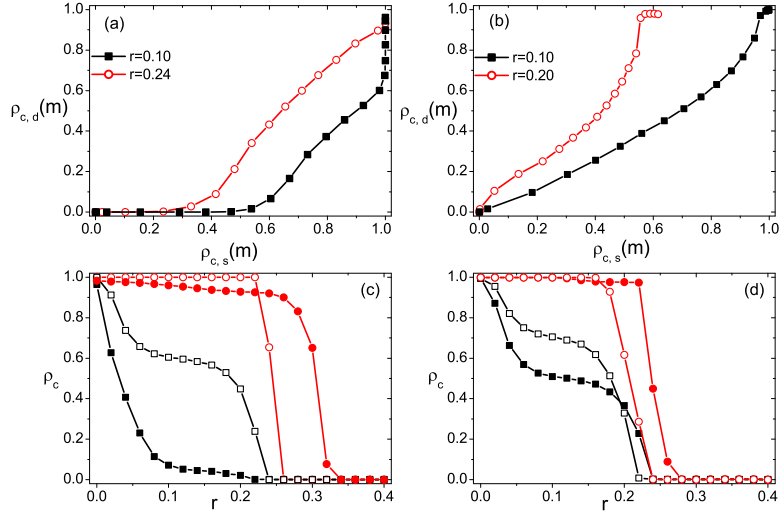
FIG. 2: (color online) The relationships between the cooperator frequencies $\rho_{c,s}$ for square lattices with self-interaction and $\rho_{c,d}$ for those with dimers. (a) $\rho_{c,d}$ is for IL-Dimer. Closed squares are for $r = 0.10$ and open circles for $r = 0.24$. (b) $\rho_{c,d}$ is for ID-Dimer. Closed squares are for $r = 0.10$ and open circles for $r = 0.20$. (c) The closed symbols are for IL-Dimer with $m = 2000$ (squares) and $m = 4400$ (circles). The open symbols are for the case without dimers and in which there are 4000 (squares) or 8800 (circles) players with self-interaction. (d) The closed symbols are for ID-Dimer with $m = 2000$ (squares) and $m = 4400$ (circles). The open symbols are for the case without dimers and in which there are 4000 (squares) or 8800 (circles) players with self-interaction. The structure of networks for the case with self-interaction is same as that in ID-Dimer, which is modified by the shortcuts between players in dimers.

[15] favors cooperation. However, how self-interaction relates to reality is unknown. As far as it is concerned that a same strategy is held by two players in a dimer, we find that the interaction between players in a dimer for IL-Dimer or ID-Dimer acts as self-interaction and either IL-Dimer or ID-Dimer provides a way to realize self-interaction for players to some extent. To make it clear, we compare the effects of IL-Dimer (or ID-Dimer) and self-interaction on cooperation. We take square lattices with $z = 4$ as examples. It has to be mentioned that the square lattices with the presence of ID-Dimer are modified by the shortcuts between players in dimers. Therefore, the networks with self-interaction to be studied are 'square lattices' with the same shortcuts as those established by dimers. To

be noted, not all players on networks have self-interaction and the number of players with self-interaction is twice as many as the number of dimers. For a given number of dimers m, we monitor the cooperator frequencies $\rho_{c,s}(m)$ for networks with self-interaction and $\rho_{c,d}(m)$ for networks with dimers. As shown in the top panel in figure 2 where the relationships between $\rho_{c,s}(m)$ and $\rho_{c,d}(m)$ are presented, $\rho_{c,s}(m)$ is positively correlated with $\rho_{c,d}(m)$ and the slope of $\rho_{c,d}$ over $\rho_{c,s}$ is roughly around 1, which means that IL-Dimer or ID-Dimer does provide a way to realize self-interaction. However, some differences exist between the systems with self-interaction and those with IL-Dimer or ID-Dimer. For example, as shown in the bottom panel in figure 2 where $\rho_{c,s}$ and $\rho_{c,d}$ against r for different m are presented, $\rho_{c,s}$ is always higher than $\rho_{c,d}$ for small r whereas $\rho_{c,s}$ becomes lower than $\rho_{c,d}$ for large r.

Additionally, the above model is investigated in square lattices and ER networks with some other mean degrees, such as $z = 8$ for square lattices and $z = 4$ for ER networks. The analogous phenomena could be obtained. Since that there is an explicit dependence between the value of the K and the outcome of the prisoner's dilemma following the Fermi update rule, we test the work with different K and find the robustness of our results against the variation of K.

Secondly, we consider the effects of dimers on a population with two communities: one is a square lattice with $z = 4$ and the other with $z = 8$. As stated in the model section, two players in each dimer belong to different communities and are randomly selected. Clearly, dimers on this population structure fall into the category of either ID-Dimer or ND-Dimer. The effects of dimers on cooperation for ID-Dimer and ND-Dimer are illustrated in the top panel and the bottom panel in figure 3, respectively. The left and middle columns show the contour graphs of ρ_c on the r-m space for the two communities respectively and the right column is for the whole population. Similar to the populations without community structure, cooperation is improved for ID-Dimer and a high level of cooperation may be reached. A resonance-like behavior with the variation of the number of dimers could be observed when $r > 0.1$, which is similar to that in [40] which was discussed particularly. In contrast with the populations without community structure, ND-Dimer indeed enhances cooperation. For ND-Dimer, though cooperation is a little downgraded in the community with $z = 8$ in which ρ_c is higher, cooperation, both in the whole population and in the community with $z = 4$ in which ρ_c is lower, is enhanced in comparison with that in the absence of dimers. As mentioned above, the deterioration of cooperation in a square lattice
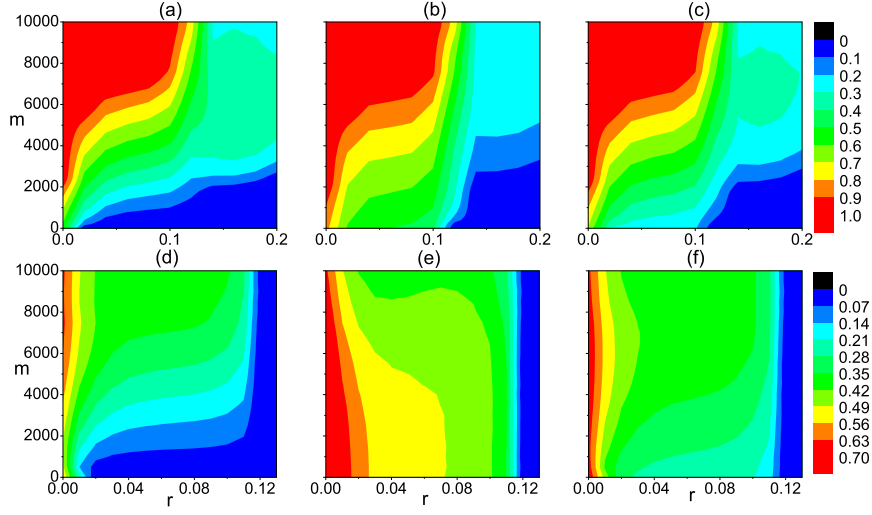
FIG. 3: (color online) The contour graphs of the cooperation level ρ_c, as a function of r and the number of dimers m in two communities with random dimer. The top panel is for ID-dimer and the bottom one is for ND-dimer. (a, d) $z = 4$. (b, e) $z = 8$. (c, f) For the averaged ρ_c over two communities.

with ND-Dimer originates from two factors involving C-dimers: players in C-dimers locating inside C-clusters can not support their partners at the boundaries of C-clusters, yet support the defectors neighboring their partners indirectly. However, for ND-Dimer defined over two communities, the deterioration of cooperation by the first factor is lost and, consequently, the downgrade of cooperation by ND-Dimer in the community $z = 8$ with a high level of cooperation becomes weak. On the other hand, the requirement that the players in dimers hold a same strategy may enhance cooperation if one of players is in a C-cluster. Combining these together, we observe a weak deterioration of cooperation in one community whereas the enhancement of cooperation in the other one and in the whole population.

An interesting comparison can be made between the results in this work and those in [40]. Both works consider the evolution of cooperation in a population with two interacting communities, but the ways of interaction are different. Either interaction through ID-Dimer or interaction through ND-Dimer may lead to two different evolutions of cooperation in comparison with those in [40]. For interaction with type of ID-Dimer, interaction between communities always improves cooperation, which is independent of the cooperation levels in

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isolated communities. Actually, even when cooperation is extinct in both isolated communities, cooperation may still reach a high level. For interaction with type of ND-Dimer, the cooperation levels in two communities may decrease simultaneously for weak interaction, and a resonance-like behavior of cooperation against interaction strength may appear for that the cooperation levels in two communities are not in the intermediate range.

IV.   CONCLUSIONS

In conclusion, we introduce dimers in which two players hold the same strategy to an evolutionary PDG on structured populations. The effects of dimers on cooperation are investigated. We find that influences of dimers on cooperation depend on the type of dimers and the population structure. For example, ID-Dimer always enhances cooperation and a high level of cooperation may be reached with a large number of dimers. However, the influences of ND-Dimer on cooperation depend on the population structure strongly: deterioration of cooperation on square lattices, little variation of cooperation on ER networks, and enhancement of cooperation on the population with two communities. Some interesting discussions between the results in this work and previous studies are made. Firstly, we discuss the relationship between dimers and self-interactions and find that the effect of dimers is similar to that of self-interaction. Secondly, we compare the interaction through interconnections between communities, where interaction is realized by playing games between players in different communities, with that through dimers, where interaction is realized through holding a same strategy by two players in a dimer. For the interaction through dimers, rich phenomena are observed.

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