The Čerenkov-Compton effect in quantum electrodynamics

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May 1, 2019

Abstract

The two-photon production by motion of a charged particle in a medium is considered. This process is called Čerenkov-Compton effect (ČCE) because the production of photons is calculated from the Feynman diagram for the Compton effect in a medium. This process is not forbidden in quantum electrodynamics of dielectric media.The cross section of this process in Mandelstam variables is calculated for pair of photons with one moving at the opposite direction to the electron motion and the second one inside Čerenkov cone. The opposite motion is not caused by the collision with a particle of a medium. The relation of this process to the CERN experiments is considered.

1 Introduction

The fast moving charged particle in a medium when its speed is faster than the speed of light in this medium produces electromagnetic radiation which is called the Vavilov-Čerenkov radiation. The prediction of Čerenkov radiation came long ago. Heaviside (1889) investigated the possibility of a charged object moving in a medium faster than electromagnetic waves in the same medium becomes a source of directed electromagnetic radiation. Kelvin (1901) presented an idea that the emission of particles is possible at a speed greater than that of light. Somewhat later, Sommerfeld (1904) proposed the hypothetical radiation with a sharp angular distribution. However, in fact, from experimental point of view, the electromagnetic Čerenkov radiation was first observed in the early 1900's by experiments developed by Marie and Pierre Curie when studying radioactivity emission. In essence they observed the emission of a bluish-white light from transparent substances in the neighborhood of strong radioactive source. But the first attempt to understand the origin of this was made by Mallet (1926, 1929a, 1929b) who observed that the light emitted by a variety of transparent bodies placed close to a radioactive source always had the same bluish-white quality, and that the spectrum was continuous, with no line or band structure characteristic of fluorescence. Unfortunately, these investigations were forgotten for many years. Čerenkov experiments (Čerenkov, 1934) was performed at the suggestion of Vavilov who opened a door to the true physical nature of the this effect (Bolotovskii, 2009). This radiation was first theoretically interpreted by Tamm and Frank (1937) in the framework of the classical electrodynamics. The source theoretical description of this effect was given by Schwinger et al. (1976) at the zero temperature regime and the classical spectral formula was generalized to the finite temperature situation and for the massive photons by autor (Pardy, 1989; 2002). The Vavilov-Čerenkov effect was also used by author (Pardy, 1983) to possible particle production by the Čerenkov mechanism and to possible measurement of the Lorentz contraction (Pardy, 1997).

The Čerenkov-Compton effect (ČCE) is in this article considered as the synergic two-photon production by a charged particle moving in a medium in such a way that it has some analogy of the Čerenkov and Compton mechanism (Pardy, 2001; 2004). The same process in the very different form was considered at present time by Bulanov et al. (Bulanov et al., 2019).

The two-photon process caused by the Čerenkov mechanism was considered by Frank in 1968 as the anomalous scattering of light on a particle in medium (Frank, 1968). In the Frank formulation of the process, the initial photon induces the final photon. The total process is then the two-photon emission in a medium. The classical theory of this process was elaborated by Frank and Tsytovich (1980), and Afanasiev et al. (2003). The theory of these authors is not based on the quantum field theory and Feynman diagrams.

The Čerenkov mechanism requires the velocity of particle is greater than the velocity of light in the medium. It can be determined classically supposing some microscopical structure of medium and interaction of moving charges with this microscopical structure (Jelley, 1958). On the other hand, it can be determined quantum mechanically using Feynman diagram where it is not used the specific microscopical mechanism of medium for calculation of this effect (Di Giacomo et al., 1994). Only the index of refraction is postulated. Also Schwinger source theory works without microscopical mechanism generating radiation. Our problem differs from the Frank problem in a sense, that the emission of two photons is here considered as the synergic process different from the Bulanov et al. process, of the Čerenkov mechanism and the Compton one.

We will show that this process is not forbidden in the framework of quantum field theory. We describe ČCE in terms of quantum electrodynamics.

We use the Feynman diagram for the Compton process as an extension and analogue of the Feynman diagram for the simple Čerenkov process, in order to describe it. So, we do not use the knowledge of the microscopical mechanism of this process.

This process was not discussed in textbooks and in monographies (Jelley, 1958) on the Čerenkov radiation. On the other hand, it seems that there is an experimental evidence of this effect in CERN, because every photography with the Čerenkovian circles involves the background with photons which are probably generated by the ČCE mechanism.

While Čerenkov process has the symbolic description

$$e \longrightarrow e + \gamma \tag*{(1),}$$

the Compton process has the symbolic description

$$e + \gamma \longrightarrow e + \gamma \tag*{(2).}$$

The Čerenkov-Compton process follows from the last symbolic equation as the transposition of its components and has the symbolic expression in the form:

$$e \longrightarrow e + \gamma + \gamma. \tag{3}$$

This process is physically allowed for the index of refraction $n > 1$ and for a charged particle moving through the transparent medium at speed greater than the speed of light in that medium.

The rigorous description of the physical reality of this process cannot be realized by some mechanical model, but only by means of Feynman diagrams with the corresponding mathematical theory of the Compton process. So, in other words, while the ordinary Čerenov effect enables mechanical explanation, as it was shown by Tamm and Frank (1937), the origin of the ČCE is quantum electrodynamical.

The Čerenkov effect in source theory follows from amplitude (Schwinger et al., 1976)

$$\langle 0_+|0_-\rangle = e^{\frac{i}{\hbar}W}, \tag{4}$$

where W is the action of electromagnetic field in medium. It seams that Feynman diagram methods are more appropriate to describe this process.

In order to be pedagogically clear, we derive in section 2 some elementary energetical and angle relations concerning the ČCE. Then, in the following section 3 we prove the theorem on the existence of the ČCE. In section 4, the cross-section of the ČCE is derived. Discussion in section 5 is devoted to the summary and the experimental evidence of ČCE in CERN.

2 Energetical and angle relations

In the pure Čerenkov process the four-momentum is of the form

$$p = k + p'. \tag{5}$$

In the pure Compton process, the four-momentum relation of the initial and final parameter is as follows:

$$p + k = p' + k.' \tag{6}$$

However, in case of the two-photon Čerenkov process which we denote as the Čerenkov-Compton effect, the four-momentum relation is of the form:

$$p = k + k' + p'. \tag{7}$$

This equation can be splitted into two equations for energy and momentum:

$$E = \omega + \omega' + E' \tag{8}$$

and

$$\mathbf{p} = \mathbf{k} + \mathbf{k}' + \mathbf{p}'. \tag{9}$$

where ω and ω' are energies of emitted photons and E and E' are initial and final energies of an electron, $\mathbf{k}$ and $\mathbf{k}'$ are wave vectors of the emitted photons and $\mathbf{p}$ and $\mathbf{p}'$ are initial and final three momenta of electron. We suppose in this article that $c = \hbar = 1$.

Now, let us introduce angle between $\mathbf{k}$ and $\mathbf{k}'$ as φ, an angle between $\mathbf{p}$ and $\mathbf{k}$ as χ and angle between $\mathbf{p}'$ and $\mathbf{k}'$ as α, or, $\angle(\mathbf{k}, \mathbf{k}') = \varphi$, $\angle(\mathbf{p}, \mathbf{k}) = \chi$, $\angle(\mathbf{p}', \mathbf{k}') = \alpha$.

We can derive simple energy equation using the procedure of Di Giacomo et al. (1994),:

$$\omega + \omega' = E(\mathbf{p}) - E(\mathbf{p}') = E(\mathbf{p}) - E(\mathbf{p} - (\mathbf{k} + \mathbf{k}')) = \frac{\partial E}{\partial \mathbf{p}}(\mathbf{k} + \mathbf{k}') = \mathbf{v}(\mathbf{k} + \mathbf{k}'), \tag{10}$$

or, $(k = \omega n, k' = \omega' n)$

$$\omega \cos\chi + \omega' \cos\alpha = \frac{\omega + \omega'}{vn}. \tag{11}$$

From eq. (11) it follows that for $\alpha = \pm\chi$ we get the Čerenkov threshold $\cos\chi = 1/nv$. It means that the total Čerenkovian spectrum involves the ordinary Čerenkovian spectrum of photons and the some part of the photon spectrum of the ČCE.

Eliminating $\cos\varphi$ from the last equation we get:

$$\cos\varphi = \frac{\cos\alpha}{vn} + \frac{\omega'}{\omega}\left(\frac{\cos\alpha}{vn} - \cos^2\alpha\right) + \sin\alpha\sqrt{1 - \left(\frac{1}{vn} + \frac{\omega'}{\omega}\left(\frac{1}{vn} - \cos\alpha\right)\right)^2}. \tag{12}$$

For the sake of simplicity, let us consider the experimental situation with the backward emission of photon with frequency ω', or, with $\alpha = \pi$. We do not consider the situation with $\alpha = 0$ because it may be easy to show that the final cross-section is negative and it means that such situation is not physically meaningful.

Using $\varphi = \chi - \pi$, and eq. (11), or (12) we get the following equation:

$$\cos\chi = \frac{1}{vn} + \frac{\omega'}{\omega}\left(\frac{1}{vn} + 1\right). \tag{13}$$

We see, that the last equation involves the Čerenkov threshold $\cos\chi = 1/nv$ and it means that the relation of the considered effect to the Čerenkov effect is appropriate.

From eq. (13) the following relations follows:

$$(1 + n^2\cos\chi) = 1 + \frac{n}{v} + \frac{\omega'}{\omega}\left(\frac{n}{v} + n^2\right) \tag{14}$$

and

$$1 - nv\cos\chi = -\frac{\omega'}{\omega}(1 + nv). \tag{15}$$

Formulas (14) and (15) will be used later in determination of the cross section. From eq. (13) follows the relation between frequencies ω and ω', if the initial frequency ω' is emitted in the direction $\alpha = \pi$. Using

$$\frac{1}{vn} \leq \cos\chi \leq 1, \tag{16}$$

we get

$$\omega \geq \omega'\frac{vn+1}{vn-1}; \quad vn \geq 1; \quad \alpha = \pi \tag{17}$$

and we see that when $nv \to 1$, then, $\omega/\omega' \to \infty$.

3 The existence of the ČCE

The ČCE exists if and only if a charged particle moves in a dielectric medium with the speed which is faster then the speed of light in this medium. For velocities lower then the speed of light in the medium, there is no ČCE. Let us proof it.

In general, we have:

$$p - k = p' + k', \tag{18}$$

or, after squaring the last equation, we have with $p^2 = m^2$ and $k^2 = 0$ in the laboratory system where dielectric medium is in rest:

$$-(E\omega - \mathbf{p}\cdot\mathbf{k}) = E'\omega' - \mathbf{p}'\cdot\mathbf{k}'. \tag{19}$$

Then, with $\mathbf{p} = E\mathbf{v}$ and $|k| = n\omega$ and $v' \approx v$ we have:

$$-\frac{E\omega}{E'\omega'} = \frac{1 - vn\cos\alpha}{1 - vn\cos\chi} < 0 \tag{20}$$

Now, we can decide two cases:

$$I: (1 - vn\cos\alpha) < 0; (1 - vn\cos\chi) > 0, \tag{21}$$

$$II: 1 - vn\cos\alpha) > 0; (1 - vn\cos\chi) < 0. \tag{22}$$

If we now apply the Čerenkov condition $vn > 1$, we see that the first and the second possibilities can be true for some angle α and χ. On the other hand, if we suppose velocities which are smaller than the velocity of light in a medium,or, $vn < 1$, then, neither the first condition I, nor the second condition II can be true for α and χ. So, only for velocities which are greater than velocity of light in medium the two-photon process can occur. So the denotation Čerenkov-Compton effect is appropriate.

It means, in other words, that the conditions for the existence of the ČCE is the same as the condition of the existence of the pure Čerenkov effect.

4 The cross-section of the Čerenkov-Compton effect

We shall use the notion cross-section although there is no targed. However, this notion has also a meaning of the probability of generation of two photons of the specific parameters and from this point of view the cross-section is physically meaningful. On the other hand, if we perform the time reflection $t \to -t$, then the notion cross-section is evidently physically meaningful.

The cross-section of the ČCE can be easily evaluated in the so called Mandelstam variables s, t, u which are defined, as it is well known, by the following relations in vacuum (Berestetskii et al., 1989):

$$s = (p + k)^2 = m^2 + 2p'k' \tag{23}$$

$$t = (p - p') = -2kk' \tag{24}$$

$$u = (p - k') = m^2 - 2p'k, \tag{25}$$

where the right side of equations concerns the Compton situation $p + k = p' + k'$. In the case of the Čerenkov-Compton effect when $p = k + k' + p'$, we see that the this relation can be obtained from the former relation, by transformation $k \to -k$. In such a way the Mandelstam parameters follows from eqs. (23)–(25) in the form:

$$s = (p - k)^2 = m^2 + 2p'k' \tag{26}$$

$$t = (p - p') = 2kk' \tag{27}$$

$$u = (p - k') = m^2 + 2p'k. \tag{28}$$

The equations (26)–(28) are valid in vacuum and are invariant in all inertial systems. However if we consider the process in medium then, the invariance is not valid because photon four-vector is transformed according to the specific transformation appropriate to the medium. So, we will use the laboratory system where dielectric medium is at rest and where $k^2 = 0$. The derived formulas are valid for the laboratory system.

We express the last equations in terms of the four-momenta of electron and photon $(E, \mathbf{p}), (\omega, \mathbf{k})$. So, in the laboratory system we have:

$$s = m^2 + 2E'\omega'(1 - v'n\cos\alpha) \tag{29}$$

$$t = 2\omega\omega'(1 - n^2\cos\varphi) \tag{30}$$

$$u = m^2 + 2E'\omega(1 - nv'\cos\chi). \tag{31}$$

The cross-section has the following general form in the Mandelstam variables (Berestetskii et al., 1989).

$$d\sigma = \frac{8\pi r_e^2 m^2 dt}{(s-m^2)^2}\left\{\left(\frac{m^2}{s-m^2} + \frac{m^2}{u-m^2}\right)^2 + \right.$$

$$\left.\left(\frac{m^2}{s-m^2} + \frac{m^2}{u-m^2}\right) - \frac{1}{4}\left(\frac{s-m^2}{u-m^2} + \frac{u-m^2}{s-m^2}\right)\right\}. \tag{32}$$

Now, let us determine the $d\sigma$ of ČCE for special situation where $\alpha = \pi$.

In this case $(E \approx E')$

$$s \approx m^2 + 2E\omega'(1 + vn) \tag{33}$$

$$t = 2\omega\omega'(1 + n^2\cos\chi) \tag{34}$$

$$u \approx m^2 + 2E\omega(1 - nv\cos\chi), \tag{35}$$

or,

$$s - m^2 \approx 2E\omega'(1 + nv). \tag{36}$$

$$t = 2\omega\omega'\left(1 + \frac{n}{v}\right) + 2\omega'^2\left(\frac{n}{v} + n^2\right); \quad dt|_{\omega'=const} = 2\omega'\left(1 + \frac{n}{v}\right)d\omega. \tag{37}$$

$$u - m^2 \approx -2E\omega'(1 + nv). \tag{38}$$

For differential cross-section we then have with $r_e = e^2/m$:

$$d\sigma = \frac{2\pi e^4}{E^2\omega'}\frac{1+n/v}{(1+nv)^2}d\omega. \tag{39}$$

Since e^2 has dimension energy × length, then the dimension of $d\sigma$ is length^2. We use term cross-section, although there is no target. It means that the physical meaning of this term is to be considered in terms of the probability of the process.

In case, we do not use the specification $\alpha = \pi$, we get very complicated equations for t and dt. Also the final cross section formula is very complicated and it means it is not suitable for experimental verification. In experiment, simple formulas are more attractive, than complicated ones.

The experimental content of the last formula is as follows. To given ω' which is emitted in angle $\alpha = \pi$, and detected by the pixel detector placed in the direction of $\alpha = \pi$ there exist photon with frequency ω which can be detected by the detector placed in the direction χ given by equation (13). So, Using the pixel detectors, the ČCE is possible detect with the cross-section given by the last formula. Let us remark that emission of photons in the direction $\cos\chi > 1/nv$ was observed in CERN as a byproduct of the ordinary Čerenkov effect.

Let us still remark that the situation with photon moving at the opposite direction to the electron motion has crucial heuristical meaning which has no analogy in particle physics. In particle physics only collision with targed can produce backscattering. Here we see that without scattering centers the photons can be produced at the opposite direction.

5 Discussion

We have seen in the preceding chapters that the so called Čerenkov-Compton effect can be rigorously formulated and solved in QED. The article is some modification of author preceding articles (Pardy, 2000; 2004). We have used the analogy with the Compton effect because the ordinary Čerenkov effect can be determined from the Feynman diagram with the emission of one photon (Di Giacomo, 1994), and the Compton diagram is only generalization to the situation with two photons. At the same time it is necessary to say it was not possible to use the theory of the double Compton effect (Mandl, 1952) because the initial state of of the double Compton effect is $\gamma + e$, while the initial state of the ČCE is single electron. The ČCE substantially differs from the ordinary Čerenkov effect because it cannot be explained by any mechanical model being an effect of quantum electrodynamics.

We have proved that in QED the Čerenkov-Compton effect occurs as the integral part of every ordinary Čerenkov process. While the Čerenkov effect is described in textbooks of electrodynamics and monographes, articles and preprints, and it is observed in all particle laboratories, the Čerenkov-Compton effect is still not considered as an effect of experimental relevance.

However, this effect was yet probably observed in particle laboratories and no appropriate attention was devoted to it. The situation is an analogue of the situation when Pierre and Marie Curie observed Čerenkov radiation in vessel with the radiaoactive salts and they were not aware that it was the Čerenkov radiation. Also Mallet (1926, 1928, 1929) in 1926-1929 during experiments with the gamma rays interacting with water observed the Čerenkov radiation but he was not able to define this effect as the new phenomenon.

In CERN, the particle identification system is proposed for the LHC-B experiment. It consists of RICH detectors with three materials. The photodetectors under development are multipixel hybrid photodiodes, which allow high single photon detection and high spatial resolution. According to CERN report (Forty, 1996), there are background of photons which is in this report interpreted as Rayleigh scattering of the Čerenkovian photons. It is well known that the Rayleigh scattering is an elastic scattering of photons on nonhomogenities and density fluctuations of a medium leaving the electron energy unchanged. The cross section of this process varies as the inverse fourth power of the photon wavelength (Jackson, 1998), or,

$$\sigma \propto \frac{1}{\lambda^4}. \tag{40}$$

The transition radiation is not considered because it can be neglected. We of course suppose that also the process of Čerenkov-Compton photons contributes to the background and at present time the two kinds of photons are not distinguished. The color photographies of the Čerenkov radiation informs us that the Čerenkov circles are blue and the background inside of the circle(s) is not black (Cern Courier, 1998).

We know that the pure Čerenkov radiation is linearly polarized in the plane defined by the direction of observation and path of particle motion. The polarization of the Rayleigh photons probably differs from the polarization of the Čerenkov, or, the Čerenkov-Compton photons. So, if the background is formed by the Rayleigh photons and the Čerenkov-Compton photons, it can be in principle possible spectroscopically to distinguish both groups of photons and to verify the existence of the ČCE. The second possibility how to register the Čerenkov-Compton photons is to use absolutely homogeneous medium at very low temperature. Then, the Rayleigh scattering does not occur and we can observe only the ČCE.

However, if in experiment it will be proved that background photons are not from ČCE, then it can be considered as a crucial puzzle, because the ČCE is allowed by QED. This puzzle will open evidently new view on the ČCE.

We hope that sooner or later the Čerenkov-Compton effect will be again verified and investigated in the laboratories of particle physics, where the Čerenkov effect plays the substantial role in the experiment.

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