A Scalar Equation of Motion

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Abstract

In classical mechanics, this paper presents a scalar equation of motion, which can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.

The Scalar Equation of Motion

If we consider two particles A and B of mass m_a and m_b respectively, then the scalar equation of motion, is given by:

$$\frac{1}{2} m_a m_b \left[(\mathbf{v}_a - \mathbf{v}_b)^2 + (\mathbf{a}_a - \mathbf{a}_b) \cdot (\mathbf{r}_a - \mathbf{r}_b) \right] = \frac{1}{2} m_a m_b \left[2 \int \left(\frac{\mathbf{F}_a}{m_a} - \frac{\mathbf{F}_b}{m_b} \right) \cdot d(\mathbf{r}_a - \mathbf{r}_b) + \left(\frac{\mathbf{F}_a}{m_a} - \frac{\mathbf{F}_b}{m_b} \right) \cdot (\mathbf{r}_a - \mathbf{r}_b) \right]$$

where \mathbf{v}_a and \mathbf{v}_b are the velocities of particles A and B, \mathbf{a}_a and \mathbf{a}_b are the accelerations of particles A and B, \mathbf{r}_a and \mathbf{r}_b are the positions of particles A and B, and \mathbf{F}_a and \mathbf{F}_b are the net forces acting on particles A and B.

This scalar equation of motion can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces. In addition, this scalar equation of motion is invariant under transformations between reference frames.