

On Emerging Complexity and Foundational Physics (Part 1)

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Abstract

As of today, Quantum Field Theory (QFT) and General Relativity (GR) are broadly recognized paradigms of foundational physics. However, there are growing suspicions that both paradigms fail to hold somewhere above the Standard Model scale and in the realm of primordial cosmology. Evidence collected on multiple fronts indicates that *emergence* and *complexity* are universal features of far-from-equilibrium systems having many degrees of freedom. In line with these findings, Part 1 of this report explores the complex dynamics of evolving dimensional fluctuations beyond the Standard Model scale. Part 2 outlines the role of complex dynamics in the nonintegrable sector of particle physics, Dark Matter condensation and the gravitational regime of the early Universe.

Key words: Non-equilibrium critical phenomena, Self-Organized Criticality, fractal spacetime, multifractals, emergence, complex dynamics.

1. From Dimensional Regularization to fractal spacetime

To make the paper self-contained and for the sake of clarity, we begin by iterating the arguments for fractal spacetime inspired by Dimensional Regularization program of QFT [1].

It is known that Euclidean formulation of the Path Integral in QFT enables a useful analogy between QFT and critical phenomena. To this end, consider the two-point function of massive scalar field theory. The Euclidean propagator in momentum space is given by

$$D_E(p) = \frac{1}{p^2 + m^2} \quad (1)$$

and the correlation function of the corresponding statistical system is given by,

$$\langle \varphi(x) \varphi(0) \rangle = \int \frac{d^4 p}{(2\pi)^4} \frac{\exp(ipx)}{p^2 + m^2} \quad (2)$$

in which $|p^2| = p_\mu p_\mu$ and $px = p_\mu x_\mu$. In the limit $m|x| \gg 1$, (2) is well approximated by,

$$\langle \varphi(x) \varphi(0) \rangle \approx \frac{1}{|x|^2} \exp(-m|x|) \quad (3)$$

Let us assume that the field is placed on a four-dimensional lattice of points separated by a fixed spacing $a = \Lambda_{UV}^{-1}$, in which Λ_{UV} is the cutoff scale. The spatial coordinate is then given by,

$$|x| = Na = N\Lambda_{UV}^{-1}, \quad N \gg 1 \quad (4)$$

and (3) can be written as,

$$\langle \varphi_N \varphi_0 \rangle \propto \exp[-N(m/\Lambda_{UV})] \quad (5)$$

By analogy with statistical mechanics, (5) defines the dimensionless correlation length according to,

$$\langle \varphi_N \varphi_0 \rangle \propto \exp(-N/\xi) \quad (6)$$

where,

$$\xi = \frac{\Lambda_{UV}}{m} \quad (7)$$

Dimensional Regularization in momentum space sets up a relationship between the cutoff scale Λ_{UV} and the dimensional deviation from four-space dimensions $\varepsilon(\mu) = 4 - d(\mu) \ll 1$ as in

$$\Lambda_{UV}^2 = \mu^2 \exp(1/\varepsilon) \gg \mu^2 \quad (8)$$

where μ is the running scale. The asymptotic limit $m = O(\mu) \ll \Lambda_{UV}$ leads to the raw estimate,

$$\boxed{\varepsilon(\mu) = 4 - d(\mu) = O[m^2(\mu)/\Lambda_{UV}^2] \ll 1} \quad (9)$$

By (7) and (9), we arrive at the effective approximation,

$$\boxed{\xi(\mu) \propto [\varepsilon(\mu)]^{-1/2}} \quad (10)$$

As a diverging correlation length is a characteristic feature of critical phenomena, (10) indicates that removing the dimensional regulator in QFT (that is, taking the classical continuum limit $\varepsilon \rightarrow 0$) is *analogous to tuning the corresponding statistical system towards the critical point*. In this sense, (10) underlies the idea of *criticality in continuous dimension* $d(\mu)$, conjectured to play a key role in the ultraviolet regime of field theory and primordial

cosmology. Since, by definition, fractal structures are characterized by continuous dimensions and are the underlying geometry of both critical phenomena and chaotic behavior, (10) leads to the conclusion that taking the limit $\varepsilon \rightarrow 0$ in Dimensional Regularization turns the classical spacetime into a *minimal fractal manifold*.

2. Complex dynamics of evolving spacetime dimensions

Reaction-diffusion processes are a subset of complex phenomena defined within the framework of Non-equilibrium Statistical Physics [2 – 3]. These models are typically formulated in $d+1$ dimensions, where d is the dimension of the Euclidean manifold representing the physical space and t is the time coordinate. Reaction-Diffusion models on discrete manifolds (called lattices) are characterized by the following features,

- a) local variables reside at lattice sites,
- b) reaction chains are driven by probabilistic transition rules among sites.

We consider below a toy Reaction-Diffusion model acting on a two-dimensional lattice ($d=2$), whose local variables are time-varying *dimensional deviations* $[\delta\epsilon(t)]$, referred herein to as “*dimensional pixels*”. The model is built on four premises, namely,

A1) At any given moment “ t ”, a pixel consists of a pair of lattice sites that are either *occupied* (1) or *empty* (0) and are located horizontally adjacent to each other.

A2) The representative pixel states are listed as,

$$[\delta\epsilon(t)] = \{[0,1]; [1,0]; [1,1]; [0,0]\}_t$$

A3) There are four transpositions among these binary states from time “ t ” to time “ $t+dt$ ”, that is,

$$[1] \rightarrow [0] \quad \text{self-annihilation} \quad (11a)$$

$$[1] \rightarrow [1] + [1] \quad \text{decay/percolation} \quad (11b)$$

$$[1] + [1] \rightarrow [0] \quad \text{pair annihilation} \quad (11c)$$

$$[1]+[1]\rightarrow[1] \text{ clustering} \quad (11d)$$

A4) Following ref. [1] and figs. 1-2 below, dimensional pixels undergo transition events between " t " and " $t+dt$ " as described by,

$$[1,0]\xrightarrow{D}[0,1] \quad (12a)$$

$$[1,1]\xrightarrow{u}[1,0],[0,1] \quad (12b)$$

$$[1,0],[0,1]\xrightarrow{\kappa}[1,1] \quad (12c)$$

Here, (12a) denotes a *scattering* event at rate D , (12b) a *clustering* event at rate u and (12c) a *decay* (or *percolation*) event at rate $\kappa = \lambda - \lambda_c$, with λ being a control parameter approaching its critical value λ_c .

Up to a leading order approximation, the macroscopic attributes of Reaction-Diffusion processes may be encoded in a *mean-field* (MF) equation [1], which quantifies the competition between losses and gains in density $\rho(t)$. In particular, the decay/percolation process (3c) occurs with a rate proportional to $\kappa\rho(t)$ and leads to a gain in density. By contrast, the clustering process

(12b) drops the density with a rate proportional to $u\rho^2(t)$. Ignoring diffusion (12a), the resulting MF equation takes the form

$$\boxed{\frac{\partial \rho(t)}{\partial t} = \kappa \rho(t) - u \rho^2(t)} \quad (14)$$

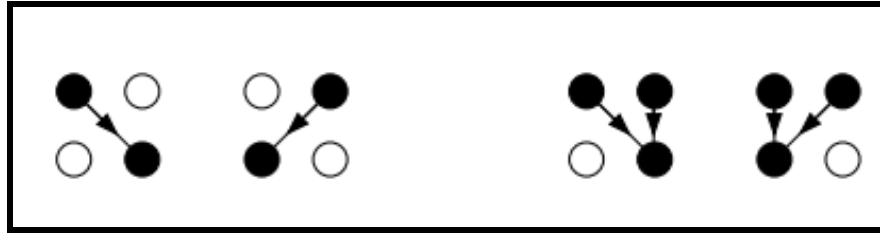


Fig. 1: *Left panel: Scattering, Right panel: Clustering ([1])*

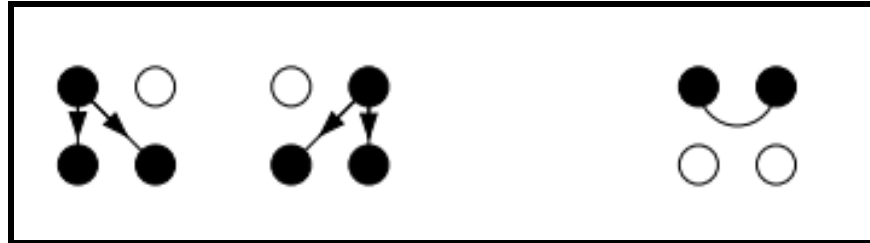


Fig. 2 *Left panel: Decay/Percolation, Right panel: Annihilation ([1])*

In the context of our paper, the control parameter $\lambda(t) = \lambda[\delta\varepsilon(t)]$ represents the *density of dimensional pixels* $\delta\varepsilon(t) \ll 1$ while $\rho(t)$ denotes the *density of active (or unstable) lattice sites*.

The MF equation (4) exhibits a two-phase configuration: an *absorbing phase* with a vanishing density of active sites ($\rho=0$) below the critical point $\lambda < \lambda_c$ and an *active phase* with a steady-state density $\rho=\kappa/u \neq 0$ above the critical point $\lambda > \lambda_c$.

If the starting point of the time evolution is a fully occupied lattice $\rho(0)=1$, the solution of (4) reads,

$$\rho(t) = \frac{\kappa}{u - (u - \kappa) \exp(-\kappa t)} \quad (15)$$

Relation (5) shows that, when the percolation rate vanishes at the critical point $\kappa = \lambda - \lambda_c = 0$, the density of unstable states drops asymptotically as in

$$\lim_{\kappa \rightarrow 0} \rho(t) = (1 + ut)^{-1} \quad (16a)$$

or,

$$\rho(t) \propto t^{-1} \quad (16b)$$

By (16a) and (16b), the number of unstable/active sites eventually goes to zero and the dynamics of dimensional deviations slows down. It follows

that, in the far infrared regime ($t \rightarrow \infty$), spacetime settles in a stationary state matching the classical limit $\delta\varepsilon = O(\varepsilon) \rightarrow 0$.

Before proceeding further, a few cautionary remarks are in order:

a) Clearly, by design, the scenario embodied in (A1) – (A4) is merely a convenient simplification. Any realistic model of pixels defined in continuous spatial dimensions must necessarily include infinite sets of non-integer pairs having the form,

$$[\delta\varepsilon(t)] = \{[\alpha_1, \beta_1]; [\alpha_2, \beta_2]; \dots [\alpha_N, \beta_N]\}_t \quad (17)$$

in which $N \rightarrow \infty$ and α_i, β_i are arbitrary numbers with

$$0 \leq \alpha_i \leq 1; \quad 0 \leq \beta_i \leq 1; \quad \alpha_i \neq \beta_i \quad (18)$$

b) It can be argued that, on Euclidean spacetime endowed with continuous dimensions, the deviations ε and their fluctuations $\delta\varepsilon$ play an identical role with the coefficients $g_{\mu\nu}$ of a corresponding non-Euclidean metric [5]. Stated differently, a flat spacetime endowed with a fractal structure may be considered as *dual* to a curved manifold. On this

basis, one may reasonably argue that classical gravitation is *implicitly accounted for* in the RD model detailed above.

c) The stability of the MF solution with respect to perturbations can be studied in a variety of ways. In a general scenario, for example, one accounts for the combined effects of diffusion (D), Gaussian noise $\varsigma(t)$ and random fluctuations $\eta(t)$ on the percolation parameter,

$$\kappa \rightarrow \kappa + \eta(t) \quad (19)$$

In this scenario, (4) gets upgraded to a Langevin type equation, namely,

$$\frac{\partial \rho(t)}{\partial t} = \kappa \rho(t) - u \rho^2(t) + D \Delta \rho(t) + \varsigma(t) + \rho(t) \eta(t) \quad (20)$$

As discussed in [2 – 3] and the complex dynamics literature, the generic model (20) has applications across a wide range of topics, including (but not limited to) percolation phenomena, epidemics spreading, forest fires, earthquake propagation, lattice dynamics with long-range correlations, spin glasses, spatiotemporal patterns in condensed matter physics,

random graph theory, galaxy clustering, large network dynamics and so on.

3. From Reaction-Diffusion processes to Self-organized Criticality (SOC)

Of particular interest is the relationship between the Reaction-Diffusion model previously outlined and Self-organized Criticality (SOC). Among the many ways to unveil this connection a straightforward approach is to supplement (4) with a *driving source* whose function is to boost dimensional instability and prevent relaxation. Adding to the percolation rate a time-independent source term E turns (4) into [4]

$$\frac{\partial \rho(t)}{\partial t} = [\kappa + E] \rho(t) - u \rho^2(t) \quad (21)$$

According to [4], since E is a conserved quantity ($\dot{E}=0$), it can be conveniently used as control parameter instead of κ . Following this scenario, the critical point is reached at $E_c = -\kappa$ and (21) replicates the *fixed sandpile model* of SOC, whose steady state is determined by E . The impact of

SOC in the non-integrable sector of particle physics and early Universe cosmology is covered in Part 2 of this report.

References (Part 1)

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