Experimental trajectories in Euclidean and hyperbolic geometry

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Abstract

The Euler tetrahedron volume formula is used to define dimensionality of space and the Euclidean straight line and area. The double disk modul (DDM) is used to define trajectories on the metrical surfaces. The geodetical paths on the planet Mars can be realized by the Mars double disk modul (MDDM). The relations of generalized Lobačevskii geometry are derived and related to the Einstein equations. It is considered spherical and pseudospherical geometry, Riemann geometry, Lobačevskii and generalized Lobačevskii geometry, Poincaré model, Beltrami model, gravity as deformation of space and Schwinger theory of gravity.

Key words: Trajectory of elementary particle, Euler's tetrahedron formula, double disk modul, spherical and pseudospherical geometry, Riemann geometry, Lobačevskii and generalized Lobačevskii geometry, Poincaré optical model, Beltrami model, gravity as deformation of a medium, Schwinger theory of gravity.

1 Introduction

Trajectories of elementary particles in the Wilson chamber, ATLAS in LHC, or in the further terrestrial and cosmical detectors are the basic ingredients of physics of elementary particles and cosmical rays. No elementary particle can exist without its trajectory. In the particle physics the trajectories of particle are determined by their parameters as mass, charge, spin, velocity and by the influence of the magnetic and electric fields on its motion. We apply here the notion of path also in geometry to define not only straight line, but all geometrical curves including the Peano line, Weierstrass function, stochastic trajectory of the Brownian particle in the thermodynamical medium and so on.

To be pedagogically clear, we start with the axioms of Euclid. The Euclid postulates in modern language are: 1. Each pair of points can be joined by one and only by one straight line segment. 2. Any straight line segment can be indefinitely extended in their direction. 3. There is exactly one circle of any given radius with any given center 4. All right angles are congruent to one another 5. Given a line and a point not on it, there is exactly one line passing through the given point that is parallel to the given line.

Proclus (ca 400 A.D.) defined the parallel line as a set of points at constant distance from given line on one side from a straight line. At present time we know that this is the definition of the equidistant line.

If the straight line and point in the Euclidean geometry is not correctly defined, then consequences of such definitions are not correct in general.

Jordan introduced the line in geometry as the path of a point moving in a space and making a trace in the space. We here respect the Jordan definition with the restriction that our trajectories are performed by so called double disk modul.

We introduce the double disk modul - DDM - which is the basic element of the kinematic geometry and dynamics of the nonholonomous systems (Nejmark et al., 1967).

The mathematical textbooks, or, monographs are written in the clear form. We use here the exact definition and exact theorem formalism. The theorems are presented without proofs because they are possible to prove by insight, or, by the elementary logic. Every theorem can be improved. It can be proved by the logical way and it can be also proved by computer with the artificial intelligence (AI). Every set of elements can be well ordered. Our set of elements are definitions and theorems. The arrangement of our definitions and theorems is not in the final form, because new and new theorems can be created by the methods of creativity (Pardy, 2005) and by the AI. To our knowledge, the world dictionary of the mathematical theorems was not still published.

2 The Euclidean area and volume

Theorem: The point A and B can be joined by the infinite number of lines passing from A to B.

Theorem: There is the shortest distance between A and B forming the segment of the straight line passing from A to B (Hilbert, 1902).

Theorem: The shortest distance between point A and B can be physically realized by the flexible but non-elastic fibre.

Theorem: The segment AB of a straight line can be prolongated in the direction AB, or BA in order to generate the straight line.

Theorem: The prolongation can be easily performed by ruler.

Theorem: If point C is a such point that it does not lie on the segment AB, then if we define AB = c, BC = a, CA = b, then the area of the triangle ABC is given by the Heron formula

$$P = \sqrt{s(s-a)(s-b)(s-c)}, \quad s = \frac{1}{2}(a+b+c). \tag{1}$$

Theorem: The segment of a straight line is a triangle with zero area.

Theorem: If P = 0, then point C lies on AB , or, on the prolongation of AB. If $P \neq 0$, the point C does not lies on AB, or, on the prolongation of AB.

Theorem: The prolongation can be repeated infinite times to create the straight line.

Theorem: The Euclid plane is formed by three points. It means if the point C is not on the prolongation of line AB, then ABC is an triangle and triangle is the element of Euclid plane.

Theorem: The Euclidean sheet is the mathematical object defined by the Hilbert distance axioms.

Theorem: There are only points, straight lines (straight trajectories) and curved lines (curved trajectories) in the Euclidean sheet defined by the Hilbert distance axioms.

Theorem: Adding the point D to the configuration ABC we get tetrahedron with triangles sides d_{ik}.

Theorem: The volume of the tetrahedron ABCD is given by the Euler formula (Callandreau, 1949):

$$V_E^2 = \frac{1}{288}\begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & d_{12} & d_{13} & d_{14} \\ 1 & d_{21} & 0 & d_{23} & d_{24} \\ 1 & d_{31} & d_{32} & 0 & d_{34} \\ 1 & d_{41} & d_{42} & d_{43} & 0 \end{vmatrix}, \tag{2}$$

where

$$d_{ik} = (x_i - x_k)^2 + (y_i - y_k)^2 + (z_i - z_k)^2, \tag{3}$$

where x_i, y_i, z_i are Cartesian coordinates of points with index i = 1, 2, 3, 4.

Theorem: If a point D is fixed in the space in such a way that $V_E = 0$, then point D lies in the plane ABC.

Theorem: It is possible to construct all points of the Euclid plane using the Euler formula.

Theorem: If point D has a such position with regard to triangle that $V_E \neq 0$, then point D belongs to the 3-dimensional space.

Theorem: The stars forming Cancer, or Crux, or Sagitta, or Libra, are the gigantic cosmological tetrahedrons.

Theorem: The chemical compounds AlF_3, Fe_3C, PH_3, NH_3 form the microscopical tetrahedrons.

Theorem: The nucleus ${}^4_2$He forms the microscopical tetrahedrons.

Theorem: The sub-nuclear tetrahedrons, still not detected by ATLAS, are created during the collisions of protons in LHC, or by collisions of particles in TEVATRON.

Theorem: The sub-nuclear tetrahedrons, still not detected, are integral parts of the cosmical rays.

3 Double disk modul (DDM) and its motion

Definition of DDM: DDM is constructed as a frame AXB with two disks at points A and B and the joint mechanism at point X - so called the bicycle modul. It is supposed for the simplicity that AX = BX = d and the deflection angle of AX from the BX is here denoted as ε. The motion of DDM is supposed to be forward, of backward with regard to vector $\overrightarrow{AX}$. The maximal ε in the direction of clock-vise and anti-clock-vise is $\pi/2$. The trajectory generated by DDM is the track of disk A.

Definition: The side-by-side DDM is defined as an analogue of the DDM with configuration A-X-B with the difference that in the side-by-side modul the elements are arranged as follows: disk A is perpendicular on AX, disk B is perpendicular on BX and AX is parallel with BX. The deflection angle $\varepsilon = 0$.

Theorem: The DDM and the side-by-side DDM is the fundamental component of the kinematic geometry and of the dynamics of the non-holonomous systems (Nejmark et al., 1967).

Theorem: The trajectories generated by the DDM can be realized physically if and only if there is some at least small interaction of the DDM with the plane. The interaction is called friction.

Theorem: The segment AB of line p can be prolongated by DDM.

Theorem: If the deflection angle ε is constant, then the trajectory of the DDM on the Euclidean sheet is a circle.

Theorem: The radius of the circle performed by the DDM with the deflection angle $\varepsilon = \pi/2$ is r = d.

Theorem: Every planar curve can be considered as the trajectory performed by the DDM with variable deflection angle ε.

Theorem: For the symmetrical DDM the trajectory of the disk A is the same as the trajectory of the disk B. Or, $T_{AB} \equiv T_{BA}$.

Theorem: $T_{AB} \neq T_{BA}$ for the non-symmetric DDM.

Theorem: Two linear segments AB and CD are of the same length if the DDM moving from A to B performs the same angle of rotation as in the case it moves from C to D.

Theorem: The trajectory generated by the DDM does not depends on the local velocity and acceleration of DDM.

Theorem: The trajectory generated by the DDM is geodetic for $\varepsilon(t) = 0$ and it is non-geodetic for $\varepsilon(t) \neq 0$.

Theorem: The trajectory generated by DDM moving without friction on the surface is not defined.

Theorem: The Brownian trajectory in statistical physics can be realized by DDM if and only if the deflection angle is stochastic function dependent on time.

Theorem: Trajectory of the disk A of the DDM performed on cone is a circle, or ellipse, or divergent spiral, or convergent spiral.

Theorem: Trajectory of the disk A of the DDM performed on cylinder is a circle, or an ellipse, or the divergent spiral, or the convergent spiral.

Theorem: If p is straight line and disk A of side-by-side DDM follows this line, then disk B, follows the line which is parallel to p.

Theorem: Every area inside of the closed line can be divided by the parallel lines. The division can be performed by side-by-side DDM.

Theorem: If the one disk of DDM is charged and the surface is dielectric medium then the Čerenkov radiation is generated if and only if the velocity of DDM is grater than the speed of light in the dielectric layer.

Theorem: If the one disk of DDM is charged and the surface is graphene sheet, the Smith-Purcell radiation is generated.

Theorem: If the one disk of DDM is charged and the surface is in magnetic field, the synchrotron radiation is generated.

Theorem: If one disk of DDM is charged and the surface is in magnetic field the trajectory of DDM is not geodetic line.

4 Plane trajectories

Theorem: Every plane curve can be realized by the infinitesimally small DDM.

Theorem: Every broken curve can be realized by the DDM.

Theorem: Every Peano curve can be realized by DDM.

Theorem: Every Feynman two dimensional path in quantum mechanics can be realized by DDM.

Theorem: Every Henstock-Kurzweil-Feynman-Pardy two dimensional path in quantum mechanics can be realized by DDM.

Definition: Brownian trajectory of a point particle in the stochastic thermodynamical medium is a fractured curve with no tangent in any point of this trajectory $\mathbf{x}(t)$.

Theorem: The Brownian motion trajectory on a sphere can be realized by DDM if and only if the deflection angle is stochastic.

Theorem: Every classical continual trajectory on sphere can be realized by DDM.

Theorem: The shortest trajectory $y = y(x)$ between two point $A(x_1, y_1)$ and $B(x_2, y_2)$ in the Euclidean plane x-y is $y = ax + b$, a, b being some constants can be find as the solution of the Bernoulli izoperimetric problem of the variational calculus with functional

$$L = \int_{x_1}^{x_2} [1 + y'^2]^{1/2} dx; \quad y' = \frac{dy}{dx} \tag{4}$$

after insertion of it into the Euler-Lagrange equation

$$\frac{\partial L}{\partial y} - \frac{d}{dx}\left(\frac{\partial L}{\partial y'}\right) = 0 \tag{5}$$

and after its solution (Lavrentjev et al., 1950).

Theorem: The massive fiber joining point $A(x_1, y_1)$ and $B(x_2, y_2)$ in the gravitational field is never of the straight line form. It is catenary.

Dirichlet principle: The geometrical form of the catenary is a such that the potential energy of catenary is minimal in the homogeneous gravitational field.

Theorem: The geometrical form of catenary from $A(x_1, y_1)$ to $B(x_2, y_2)$ is determined by the theory of the izoperimetric problem in mechanics. Namely, as problem of the minimum of the potential energy of a massive fiber with length l and linear density $\sigma = 1$, or,

$$V_{potential} = \frac{1}{l} \int_{x_1}^{x_2} y\sqrt{1 + y'^2} dx \tag{6}$$

under the condition the length of the fiber is constant, or

$$\int_{x_1}^{x_2} \sqrt{1+y'^2} dx = l. \tag{7}$$

Theorem: The DDM with the constant deflection angle ε and with AX = XB = d performs on the Euclidean sheet the circular trajectory with the radius

$$r = d/tan(\varepsilon/2). \tag{8}$$

5 Famous plane trajectories

Theorem: DDM can generate all famous plane curves of the analytical geometry. Namely: Astroid, Bicorn, Cardioid, Cartesian, Oval, Cassinian ovals, Catenary, Cayley's sextic, Circle, Cissoid, of Diocles, , Cochleoid, Conchoid, Conchoid of de Sluze, Cycloid, Devil's curve, Double folium, Drer's shell curves, Eight curve, Ellipse, Epicycloid, Epitrochoid, Equiangular Spiral, Fermat's spiral, Folium, Folium of Descartes, Freeth's nephroid, Frequency curve, Hyperbola, Hyperbolic spiral, Hypocycloid, Hypotrochoid, Involute of a circle, Kampyle of eudoxus, Kappa curve, Lam curves, Lemniscate of Bernoulli, Limacon of Pascal, Lissajous curves, Lituus, Neile's parabola, Nephroid, Newton's parabolas, Parabola, Pearls of de Sluze, Pear-shaped quartic, Plateau curves, Pursuit curve, Quadratrix of Hippias, Rhodonea curves, Right strophoid, Serpentine, Sinusoidal spirals, Spiral of Archimedes, Spiric sections, Straight Line, Talbot's curve, Tractrix, Tricuspoid, Trident of Newton, Trifolium, Trisectrix of Maclaurin, Tschirnhaus' Cubic, Watt's curve, Witch of Agnesi.

Theorem: There is a such $\varepsilon(t)$ of the DDM, that DDM generates the function which is continual but not differentiable.

Theorem: There is a such $\varepsilon(t)$ of the DDM, that DDM generates Weierstrass function which is continual but not differentiable. Namely the function

$$f(x) = \sum_{n=0}^{\infty} a^n \cos\left(b^n \pi x\right), \tag{9}$$

where $0 < a < 1$, b is add and $ab > 1 + 3\pi/2$.

Theorem: The Weierstrass function is not trajectory of the Brownian particle which is also continual but not differentiable.

6 The trajectories on the surface

Theorem: If the DDM performs geodetic trajectory from A to B on a surface, then trajectory performed from B to A is also geodetic and it is identical with the former (Parity is conserved).

Theorem: The geodetic line $y = y(x), z = z(x)$ from point $A(x_1, y_1, z_1)$ to point $B(x_2, y_2, z_2)$ on the surface $\varphi(x, y, z) = 0$ is the solution of the izoperimetric problem with the functional

$$F = \int_{x_1}^{x_2} \left\{[1 + y'^2 + z'^2]^{1/2} - \lambda\varphi(x, y, z)\right\} dx; \quad y' = \frac{dy}{dx}, z' = \frac{dz}{dx}, \tag{10}$$

where λ is the Lagrange multiplicator.

Theorem: The DDM can realize every curve on the surface, which is defined by the parametric equation (t = parameter).

$$\mathbf{r} = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k}; \quad u = u(t); \quad v = v(t). \tag{11}$$

Theorem: If $\varepsilon(t)$ in DDM is transcendental function, then the trajectory is also transcendental in some cartesian coordinate system.

Theorem: The trajectory with $-\pi/2 < \varepsilon_1(t) < 0$ is on the left side of the trajectory with $\varepsilon(t) = 0$ and the trajectory with $0 < \varepsilon_2(t) < \pi/2$ is on the right of the trajectory with $\varepsilon(t) = 0$.

Theorem: If $\varepsilon_1(t) = -\varepsilon_2(t)$, the trajectories are symmetrical with regard to the trajectory with $\varepsilon(t) = 0$.

Theorem: The DDM with $\varepsilon = \pm\pi/2$ is a compass.

Theorem: The deflection angle ε is zero if and only if the radius created by the DDM is infinite.

Theorem: The curved trajectory generated by DDM is geodetic line if and only if the deflection angle ε is zero and the plane of A and B are perpendicular to the tangential plane of the surface at the touch point.

Theorem: If the trajectory p generated by DDM starts from point A, then the trajectory q started from point $B \neq A$ is not parallel with the trajectory p in general.

Theorem: If the trajectory p generated by DDM is straight line passing through point A and the trajectory q is straight line passing through point B $\neq$ A, Then p is parallel to q if and only if there is common normal to p and q.

7 The Riemann geometry and gravity

Theorem: On the Riemann sphere with the element $ds^2 = g_{ij}dx^i dx^j$, the sum of the interior angles α, β, γ of a triangle ΔABC is $\neq 2\pi$.

Theorem: There is no linear segment on Riemann surface.

Theorem: The shortest line between point A and B on Riemann surface is the geodetic line.

Theorem: The prolongation of the geodetic segment AB on the Rieman surface can be performed by the infinitesimally narrow flexible nonelastic strip partially identical with segment AB.

Theorem: The Brownian motion trajectory on the Riemann surface can be realized by DDM if and only if the deflection angle is stochastic.

Theorem: The Tartaglia et al. (2009) idea that the curved space-time is the result of the deformation of the Minkowski space-time is correct with the definition

$$g_{\mu\nu} = (\eta_{\mu\nu} + 2\varepsilon_{\mu\nu}) \tag{12}$$

with

$$\varepsilon_{\mu\nu} = \eta_{a\nu}\frac{\partial u^a}{\partial x^\mu} + \eta_{b\mu}\frac{\partial u^b}{\partial x^\nu} + \eta_{ab}\frac{\partial u^a}{\partial x^\mu}\frac{\partial u^b}{\partial x^\nu}. \tag{13}$$

Theorem: If the gravity is the deformation of the Minkowski space-time, then it is not deformation of entropy.

8 The trajectories on a sphere

Theorem: There is no linear segment on a sphere.

Theorem: On a sphere with radius R, the sum of the interior angles α, β, γ of a triangle ΔABC, is $> 2\pi$.

Theorem: On the sphere with radius R the area of a triangle ΔABC is $R^2(\alpha + \beta + \gamma - \pi)$.

Theorem: The shortest line between points A and B on a sphere is the geodetic line.

Theorem: The prolongation of the geodetic segment on sphere cannot be performed by method of euclidean geometry.

Theorem: The prolongation of the geodetic segment AB of sphere can be performed by the strip which is infinitesimally narrow, nonelastic with flexibility perpendicular on the surface. The application of the strip is in the direction of the segment consequently many times.

Theorem: The prolongation of the geodetic segment AB can be performed mechanically by the double disk modul DDM starting its motion on the geodetic line.

Theorem: Every classical continual trajectory (curve) on the sphere can be realized by DDM.

Theorem: The DDM trajectory can be simulated by the computer program.

Theorem: Trajectory of the T_A of the disk A performed on a sphere under the condition that the deflection angle is a constant $\varepsilon = const$, is a circle, but the trajectory is not the geodetic line.

Theorem: The disks A and B of the side-by-side DDM form two trajectories which are equidistant.

Theorem: The disks A and B of the side-by-side DDM forms two trajectories which are equidistant but not parallel in general.

Theorem: It is possible to create the trajectory q which is parallel to the trajectory p by the DDM.

9 The Lobačevskii geometry

The Lobačevskii geometry is the integral part of the general geometry called non-euclidean geometry. The name non-Euclidean was used by Gauss to describe a system of geometry which differs from Euclid's in its properties of parallelism. Such a system was developed independently by Bolyai in Hungary and Lobačevskii in Russia, about 120 years ago. Another system, differing more radically from Euclid's, was suggested later by Riemann in Germany and Schlafli in Switzerland. The subject was unified in 1871 by Klein, who gave the names parabolic, hyperbolic, and elliptic to the respective systems of Euclid, Bolyai-Lobačevskii, and Riemann-Shlafli (Coxeter, 1998).

Definition: (of the angle of parallelism by Lobačevskii) Given a point P and a line q. The Intersection of the perpendicular through P let be Q and PQ = x. The intersection of line p passing through P, with q, let be R and QR = k. Then the angle RPQ for perpendicular distance x

$$\Pi(x) = 2\tan^{-1} e^{-x/k} \tag{14}$$

is known as the Lobačevskii formula for the angle of parallelism.(Coxeter, 1998; Lobačevskii, 1914).

Theorem: It is not excluded that the Lobačevskii geometry was created using the appropriate solid state model.

Theorem: The Tartaglia et al. idea (Tartaglia, et al., 2009) is the analogue of the Beltrami trial to find realistic model for the Lobačevskii geometry.

Theorem: Optical geometry with the variable index of refraction is not the Euclidean geometry.

Theorem: Lobačevskii geometry can be realized by the appropriate plane deformation.

Theorem: Lobačevskii geometry is the specification of the Riemann geometry.

Theorem: If the surface is the Euclidean plane with Euclidean metric, then it is not possible to generate the Lobačevskii geometry on it by DDM with constant deflection angle $\varepsilon(t) \equiv 0$.

Theorem: The Poincaré model of the Lobačevskii geometry is the physical one.

Theorem: According to Hilbert (Hilbert, 1903), it is not possible to realize the Lobačevskii geometry globally on sphere with the constant negative curvature.

Theorem: The Beltrami realization of the Lobačevskii geometry is only partial.

Theorem: The Lobačevskii geometry is the partial geometry on the pseudosphere with the parametric equations

$$x = a \sin u \cos v, \quad y = a \sin u \sin v, \quad z = a\left(\ln \tan \frac{u}{2} + \cos u\right). \tag{15}$$

(Kagan, 1947, ibid. 1948; Kagan, 1955; Efimov, 2004; Norden, 1956; Klein, 2004; Fuks, 1951; Manning, 1963).

Theorem: The pseudosphere is the surface generated by the rotation of the tractrix with equation

$$x = a \sin u, \quad y = 0, \quad z = a\left(\ln \tan \frac{u}{2} + \cos u\right). \tag{16}$$

Theorem: The pseudosphere is in the half geodetic coordinates given by the squared element

$$ds^2 = du^2 + \cosh^2 \frac{u}{a} dv^2. \tag{17}$$

Theorem: The pseudosphere is in the izothermical coordinates given by the Poincaré squared element

$$ds^2 = \frac{a(dx^2 + dy^2)}{y^2}. \tag{18}$$

Theorem: The Leibniz solution of the tractrix problem is as follows:

$$y = a\frac{\ln(a + \sqrt{a^2 - x^2})}{x} - \sqrt{a^2 - x^2}. \tag{19}$$

Theorem: The Lobačevskii formulas for triangles in his geometry follows from the spherical formulas

$$\cos\frac{a}{r} = \cos\frac{b}{r}\cos\frac{c}{r} + \sin\frac{b}{r}\sin\frac{c}{r}\cos A, \tag{20}$$

$$\frac{\sin A}{\sin a/r} = \frac{\sin B}{\sin b/r} + \frac{\sin C}{\sin c/r}, \tag{21}$$

$$\cos A = -\cos B\cos C + \sin B\sin C\cos(a/r). \tag{22}$$

by transformation $r \to ir$, where r is the radius of sphere and a, b, c are lengths of sides of the triangle on the sphere and A, B, C are appropriate angles.

Theorem: The Lobačevskii relations for the triangle on his plane are as follows:

$$\cosh\frac{a}{r} = \cosh\frac{b}{r}\cosh\frac{c}{r} + \sinh\frac{b}{r}\sinh\frac{c}{r}\cosh A, \tag{23}$$

$$\frac{\sin A}{\sinh a/r} = \frac{\sin B}{\sinh b/r} + \frac{\sin C}{\sinh c/r}, \tag{24}$$

$$\cos A = -\cos B\cos C + \sin B\sin C\cosh(a/r). \tag{25}$$

Theorem: If the $g_{\mu\nu}$ is the metrical tensor of the Lobačevskii geometry, then the tensor of energy-momentum follows from the Einstein gravitational equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}. \tag{26}$$

Theorem: The generalized Lobačevskii formulas for triangles in generalized geometry follow from the spherical formulas (20), (21), (22) by transformation $r \to r + i\varrho$. The new formulae are as follows:

$$\cos\varphi_a\cosh\chi_a + \sin\varphi_a\sinh\chi_a =$$

$$[\cos\varphi_b\cosh\chi_b + \sin\varphi_b\sinh\chi_b][\cos\varphi_c\cosh\chi_c + \sin\varphi_c\sinh\chi_c] \quad +$$

$$[\sin\varphi_b\cosh\chi_b + \cos\varphi_b\sinh\chi_b][\sin\varphi_c\cosh\chi_c + \cos\varphi_c\sinh\chi_c]\cos A, \tag{27}$$

$$\frac{\sin A}{\sin\varphi_a\cosh\chi_a + \cos\varphi_a\sinh\chi_a} =$$

$$\frac{\sin B}{\sin\varphi_b\cosh\chi_b + \cos\varphi_b\sinh\chi_b} \quad +$$

$$\frac{\sin C}{\sin\varphi_c\cosh\chi_c + \cos\varphi_c\sinh\chi_c}, \tag{28}$$

$$\cos A = -\cos B\cos C \quad +$$

$$\sin B\sin C[\cos\varphi_a\cosh\chi_a + \sin\varphi_a\sinh\chi_a], \tag{29}$$

where

$$\varphi_a; \varphi_b; \varphi_c; = \frac{ar}{r^2+\varrho^2}; \quad \frac{br}{r^2+\varrho^2}; \quad \frac{cr}{r^2+\varrho^2} \tag{30}$$

and

$$\chi_a; \chi_b; \chi_c; = \frac{a\varrho}{r^2+\varrho^2}; \quad \frac{b\varrho}{r^2+\varrho^2}; \quad \frac{c\varrho}{r^2+\varrho^2} \tag{31}$$

and ϱ is the new parameter of the triangle on the generalized sphere and A, B, C are appropriate triangle angles.

Theorem: If the $g_{\mu\nu}$ is the metrical tensor of the Lobačevskii-Pardy geometry, with $r \to r + i\varrho$, then the tensor of energy-momentum follows from the Einstein gravitational equations (26).

Theorem: It is not excluded that the Lobačevskii-Pardy triangle relations correspond to processes in the particle physics.

Theorem: The space of velocities in the special theory of relativity is the space of the Lobačevskii geometry (Fok, 1961; Landau et al., 1988).

Theorem: If $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$, are velocities of the three massive points in the special theory of relativity, then these velocities form the triangle in the Lobačevskii space of velocities (Fok, 1961).

Theorem: The trajectories of particle components in the decay of the neutral π-meson (Steinberger at al., 1950)

$$\pi^0 \quad \longrightarrow \quad \gamma \quad + \quad \gamma \tag{32a}$$

creates the Lobačevskii angle of parallelism $\Pi < \pi/2$.

Theorem: The trajectories of particle components in the decay of the neutral K_1^0-meson (Cabbibo et al., 1960; Barger, 1964)

$$K_1^0 \quad \longrightarrow \quad \gamma \quad + \quad \gamma \tag{32b}$$

creates the Lobačevskii angle of parallelism $\Pi < \pi/2$.

Theorem: The trajectories of particle components in the decay of the arbitrary neutral meson

$$neutral \quad meson \quad \longrightarrow \quad \gamma \quad + \quad \gamma \tag{32c}$$

creates the Lobačevskii angle of parallelism $\Pi < \pi/2$.

10 The Poincaré model of the Lobačevskii geometry

Fermat's Theorem: The trajectory of light from point A to B in the optical medium is of the shortest optical length.

Theorem: The trajectory of the optical ray from point A to point B with reflection on the mirror in point C is of the shortest optical length in classical optics.

Theorem: The Descartes statement that the angle of the ray reflection by the mirror is equal to the angle of incident ray on the mirror is correct in classical optics.

Theorem: The Descartes statement that the angle of reflection ray is equal to the angle of incident ray on the mirror is not correct in the crystal optics.

Theorem: The trajectory of light passing from $A(x_1, y_1)$ to $B(x_2, y_2)$ is determined by by the Fermat principle, which states that the time from $A(x_1, y_1)$ to $B(x_2, y_2)$ is the result of the minimum of the functional

$$T(y, y') = \int_{x_1}^{x_2} \frac{ds}{v(y)} = \int_{x_1}^{x_2} \frac{\sqrt{1+y'^2}}{v(y)} dx, \tag{33}$$

where $v(y)$ is the velocity of light.

Theorem: The functional $T(y, y')$ in the Euler-Lagrange equations

$$T(y, y') - y'T_{y'}(y, y') = C \tag{34}$$

gives for $v = Ay$ the solution in the form of circles forming the Poincaré model of the Lobačevskii geometry:

$$(x - C)^2 + y^2 = r^2. \tag{35}$$

Theorem: The Poincaré circles can be generated by the DDM.

Theorem: If $v = Ay$ in the optical medium, then the plane (x, y) is the Poincaré realization of the Lobačevskii hyperbolic geometry.

Theorem: The Poincaré theorems for circles in his model are analogue of the theorems for straight lines in the Euclidean geometry.

Theorem: The following theorems are valid in the Poincaré model of the Lobačevskii geometry:

Theorem 1:. Only one half-circle passes through two points A, B in the Poincaré plane.

Theorem 2: The curvilinear segment AB in the Poincaré plane is of the shortest length.

Theorem 3: The parallels are two half-circles with the intersections on the x-axes.

Theorem 4: If point $A \notin q$ then there are $q_1 \parallel q$, $q_2 \parallel q$ passing through A, with $q_1 \neq q_2$.

Theorem 5: If point $A \notin q$, $q_1 \parallel q, q_2 \parallel q$, then q_1, q_2 divide the Poincaré plane in four different sectors I, II, II, IV.

Theorem: The optical distance between point A and B is not equivalent to the mechanical distance realized by the nonelastic flexible fibre, or by the DDM.

Theorem: The geometrical form of the optical trajectory between A and B is not identical with the geometrical form of the string stretched between A and B.

Theorem: Einstein deflection of light by the curved space time is possible to interpret as the interaction of light with modified vacuum by the gravitational field.

Theorem: All gravity effects of Einstein theory with the metrical tensor $g_{\mu\nu}$ follows from the theory where the interaction of light with the gravitational field is described by the Schwinger action for the massless spin 2 fields (Schwinger, 1976):

$$W(T) = \frac{1}{2} \int (dx)(dx') \quad \times$$

$$\left(T^{\mu\nu}(x) D_+(x - x') T_{\mu\nu} x') - \frac{1}{2} T(x) D_+(x - x') T(x') \right). \tag{36}$$

where $T^{\mu\nu}$ is the tensor of energy-momentum, $T = T^{\nu}_{\nu}$ and $D_+(x - x)$ is the causal propagator of the elementary particle with zero mass.

Theorem: Spin 2 as the quantum phenomenon is not possible to derive from Einstein gravity. Similarly the Maxwell-Boltzmann statistical distribution is not possible to derive from the thermodynamic state equation $pV = RT$, and quantum mechanics is not possible to derive from classical mechanics.

Theorem: The Lobačevskii geometry represented by the trajectories in the optical medium where the velocity of light is $v = const.y$ is equivalent to the existence of the index of refraction $n = const.y/c$ in this medium, c being the velocity of light in vacuum.

Theorem: The Poincaré model of geometry where the light velocity is $v = const.y$ is the interaction model of light with the optical medium.

11 Discussion

We have seen in the article how to create the trajectories and geodetic lines by so called double disk modul (DDM) which is the method replacing the older geometrical method in defining the geodetic lines. The modul in the electronic form can be used to realize the geodetical paths on the planet Mars by the Mars double disk modul (MDDM) - and NASA is able to prove it.

The article involves great amount of the geometrical theorems which were formulated using the DDM. Every theorem can be proved by the rigorous mathematical way but with regard to the simplicity of the theorems, the proofs were not performed. We know that the theorems can be created by the logical combinations of the mathematical ideas and mathematical objects. However, according to Poincaré, only some of such combinations are useful. Namely, only beautiful mathematical theorems are useful.

The article is in no case an analogue of the Pascal, Spinoza, Descartes, Husserl, or Wittgenstein meditations, because their meditations cannot be proved by the mathematical way and they are formulated only on the philosophical platform.

Our approach to the geometry, where the trajectories are created by DDM, is in some sense experimental. We are aware that it is not possible to ascertain that a line is straight without making some measurement, or, without sliding along this line an instrument called a ruler which is a sort of measuring instrument. Or, we know, that the Poincaré model of the Lobačevskii geometry is the optical model based on the index of refraction, which is the consequence of the interaction of light with medium.

Also, space, when considered independently of measuring instruments, has neither metric nor projective properties; it has only topological properties. It is amorphous (Poincaré, 1963). That which cannot be measured cannot be an object of science.

Analogically, the properties of time are those of the measuring instruments. The Bergson definition of time as duration could not be tested by an instrument. The Bergsonian duration, is psychological time and it can be influenced by pharmaceutical interaction.

Some theorems in this article will have the substantial influence of the scientific potential of the society and will have substantial influence on the growth of world economy. At the same time it is not excluded that the kinematic geometry based on DDM will start the reformulation of the differential geometry.

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