

Wavepacket detection with the Unruh-DeWitt model

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In this paper we deal with several issues in the localisation of the Unruh-DeWitt detector model. In its original formulation as a pointlike detector, the Unruh-DeWitt model has been used to study extensively the physics of quantum fields in presence of accelerations or curved backgrounds. Natural extensions of the model have tried to take into account the spatial profile of such detectors but all of them have met a series of problems in their spectral response which render them useless to study some of the most interesting physical scenarios. This paper analyses the spectral response of spatially smeared Unruh-DeWitt detectors, discusses the kind of spatial profiles which are useful for the study of relevant scenarios and study in which cases the Unruh-DeWitt model can be effectively used to describe atoms interacting with the EM field.

I. INTRODUCTION

The Unruh-DeWitt model describes a monopole detector coupled to a massless scalar field, moving in the four-dimensional Minkowski space. Since its inception, it has been used to study the response of detectors experiencing acceleration, to provide a proof for the Unruh effect, and particularly as one of the main tools to probe dynamics of entanglement in the context of the recent field of Relativistic Quantum Information (RQI).

Usually, the detector is considered a quantum system with two internal states, ground state $|g\rangle$ and excited state $|e\rangle$, with Ω being (taking $\hbar = 1$) the energy difference between the two levels. The detector is then coupled to the real massless scalar field ϕ according to the following interaction Hamiltonian:

$$H_{\text{int}} = \lambda \xi(\tau) \mu(\tau) \phi(x(\tau)) \quad (1)$$

where λ is the coupling strength, ξ is a switching function which activates during the interaction time, $\mu(\tau)$ the monopole momentum operator and $x(\tau)$ the worldline of the atom.

In spite of the differences between this monopole-scalar field interaction and QED (for instance in the behaviour at very extreme frequencies which may quantitatively vary), it qualitatively models the matter-radiation interaction in some specific settings [1] (see section V for further details), while it very accurately models the interaction of internal degrees of freedom of atoms with phonon fields (for example the spin-phonon interaction of ions in a Coulomb crystal, collective excitations of Bose-Einstein condensates [2] and other solid state and analog systems).

This detector has been commonly analysed under the pointlike approximation, i.e. it has no extension and it interacts with the field only in the exact geometric point of the space-time where it is placed. While this assumption—which will always be an approximation since any physical detector has a finite size—seems to be valid in many scenarios, it is not valid in general even for physically interesting scenarios, and is particularly problematic in some specific settings that we will discuss below. Also,

it presents UV divergences as any pointlike interaction and cannot be guaranteed to hold for any context where we consider several detectors undergoing relativistic motion where the pointlike approximation may be violated from some reference frames. Moreover, additional problems with the pointlike nature of the detector arise. For instance, there are various regularisation schemes which yield different transition probabilities [3].

For all these reasons, and keeping in mind that any realistic particle detector has a finite size, it is important to model and understand particle detectors that present a spatial smearing. However, previous localisation models presented a series of issues when it comes to analysing non-vacuum field states. In this note we intend to provide a pedagogical description of the use of a spatially smeared Unruh-DeWitt model and we will discuss how to overcome the problems of these particle detectors when analysing signals by means of a small but essential modification of the spatial profiles employed in the past. We will also analyse how to implement it for spatially smeared uniformly accelerated detectors and to what extent an Unruh DeWitt detector is a reliable model of electromagnetic atomic transitions.

The paper is organised as follows: in section II we present the localisation issues of the canonical Unruh-DeWitt detector employed in the literature. In section III we propose a way around these difficulties by modifying the spatial profile of the smeared Unruh-DeWitt detector. In section IV we discuss how to use these detectors to analyze arbitrary signals in accelerated settings. Section V shows how to relate the spatial profile of the Unruh-DeWitt model to the wavefunctions of physical systems under standard QED interactions. Finally, section VI contains our conclusions.

II. LOCALISATION ISSUES OF THE UNRUH-DEWITT DETECTOR

The first Unruh-DeWitt localization model was introduced by Schlicht [3] to solve the problems with the non-equivalence of regulators derived from the pointlike na-

ture of the detector. In particular, he proposed a localised spatial profile for the detector (which for computational convenience was chosen to be Lorentzian). This localisation model was further studied by Langlois [5] first, and then by Satz and Louko [6, 7], who envisioned a more general scheme which allowed general spatial profiles to be considered undergoing arbitrary movement throughout spacetime. In these works the interaction Hamiltonian is defined as follows:

$$H_I = g \int_0^\infty \frac{d\mathbf{k}}{\sqrt{2\omega_{\mathbf{k}}(2\pi)^3}} \int d\mathbf{x} F(\mathbf{x}) (\sigma^+ e^{i\Omega t} + \sigma^- e^{-i\Omega t}) \times \left(a_{\mathbf{k}}^\dagger e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega t)} + a_{\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} \right) \quad (2)$$

Where $F(\mathbf{x})$ is the spatial smearing of the detector that is supposed, for simplicity and without loss of generality, at rest and centred in $\mathbf{x} = 0$, and Ω represents the frequency gap of the two-level system, in other words, the transition energy between the ground and excited state of the detector. The detector is supposed to be tuned to this frequency, i.e. it is more likely that the detector absorbs field quanta of this frequency than anything else, as we will discuss below. In the case that the detector is point-like $F(\mathbf{x}) = \delta(\mathbf{x})$, this model becomes the standard Unruh-Dewitt detector introduced in [8].

Previous works dealing with the localised Unruh-Dewitt model just considered the behavior of the detector interacting with the Minkowski vacuum, which is known to have equivalent behavior for all frequencies [6, 7]. In that respect, the problems of the model dealt with in this manuscript have not been studied yet. We will discuss below how they can build up when one tries to process physical signals and photon wavepackets with such a detector.

For most recent analyses [3, 5–7] a real symmetric profile function was chosen. In particular, the spatial profile used for most calculations was a Lorentzian. To illustrate here the problem in the most simple way we will consider a Gaussian profile, but all results apply equivalently to the Lorentzian case or to any other spatial profile.

From the Hamiltonian (2), the integral over \mathbf{x} takes the form of a trivial Fourier transform

$$H_I = g \int_0^\infty \frac{d\mathbf{k}}{\sqrt{2\omega_{\mathbf{k}}(2\pi)^3}} (\sigma^+ e^{i\Omega t} + \sigma^- e^{-i\Omega t}) \times \left(\hat{F}(\mathbf{k}) a_{\mathbf{k}}^\dagger e^{i\omega_{\mathbf{k}} t} + \hat{F}(-\mathbf{k}) a_{\mathbf{k}} e^{-i\omega_{\mathbf{k}} t} \right) \quad (3)$$

where we have made the dispersion relation explicit $\omega_{\mathbf{k}} = c|\mathbf{k}|$ and

$$\hat{F}(\mathbf{k}) = \int d\mathbf{x} F(\mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}} \quad (4)$$

is the Fourier transform of the spatial profile.

We can rewrite the Hamiltonian in a way in which the

resonant and anti-resonant terms are made explicit:

$$H_I = g \int \frac{d\mathbf{k}}{\sqrt{2\omega_{\mathbf{k}}(2\pi)^3}} \left[\hat{F}(\mathbf{k}) \left(a_{\mathbf{k}}^\dagger \sigma^- e^{i(\omega_{\mathbf{k}}-\Omega)t} + \text{H.c.} \right) + \hat{F}(-\mathbf{k}) \left(a_{\mathbf{k}}^\dagger \sigma^+ e^{i(\omega_{\mathbf{k}}+\Omega)t} + \text{H.c.} \right) \right] \quad (5)$$

The time evolution operator is computed as the time ordered exponential of the Hamiltonian. When integrating over times, the exponential factors in the Hamiltonian above are highly oscillating except when $\omega_{\mathbf{k}} = c|\mathbf{k}| \approx \pm\Omega$ (stationary phase). This is the mathematical reason why a detector is tuned to the frequency of the energy gap between the ground and the excited state, as it is very well known from the study of the matter-radiation interactions [1, 9]. In plain words, if we want to stimulate the transition between ground and excited state we have to 'beam' the detector with radiation tuned to the natural frequency of the transition (on resonance). Otherwise, the probability of transition quickly decreases with the detuning between this natural frequency and the frequency of the radiation stimulating the transition.

Here is the issue. If we choose $F(x)$ to be a localised smooth function such as a Gaussian or a Lorentzian, which is the case for most realistic atoms, the frequency profile $F(\mathbf{k})$ will be a localised function centred in $\mathbf{k} = 0$. Being this so, its evaluation at Ω/c will give a negligible value, for Ω sufficiently large.

The reason why this issue does not arise in electronic transitions for atoms at rest is because, for most cases, Ω is small enough. For instance, electronic transitions in the hydrogen atom have an Ω in the visible range of the spectrum, whereas the Fourier transform of the spatial profile has a width of $\sim a_0^{-1}$, which extends up to the X-ray spectrum.

However, when we consider accelerated detectors, the gap frequency Ω very soon leaves the support of the detector frequency profile. This is so, even if we compensate the Doppler shift of the wavepacket so that it is always peaked at a frequency $\approx \Omega$ (from the perspective of the detector), where we would expect the resonance at every time. If the spatial profile function does not have information about the energy gap between the ground and excited state of the detector, the response of the detector to the resonance frequency (the frequency which, by far, mostly contributes to the estimated transition from the ground and excited state) will be exponentially dampened by the Gaussian or Lorentzian tails. That implies that an accelerated detector would be, in practical terms, incapable of detecting a wavepacket centred on its natural frequency. If we are to analyse signals with Unruh-Dewitt detectors, the model should be accordingly modified to avoid this issue.

To illustrate the problem let us consider the most simple 1-D case, and a detector with a Gaussian spatial profile. We can take $F(x)$ to be a normalised Gaussian profile with characteristic length L :

$$F(x) = \frac{1}{L\sqrt{2\pi}} \exp\left(-\frac{x^2}{2L^2}\right) \quad (6)$$

And so its Fourier Transform $\hat{F}(k)$ will be a Gaussian localised around $k = 0$

$$\hat{F}(k) = \exp\left(\frac{-k^2 L^2}{2}\right) \quad (7)$$

Any frequencies such that $\omega_k \gg 0$ would be exponentially dampened in the integral over k by the weight $\hat{F}(k)$. In particular, if $\Omega \gg 0$, the stationary phase contribution $\omega_k = \pm\Omega$ will be zero due to $F(\pm\Omega/c) \approx 0$, effectively cancelling any non-trivial time evolution.

So, as it is illustrated in fig. 1, if $\Omega \gg cL^{-1}$ the detector will not ever detect any signal even if it is a powerful pulse tuned to the transition frequency. Therefore, in order to be able to study relativistic settings, some modifications must be made to the model.

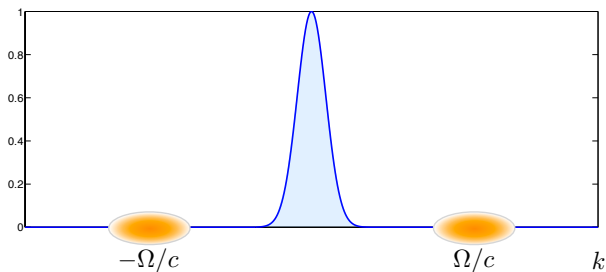


Figure 1. A highly localised $\hat{F}(k)$ centred in 0 would practically suppress the possibility of detection for the resonance frequencies to which the detector is most responsive, $k = \pm\Omega/c$. This results in a vanishing transition probability no matter what frequency we use to illuminate the detector.

One could argue that if the detector is very small with respect of the wavelength to which it is tuned (as it is the case of atoms), the Gaussian profile $\hat{F}(k)$ may cover the resonance regions. However, as seen in figure 2, if we analyze the probability of transition as a function of the frequency of the radiation with which the detector interacts, its spectral response will be asymmetric in the detuning between the detector natural frequency and the frequency of the radiation stimulating the transition $\Delta = \omega_k - \Omega$.

In other words, if the transition frequency is Ω and the radiation stimulating the transition is detuned from the energy gap of the detector by a small factor δ , the probability of transition will be positively weighed by $\hat{F}(k)$ if $\omega_k = \Omega - \delta$, and dampened if $\omega_k = \Omega + \delta$.

Although a similar asymmetry occurs in realistic atomic transitions (as detailed in section V), the effect is so small that it can be neglected in most circumstances. In practice, no such effects are observed neither in atomic detectors nor in any other settings where quantum systems (like harmonic oscillators) are coupled to quantum fields.

When the size of the detectors increases as to become comparable with the wavelength to which they are tuned, e.g. quantum microwave antennas (for example flux or charge qubits in cQED), Rydberg atoms and cavity based

detectors [10, 11], the detector response is also symmetric in frequencies so the use of the Unruh DeWitt detector presented above to model those scenarios (where the spatial profile is related to the natural dimension of the detector) can be problematic.

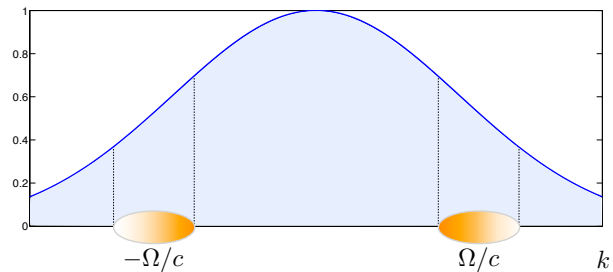


Figure 2. A not-so localised $\hat{F}(k)$ centred in 0 would introduces an asymmetry in the detection of frequencies $\omega_k = \Omega \pm \delta$ $k = \pm\Omega/c$

III. MODULATED OSCILLATIONS IN THE SPATIAL PROFILE

In most realistic settings, the spectral response function of two level emitters is symmetric with respect to the resonance frequency, thus a small detuning should produce similar effects no matter if it is positive or negative. Also, as we discussed above, if the two level system size is comparable with the wavelength it is tuned to, the localized Unruh-DeWitt model employed in the literature will dramatically fail to detect anything, even if it is the case of an intense pulse of radiation centred in the natural frequency of the detector's transition.

Taking these issues into account, we propose a modification of the way in which the Unruh-Dewitt detector is spatially smeared. We will do so by feeding the spatial profile with information about the resonance frequency. For that matter, we will introduce a spatial profile which is strongly localized by a function $S(x)$, modulated by internal oscillations associated with the frequency the two level system is tuned to.

If the spatial profile is

$$F(x) = S(x) \cos\left(\frac{\Omega x}{c}\right) \quad (8)$$

then the spectral profile would be

$$\hat{F}(k) = \frac{1}{2} \left[\hat{S}(k - \Omega/c) + \hat{S}(k + \Omega/c) \right] \quad (9)$$

which is a localised profile in frequencies around the two resonance regions. If we take $S(x)$ to be the Gaussian profile (6) then

$$\hat{F}(k) = \frac{1}{2} \left(e^{\frac{1}{2}(k-\Omega/c)^2 L^2} + e^{-\frac{1}{2}(k+\Omega/c)^2 L^2} \right) \quad (10)$$

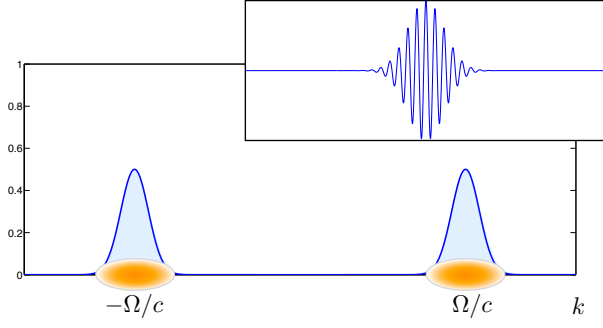


Figure 3. A localised $\hat{F}(k)$ can be not centered in 0 by introducing a oscillating term in the spatial profile seen in the inset. The figure shows symmetric detection zones may appear centered in the frequencies $k = \pm\Omega/c$

which, as seen in figure 3, covers symmetrically the resonance regions.

By doing this we have the desired spectral response no matter the value of Ω , and the detector is spatially localized around $x = 0$ with a characteristic proper length L .

IV. ACCELERATED DETECTORS

In order to provide a complete description of the localised detector model proposed in this note, in this section we will describe how to use this model to analyse arbitrary signals with a spatially smeared uniformly accelerated detector.

There is a well known problem with accelerating rigid bodies: the proper distance between two points of a solid accelerating with the same relativistic acceleration increases with time, eventually destroying the solid when the internal tension it supports is overrun by the relativistic effects.

The reasonable hypothesis for a physical detector is that it has to keep internal coherence. This means that the internal forces that keep the detector together will prevent it from being further smeared due to relativistic effects up to some reasonable acceleration regimes. That means that, effectively, every point of the detector will accelerate with a different acceleration in order to keep up with the rest of its points. The natural formalism to treat this detector is the use of the well-known Fermi-Walker coordinates [3, 12].

Thus, the interaction Hamiltonian of a smeared uniformly accelerated rigid detector is

$$H_I(t) = g \int \frac{dk}{\sqrt{2\omega_k(2\pi)}} \int d\chi F(\chi) (\sigma^+ e^{i\Omega\tau} + \sigma^- e^{-i\Omega\tau}) (a_k^\dagger e^{i(\omega_k t(\chi, \tau) - kx(\chi, \tau))} + a_k e^{-i(\omega_k t(\chi, \tau) - kx(\chi, \tau))}) \quad (11)$$

where $\chi = (\chi, 0, 0)$ and τ are the Fermi-Walker coordinates associated with the trajectory of the detector.

These coordinates have the particularity that at every point on the trajectory $x(\tau) = (ct(\tau), x(\tau), 0, 0)$ the hyperplane which is orthogonal to the 4-velocity $u(\tau) = (ct(\tau), \dot{x}(\tau), 0, 0)$ is the three-dimensional space which consists of all the events which are simultaneous to $x(\tau)$, where simultaneity is judged from the comoving inertial frame. We assume that we move only in one direction, so that $\chi_1 = \chi, \chi_2 = y = 0, \chi_3 = z = 0$.

If we attach a dreibein to every such hyperplane

$$e_{\chi_1} = (c^{-1}\dot{x}(\tau), \dot{t}(\tau), 0, 0) \\ e_{\chi_2} = (0, 0, 1, 0), \quad e_{\chi_3} = (0, 0, 0, 1), \quad (12)$$

we can characterise every event x_e in a neighborhood of the trajectory with (τ_e, χ_e) .

These coordinates guarantee a rigid detector (where rigidity means that its 3-geometry as seen from its own momentary rest system is unchanged in the course of proper time). In contrast, in a Rindler frame (standard approach for pointlike detectors) every point of the detector accelerates with a different proper acceleration, so they cannot account for rigid detectors that have internal coherence. In the F-W frame the detector will accelerate coherently, so this models very well what would happen to an accelerated rigid-body.

The change of coordinates between the inertial system to the Fermi-Walker frame is given by

$$\mathbf{x}(\tau, \chi) = \mathbf{x}(\tau) + \chi^i \mathbf{e}_i(\tau), \quad t(\tau, \chi) = t(\tau) + \frac{\chi^i e_i^0}{c} \quad (13)$$

For the uniformly accelerated observer, the trajectory (parametrised in terms of comoving time) is

$$x(\tau) = \left[\frac{c^2}{a} \sinh\left(\frac{a\tau}{c}\right), \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right), 0, 0 \right] \quad (14)$$

The only relevant component of the dreibein is

$$e_{\chi_1} = \left[\sinh\left(\frac{a\tau}{c}\right), \cosh\left(\frac{a\tau}{c}\right), 0, 0 \right] \quad (15)$$

So, directly from (13) we read the change of coordinates

$$t(\tau, \chi) = \left(\frac{c}{a} + \frac{\chi_1}{c} \right) \sinh\left(\frac{a\tau}{c}\right) \\ \mathbf{x}(\tau, \chi) = \left[\left(\frac{c^2}{a} + \chi_1 \right) \cosh\left(\frac{a\tau}{c}\right), \chi_2, \chi_3 \right] \quad (16)$$

Within this scheme we compute the probability of excitation of an accelerated detector responding to an arbitrary signal. In first order perturbation theory,

$$P = |g|^2 \int_{\tau_0}^{\tau} d\tau' \int_{\tau_0}^{\tau} d\tau'' e^{i\Omega(\tau' - \tau'')} \langle y | \Psi(\tau'') \Psi(\tau') | y \rangle \quad (17)$$

$$\Psi(\tau) = \int \frac{F(\chi) dk d\chi}{\sqrt{2c|k|(2\pi)}} \langle y | \left(a_{\mathbf{k}} e^{i(\mathbf{k} \cdot \mathbf{x}(\chi, \tau) - c|k|t(\chi, \tau))} + \text{H.c.} \right) | y \rangle \quad (18)$$

$|y\rangle$ is a general superposition of plane-wave field modes corresponding to a Minkowskian-shaped wavepacket prepared in the lab that we want to analyse with our detector

$$|y\rangle = \left(\int dk y(k) a_k^\dagger \right) |0\rangle \quad (19)$$

Let us evaluate the time-correlation function $W_y(\tau', \tau'') \equiv \langle y | \Psi(\tau'') \Psi(\tau') | y \rangle$. The two χ integrals can be rewritten in terms of Fourier transforms greatly simplifying the expression of $W_y(\tau', \tau'')$. To do this we first note that

$$kx(\chi, \tau) - c|k|t(\chi, \tau) = L(k, \tau) \left(\chi + \frac{c^2}{a} \right) \\ L(k, \tau) = k \cosh \left(\frac{a\tau}{c} \right) - |k| \sinh \left(\frac{a\tau}{c} \right) \quad (20)$$

Now if we define $G^\pm(k, \tau) = \hat{F}[\pm L(k, \tau)]$, where $\hat{F}(k)$ is the Fourier transform of $F(\chi)$ as in (4), we can rewrite $W_x(\tau', \tau'') =$

$$= \int \frac{\bar{y}(k)y(\kappa)dkd\kappa}{2(2\pi)c\sqrt{|k||\kappa|}} G^+(k, \tau'') G^-(\kappa, \tau') e^{i\frac{c^2}{a}[L(k, \tau'') - L(\kappa, \tau')]} \\ + \int \frac{|y(\kappa)|^2 dk d\kappa}{2(2\pi)c|k|} G^+(k, \tau') G^-(k, \tau'') e^{i\frac{c^2}{a}[L(k, \tau'') - L(k, \tau')]} \\ + \int \frac{\bar{y}(k)y(\kappa)dkd\kappa}{2(2\pi)c\sqrt{|k||\kappa|}} G^+(\kappa, \tau') G^-(k, \tau'') e^{i\frac{c^2}{a}[L(k, \tau'') - L(\kappa, \tau')]}$$

which can be further simplified if $F(k) = F(-k)$ (true for a Gaussian or Lorentzian profile), then we get $G^+ = G^- = G$ (although in general $G(k) \neq G(-k)$), and if the frequency profile of the signal $y(\omega)$ we want to analyse is chosen to be real, we can rewrite $W_x(\tau', \tau'') =$

$$= \int \frac{y(k)y(\kappa)dkd\kappa}{(2\pi)c\sqrt{|k||\kappa|}} G(k, \tau'') G(\kappa, \tau') \cos \left[\frac{L(\kappa, \tau') - L(k, \tau'')}{ac^{-2}} \right] \\ + \int \frac{[y(\kappa)]^2 dk d\kappa}{2(2\pi)c|k|} G(k, \tau') G(k, \tau'') e^{i\frac{c^2}{a}[L(k, \tau'') - L(k, \tau')]},$$

providing an operative expression for the response of a localized accelerated detector to a given signal.

V. THE UNRUH DEWITT DETECTOR TO MODEL EM TRANSITIONS

An Unruh-DeWitt detector is an ad-hoc model commonly used to study academic problems in field theory and noninertial settings. The model is built specifically for its useful properties and simplicity. While desirable traits are good guidelines for model building, one should always keep the physics in mind. This section is concerned with the build up of a smeared Unruh-DeWitt detector out from first principles and standard QED interactions.

First, note that the simple scalar field model (1) cannot be directly used to relate the Unruh-DeWitt model to

electromagnetic phenomena due to the vector character of the photon field. The vector version of an Unruh-DeWitt interaction with a smeared field operator would be

$$H_I = \sum_{\lambda=+,-} \int d\mathbf{x} \lambda [\mathbf{F}(\mathbf{x}) \sigma^+ + \mathbf{F}^*(\mathbf{x}) \sigma^-] \cdot \mathbf{A}(\mathbf{x}) \quad (21)$$

where we have omitted any switching function, as the electromagnetic interaction cannot be switched, and where σ^- is the two-level system lowering operator, as is common in the literature. We have also allowed for a complex profile function. The detector is assumed to be inertial; we discuss the treatment of an accelerated Unruh-DeWitt detector in appendix IV.

The physical system the Unruh-DeWitt detector tries to emulate is that of a two-level atom coupled to a quantum field through a dipolar interaction. The Hamiltonian for such a system is well-known and it is simply

$$H_I^{\text{QED}} = e \mathbf{p}_D \cdot \mathbf{A}(\mathbf{x}, 0) \\ = \mathbf{p}_D \cdot \sum_{\lambda=+,-} \int \frac{d\mathbf{p}}{\sqrt{2p}} \left[\epsilon_{\mathbf{p}, \lambda} a_{\mathbf{p}, \lambda}^\dagger e^{-ipx} + \epsilon_{\mathbf{p}, \lambda}^* a_{\mathbf{p}, \lambda} e^{ipx} \right], \quad (22)$$

where \mathbf{p}_D is the detector momentum and in the last two equalities we assume a (1+1)-dimensional setting. In this setting, p_D is itself an operator, the momentum operator of the valence electron of the two-level system. There is a simple way to relate (22) to (21); we simply write down the operator in (22) in terms of field operators and atomic Pauli matrices. There are four possible matrix elements for the $\mathbf{p}_D \mathbf{A}(\mathbf{x}, 0)$ operator in terms of the relevant wavefunctions, $\Psi_g(\mathbf{x})$ for the ground state and $\Psi_e(\mathbf{x})$ for the excited state of the detector, which can be neatly written into matrix form as,

$$H_I^{\text{QED}} = \alpha \mathbf{I} + \beta \sigma_z + \gamma \sigma_x + \delta \sigma_y, \\ \alpha = e \sum_{\lambda=+,-} \int \frac{d\mathbf{p}}{\sqrt{2p}} \left[a_p^\dagger \frac{G_{gg}^\lambda(\mathbf{p}) + G_{ee}^\lambda(\mathbf{p})}{2} + \text{H.c.} \right], \\ \beta = e \sum_{\lambda=+,-} \int \frac{d\mathbf{p}}{\sqrt{2p}} \left[a_p^\dagger \frac{G_{gg}^\lambda(\mathbf{p}) - G_{ee}^\lambda(\mathbf{p})}{2} + \text{H.c.} \right], \\ \gamma = e \sum_{\lambda=+,-} \int \frac{d\mathbf{p}}{\sqrt{2p}} \left[a_p^\dagger \frac{G_{ge}^\lambda(\mathbf{p}) + G_{eg}^\lambda(\mathbf{p})}{2} + \text{H.c.} \right], \\ \delta = e \sum_{\lambda=+,-} \int \frac{d\mathbf{p}}{\sqrt{2p}} \left[a_p^\dagger \frac{G_{ge}^\lambda(\mathbf{p}) - G_{eg}^\lambda(\mathbf{p})}{2i} + \text{H.c.} \right], \quad (23)$$

with

$$G_{ij}^\lambda(\mathbf{p}) = \int d\mathbf{x} e^{-ipx} \epsilon_{\mathbf{p}, \lambda} \cdot (\Psi_i^*(\mathbf{x}) [-i \nabla \Psi_j(\mathbf{x})]). \quad (24)$$

If we performed the same calculation with the interaction (21), we would have obtained

$$G_{ij}^\lambda(\mathbf{p}) = [\delta_{ig}\delta_{je} + \delta_{ie}\delta_{jg}] \int d\mathbf{x} e^{-i\mathbf{p}\cdot\mathbf{x}} \boldsymbol{\epsilon}_{\mathbf{p},\lambda} \cdot \mathbf{F}(\mathbf{x}). \quad (25)$$

We have thus expressed the physical interaction hamiltonian H_I^{QED} in the language of (21). If for a moment we only consider the σ_x and σ_y terms, we may compare directly to (21). From (24) and (25) we find that the two Hamiltonians are equivalent with a smearing function

$$\mathbf{F}(\mathbf{x}) = -i\Psi_e^*(\mathbf{x})\nabla\Psi_g(\mathbf{x}). \quad (26)$$

We have thus made a first connection between (21) and the physics - we got our smearing function in terms of the atomic wavefunctions of the two-level system. Note that the terms with \mathbf{I} and σ_z do not vanish and can never do so unless $\Psi_e = \Psi_g = 0$.

So while the smearing function is indeed determined in a simple way from the physics of the system, we still have two terms not included in the original Unruh De Witt model.

Dealing with α is easiest as it can be reabsorbed into the free field Hamiltonian H_F ,

$$H_F + \alpha = \int d\mathbf{p} \left[(|p|a_{\mathbf{p}}^\dagger a_{\mathbf{p}} + \frac{1}{\sqrt{2p}} \left(a_{\mathbf{p}}^\dagger \frac{G_{gg}^\lambda(\mathbf{p}) + G_{ee}^\lambda(\mathbf{p})}{2} + a_{\mathbf{p}} \frac{G_{gg}^\lambda(\mathbf{p})^* + G_{ee}^\lambda(\mathbf{p})^*}{2} \right) \right] \quad (27)$$

and so defining new modes

$$b_{\mathbf{p}} = a_{\mathbf{p}} + \frac{e}{(2p)^{3/2}} [G_{gg}^\lambda(\mathbf{p}) + G_{ee}^\lambda(\mathbf{p})] \quad (28)$$

and neglecting the usual infinite zero-point contribution, we deal with the α term. We only have to substitute the $a_{\mathbf{p}}$ in terms of the $b_{\mathbf{p}}$ in γ , which amounts to the addition of a constant term to γ ,

$$\alpha_\gamma = \frac{e^2}{4} \Re \left\{ \int \frac{d\mathbf{p}}{p} [G_{gg}^\lambda(\mathbf{p})^* + G_{ee}^\lambda(\mathbf{p})^*] (G_{ge}^\lambda(\mathbf{p}) + G_{eg}^\lambda(\mathbf{p})) \right\}. \quad (29)$$

This will induce an extra $\alpha_\gamma \sigma_x$ term in the Hamiltonian, which will be relevant or not depending on how α_γ compares with Ω , the detector system gap. As α_γ/e is typically of order 1 or less, this term will not be important if we are in a perturbation theory regime where the coupling e is assumed to be small. The same considerations apply to α_δ . The analogous correction to β ,

$$\alpha_\beta = \frac{e^2}{4} \Re \left\{ \int \frac{d\mathbf{p}}{p} [G_{gg}^\lambda(\mathbf{p})^* + G_{ee}^\lambda(\mathbf{p})^*] (G_{gg}^\lambda(\mathbf{p}) - G_{ee}^\lambda(\mathbf{p})) \right\}, \quad (30)$$

can be reabsorbed into Ω .

Dealing with β is a more challenging matter. We cannot do the same as before because even though we could make the Hamiltonian look like that of a free field plus an Unruh-DeWitt interaction, the detector and field operators would not commute and hence even without the interaction the theory would not be a free theory.

There is one special circumstance in which β vanishes: in systems with a strong spin interaction, so that the gap comes from the spin dependence of the energy levels. This could happen, for instance, in states of an atom within a strong magnetic field. In this case the atomic wavefunctions of the ground and excited states are the same and therefore $\beta = 0$ exactly. The energy gap would be $\hbar\Omega = \mu_B B$. The coupling constant to the electric field is $\approx ed$ where d is a typical dimension of the atom, so in order to be in perturbation theory regime we require electric fields of order $E < \mu_B B / ed$.

As a particular example, consider the smearing function for a hydrogen atom in its 1s state subjected to a magnetic field. According to (26), it is

$$\mathbf{F}(\mathbf{x}) = -i \frac{e^{-r/a_0}}{\pi a_0^4} \mathbf{u}_r. \quad (31)$$

Note in particular that the Fourier transform of this function is peaked at 0, so it will not be useful to study the quantum effects of acceleration, as argued above.

VI. CONCLUSIONS

We have discussed that in order to respond to a given frequency, the spatial profile cannot be chosen arbitrarily. Some information about the spectral response of the detector must be fed to the spatial profile, or otherwise the detector will not have the expected behaviour and may dramatically fail to detect radiation on resonance with the two-level system transition. We have also related the spatial profile of the detector system with physical properties of the detector.

To solve the problems, we suggest to introduce a spatial oscillation of the profile, which will make the detector tune to the resonance frequency regardless of its size and configuration.

Not all the spatial profiles for the Unruh-DeWitt model would be compatible with the experimental response of accelerated particle detectors: the existence of some monopole (or dipolar) momentum that couples the atom to the field with a given characteristic transition frequency requires those oscillations introduced in the spatial profile to reproduce spectra centred in the characteristic transition frequency of the detector. If one thinks of that profile as being something like a charge distribution, then those oscillations would be the responsible for the appearance of the momentum that correctly couples it to the field.

Completing our proposal, we have explained how to use this formalism while calculating the probability of detection of a wavepacket for an accelerated detector and we

have related the smeared Unruh-DeWitt model to the usual $\mathbf{p} \cdot \mathbf{A}$ form of the QED interaction coupling atoms to the electromagnetic field. We find that this interaction can indeed be rewritten as an smeared Unruh-DeWitt interaction and provide an explicit expression for the spatial profile of the detector, which is associated with the electronic wavefunction of the relevant orbitals, although this profile is complex in general.

Finally note that, in parallel with this work, an analysis of the transition rates of smeared Unruh-Dewitt detectors coupled to different kinds of physical field modes

and undergoing different relativistic motion is being carried out by Lee and Fuentes, and it is scheduled to appear elsewhere simultaneously with this manuscript [4].

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