The Source Theory of the Formfactor

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Abstract

We consider the situation when particles interact during their propagation between sources. There are two possibilities of the interaction. In the first case the ψ-particles interact by the exchange of the virtual space-like particle φ. In the second case the two outgoing ψ-particles recombine to produce the modified propagation function. The consequence of such processes is the non-local distribution of charge, or, the existence of the formfactor in the Rosenbluth theory.

1 Introduction

The structure of the proton is one of the defining problems of QCD. The electromagnetic structure can be expressed in terms of the electric and magnetic form factors, G_E and G_M, which depend only on the four momentum transfer squared, Q^2. They have traditionally been determined utilizing the Rosenbluth formula (Qattan et al., 2005) for elastic $e-p$ scattering:

$$\frac{d\sigma}{d\Omega_e} = \frac{\sigma_{Mott}}{\varepsilon(1+\tau)}[\tau G_M^2(Q^2) + \varepsilon G_E^2(Q^2)], \tag{1}$$

where is $\tau = Q^2/4M_p^2$ and $\varepsilon = 1/[1+2(1+\tau\tan^2(\theta_e/2)]$ is the virtual photon polarization parameter, M_p is the proton mass, and θ_e is the electron scattering angle. The formfactors are related to the spatial distributions of the charge and magnetization in the proton, and in the non-relativistic limit are simply the Fourier transforms of these distributions (Qattan et al., 2005).

Let us remark with Muirhead that the experimental data can be analysed to yield an electromagnetic shape for the nucleon, although the physical meaning of the result is ambiguous especially for high momentum transfers (Muirhead, 1965).

2 The source method formulation of the formfactor

Our goal is describe the formfactor using the source methods in quantum field theory (Dittrich, 1978; Schwinger, 1970; 2018), where particles interact during their propagation between sources. There are two possibilities of the interaction. In the first case the ψ-particles interact by the exchange of the virtual space-like particle φ. In the second case the two outgoing ψ-particles recombine to produce the modified propagation function. The consequence of such processes is the non-local distribution of charge, or, the existence of the formfactor (Dittrich, 1978).

So, we will consider the situation when particles interact during their propagation between sources. There are two simple possibilities of the interaction between particles.

We consider the process a) of the emission of the two ψ-particles which interact during propagation by the exchange of the virtual space-like particle φ. The process b) describes the situation where two outgoing ψ-particles recombine to produce the modified propagation function. We can extract the processes and unify them.

The vacuum-vacuum amplitudes of both processes a) and b) are generated by the interaction Lagrangian. We take it in the form:

$$\mathcal{L}_{int} = g\psi^*\psi\varphi = j\varphi; \quad j = g\psi^*\psi. \tag{2}$$

In order to represent the both processes by means of the mathematical symbols, let us notice that with regard to the principle of superposition, it is

$$\psi = \psi_1 + \psi_2 \tag{3}$$

for the total field ψ. Then,

$$j = j_{11} + j_{22} + j_{12}, \tag{4}$$

where

$$j_{12} = g(\psi_1^*\psi_2 + \psi_1\psi_2^*) \tag{5}$$

$$j_{11} = g\psi_1^*\psi_1; \quad j_{22} = g\psi_2^*\psi_2. \tag{6}$$

Now, we extract from the total vacuum amplitude

$$\langle 0_+|0_-\rangle = \frac{i}{2}\int j^*\Delta_+^\mu j, \tag{7}$$

those terms, which correspond to the processes a) and b). Or,

$$\langle 0_+|0_-\rangle^{(a)} = \int (dx)(dx')j_{12}^*(x)\Delta_+^\mu(x-x')j_{12}(x') \tag{8}$$

$$\langle 0_+|0_-\rangle^{(b)} = i\int (dx)(dx')j_{11}^*(x)\Delta_+^\mu(x-x')j_{22}(x'). \tag{9}$$

All the other terms in (7) have indexes which do not correspond to the emission, or, absorption processes.

The effective source of emission is given as

$$iH_2(x)H_2^*(x')|_{eff\ emis} = g\delta(x-x')\varphi_2(x). \tag{10}$$

On the other hand, the absorption process is involved in the term

$$e^{i\int H_1^*\psi_2}\, e^{i\int H_1\psi_2^*} \to -\int \psi_2^*(x)H_1^*(x)H_1(x')\psi_2(x'), \tag{11}$$

which is to be compared with

$$\langle 0_+|0_-\rangle^{(a)} = \frac{i}{2}g^2\int (dx)(dx') \quad \times$$

$$\left(\psi_1\psi_2^* + \psi_1^*\psi_2\right)(x)\Delta_+(x-x')\left(\psi_1^*\psi_2 + \psi_1\psi_2^*\right)(x') \tag{12}$$

in order to get

$$iH_1^*(x)H_1(x')|_{eff\ abs} = g^2\psi_1^*(x)\Delta_+^\mu(x-x')\psi_1(x'). \tag{13}$$

The vacuum amplitude of the two-particle exchange via a μ-particle is given by eq. (11) as follows:

$$\langle 0_+|0_-\rangle = \int \psi_2 iH_1^*H_1\psi_2^* \quad =$$

$$-\int H_1^*(x)\Delta_+^m(x-x')H_2(x')H_2^*(y')\Delta_+^m(y-y')H_1(y) \quad \to$$

$$-\int iH_1^*(x)H_1(y)|_{eff\ abs}\ \Delta_+^m(y'-y)\Delta_+^m(x-x')iH_2^*(y')H_2(x')|_{eff\ emiss}, \tag{14}$$

which gives after insertion the effective sources from (14) and (13) the following result:

$$\langle 0_+|0_-\rangle \quad = \int \psi_1^*(x)g\Delta_+^m(x-\xi)g\varphi_2(\xi)\Delta_+^m(\xi-x')g\psi_1(x')\Delta_+^\mu(x'-x). \tag{15}$$

Now, we use the momentum representation for symbols in the vacuum amplitude. In such a way,

$$\Delta_+^m(x-\xi) = i\int d\omega_p e^{ip(x-\xi)} \tag{16}$$

and

$$x' < x'': \quad \psi_1(x') = i\int (dx'')\left[\int d\omega_p e^{ip(x'-x'')}\right]H_1(x'') \quad =$$

$$i\int d\omega_p e^{-ipx''}\int (dx'')e^{ipx''}H_1(x'') \quad =$$

$$i\int d\omega_p e^{-ipx'}H_1(-p)=\sum_p\sqrt{d\omega_p}e^{-ipx'}i\sqrt{d\omega_p}H_1(-p)=\sum_p d\omega_p e^{ipx'}\langle 0_+|1_p\rangle^{H_1} \tag{17}$$

$$\psi_1^*(x)=i\int d\omega_p e^{-ipx}H_1^*(-p) \tag{18}$$

$$\int d\xi\varphi_2(\xi)e^{i\xi(p+p')}=\varphi_2;\quad k=p+p'. \tag{19}$$

Inserting Δ_+^m, $\psi_1^*(x)$ and $\psi_1(x')$ into $\langle 0_+|0_-\rangle$ in eq. (15) we find the expression

$$\int(dx)(dx')e^{i(p-p_1)}\Delta_+^\mu(x-x')e^{i(p'-p_1')x'}=\frac{1}{(p_1-p)^2+\mu^2}\int(dx)e^{i(p+p'-p_1-p_1')x}=$$

$$\frac{1}{(p_1-p)^2+\mu^2}(2\pi)^4\delta(p+p'-p_1-p_1'). \tag{20}$$

Inserting

$$1=\int(2\pi)^3\delta(p+p'-k)\frac{(dk)}{(2\pi)^3};\quad \frac{(dk)}{(2\pi)^3}=d\omega_k dM^2 \tag{21}$$

into eq. (20), we get the combination

$$(2\pi)^4\delta(p+p'-p_1-p_1')=(2\pi)^4\delta(p_1+p_1'-k)d\omega_k dM^2\delta(k-p-p') \tag{22}$$

and finally

$$\langle 0_+|0_-\rangle^{(a)}\int d\omega_k dM^2 d\omega_{p_1}d\omega_{p_1'}(2\pi)^4\delta(p_1+p_1'-k)\quad\times$$

$$g\varphi_2(k)H_1^*(-p_1)I(M^2)H_1(-p_1'), \tag{23}$$

where

$$I(M^2)=g^2\int d\omega_p d\omega_{p'}(2\pi)^3\delta(p+p'-k)\frac{1}{(p_1-p)^2+\mu^2} \tag{24}$$

is a scalar function and it can be evaluated in any frame, for instance in the rest frame of k.In this frame we have $\mathbf{k}=0, k^0=M$, because $k^2=-M^2$ and $(p+p')^2=k^2$.

The evaluation of $I(M^2)$ gives (Dittrich, 1978).

$$I(M^2)=\frac{g^2}{16\pi^2}\frac{\left(1-\frac{4m^2}{M^2}\right)}{M^2-4m^2}\ln\frac{m^2-4m^2-\mu^2}{\mu^2}. \tag{25}$$

Now, we can write the vacuum amplitude of the situation in a) in the form

$$\langle 0_+|0_-\rangle^{(a)}=\int d\omega_k dM^2 d\omega_{p_1}d\omega_{p_1'}\int(dx)e^{i(k-p_1-p_1')x}\quad\times$$

$$g\varphi_2(k)H_1^*(-p_1)I(M^2)H_1(-p_1')\quad=$$

$$i\int dM^2 I(M^2)\int (dx)(dx')j(x)\left[\Delta_+(x-x',M^2)+C.T.\right]\varphi(x'), \tag{26}$$

where C. T. is contact term determined by the requirement that the action appropriate to photons $k^2=0$ must not be altered. This can be achieved by relation

$$\Delta_+(x-x',M^2)-\frac{1}{M^2-\mu^2}\delta(x-x') \quad =$$

$$-\frac{1}{M^2-\mu^2}(-\partial^2+\mu^2)\Delta_+(x-x',M^2), \tag{27}$$

or, in the momentum representation

$$\frac{1}{k^2+M^2-i\varepsilon}-\frac{1}{M^2-\mu^2}=-\frac{k^2+\mu^2}{M^2-\mu^2}\ \frac{1}{k^2+M^2-i\varepsilon}. \tag{28}$$

In such a way, we have for $\langle 0_+|0_-\rangle^{(a)}=iW^{(a)}$, or,

$$iW^{(a)}=i\int dM^2 I(M^2)\int (dx)(dx')j(x) \quad \times$$

$$\left[\frac{-\partial^2+\mu^2}{M^2-\mu^2}\Delta_+(x-x',M^2)\right]\varphi(x'), \tag{29}$$

or,

$$W^{(a)}=\int (dx)(dx')g\psi^*(x)\psi(x)\left[\int dM^2 I(M^2)\frac{\partial^2-\mu^2}{M^2-\mu^2}\Delta_+(x-x',M^2)\right]\varphi(x'), \tag{30}$$

which can be interpreted as the non-local coupling between fields $\varphi(x')$ and the density $\psi^*(x)\psi(x')$.

If we write the eq. (30) in the form

$$W^{(a)}=i\int\ (dx)(dx')j(x)h(x-x')\varphi(x'), \tag{31}$$

where

$$h(x-x')=\int\ dM^2 I(M^2)\left(\frac{\partial^2-\mu^2}{M^2-\mu^2}\right)\Delta_+(x-x',M^2), \tag{32}$$

we see that eq. (31) corresponds to an excitation - exchange of mass M which propagates from x' to x.

The momentum space representation of eq. (32) is

$$h(k^2)=-(k^2+\mu^2)\int dM^2\frac{M^2}{M^2-\mu^2}\frac{1}{k^2+M^2-i\varepsilon} \tag{33}$$

with

$$h(k^2)=0; \quad k^2=-\mu^2. \tag{34}$$

It is obvious that the integral in (33) exists.

The graphical representation of the considered situation can be pictured. (Dittrich, 1978).

The integral part of the global scattering process can be pictured by the diagram (Dittrich, 1978) and can be described by the non-local coupling

$$i \int j(x)\tilde{\varphi}(x), \tag{35}$$

where, with regard to the considered situation, the field $\tilde{\varphi}(x)$ is

$$\tilde{\varphi} = \int \tilde{\Delta}^{\mu}_{+}(x-x')K(x'), \tag{36}$$

where

$$\tilde{\Delta}^{\mu}_{+}(x-x') = \Delta^{\mu}_{+}(x-x') \quad +$$

$$\frac{g^2}{(4\pi)^2}\int_{(2m)^2}^{\infty} dM^2 \frac{(1-4m^2/M^2)^{1/2}}{M^2-\mu^2}\Delta_{+}(x-x',M^2). \tag{37}$$

The total action is then

$$W = i\int (dx)(dx')j(x)h(x-x')\varphi(x') + i\int (dx)j(x)\tilde{\varphi}(x), \tag{38}$$

which has the approximate form $\varphi \to \tilde{\varphi}$ as follows

$$W = i\int (dx)(dx')j(x)\left[\delta(x-x') + h(x-x')\right]\tilde{\varphi}(x') \quad =$$

$$i\int (dx)(dx')j(x)h(x-x')F(x-x')\tilde{\varphi}(x), \tag{39}$$

where

$$F(x-x') = \delta(x-x') - \int dM^2 \frac{I(M^2)}{M^2-\mu^2}(\partial^2-\mu^2)\Delta_{+}(x-x',M^2), \tag{40}$$

or, in the momentum space

$$F(k^2) = 1 - (k^2-\mu^2)\int dM^2 \frac{I(M^2)}{M^2-\mu^2}\,\frac{1}{k^2+M^2-i\varepsilon}, \tag{41}$$

with bypass result $F(-\mu^2) = 1$. For $k^2 = M^2$, we get from the last formula

$$F(-M^2) = 1 + \frac{M^2-\mu^2}{\pi}\int dM'^2 \frac{1}{M'^2-\mu^2}\,\frac{g^2}{16\pi} \quad \times$$

$$\frac{\left(1-\frac{4m^2}{M'^2}\right)^{1/2}}{M'^2-4m^2}\,\ln\left(\frac{M'^2-4m^2-\mu^2}{\mu^2}\right)\frac{1}{M'^2-M^2-i\varepsilon}. \tag{42}$$

The mathematical object $F(x-x')$ is called formfactor of elementary particle and its physical meaning follows from the definition of $\tilde{j}$ by the equation

$$\tilde{j}(x') = \int (dx)j(x)F(x-x'), \tag{43}$$

which enables to express W as

$$W = i \int \tilde{j}(x')\tilde{\varphi}(x'). \tag{44}$$

As the current $\tilde{j}(x)$ is obtained from the original current $j(x)$ by integration with the weight function $F(x - x')$, we can say that this function represents the additional distribution of charge corresponding to the original ideal situation.

Discussion

The electric (G_E) and magnetic (G_M) formfactors of the proton, the so-called Sachs formfactors, in elastic scattering of the electron on the proton have been experimentally studied since the mid-1950s. In the case of unpolarized electrons and protons, all experimental data on the behavior of the formfactors of the proton were obtained using the Rosenbluth formula (Rosenbluth, 1950) for the differential cross section for the $e + p \rightarrow e + p$ elastic scattering in the laboratory reference frame, where the initial proton is at rest.

It is usually stated in the modern literature, and in textbooks on the physics of elementary particles by nobelist Halzen (Halzen et al., 1984), that the Sachs formfactors are simply convenient because they allow the representation of the Rosenbluth formula in the simple and compact form of the sum of two terms containing only G_E, E and G_M These formal reasons for advantages of the Sachs formfactors are included, for instance in known monographs (Akhiezer et al., 1965; 1969; Berestetskii, et al., 1989), are not criticized, and are accepted.

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