# The gravity waves from the binary galaxies the new deal of the James Webb Space Telescope 

Miroslav Pardy<br>Department of Physical Electronics<br>Masaryk University, Kotlářská 2, 61137 Brno, Czech Republic<br>e-mail:pamir@physics.muni.cz

August 7, 2023


#### Abstract

The energy spectrum of gravitons emitted by the black hole binary as a gravity waves is calculated in the first part of the article. Then, the total quantum loss of energy, is calculated in the Schwinger theory of gravity. Using analogy with the binary stars, we calculate the graviton spectrum of the binary galaxies discovered by the JWST of NASA.


Keywords Binary stars, binary galaxies, JWST, Graviton, Schwinger source theory.

## 1 Introduction

A binary star is a system of two stars that are gravitationally bound to and in orbit around each other. Binary stars - a single object to the naked eye - are often resolved using a telescope as separate stars, in which case they are called visual binaries.

Many visual binaries have long orbital periods of several centuries and therefore have orbits which are uncertain, or, poorly known. They may also be detected by indirect techniques, such as spectroscopy - spectroscopic binaries, or, astrometry - astrometric binaries.

If a binary star happens to orbit in a plane along our line of sight, its components will eclipse and transit each other; these pairs are called eclipsing binaries, or, together with other binaries that change brightness as they orbit, photometric binaries. If components in binary star systems are close enough they can gravitationally distort their mutual outer stellar atmospheres.

Double stars, a pair of stars that appear close to each other, have been observed since the invention of the telescope. Mizar, in the Ursa Major, was observed to be double by Giovanni Battista Riccioli in 1650 and earlier by Benedetto Castelli and Galileo. The double star Acrux, in the Southern Cross, was discovered to be double by Father Fontenay in 1685. William Herschel began observing double stars in 1779.

Over the years, many more double stars have been cataloged and measured. As of June 2017, the Washington Double Star Catalog, a database of visual double stars compiled by the

United States Naval Observatory, contains over 100000 pairs of double stars, including optical doubles as well as binary stars. Our goal is not to consider the binary stars but binary galaxies discovered by JWST.

## 2 Binary galaxies

A galaxy is a system of stars, stellar remnants, interstellar gas, dust, and dark matter bound together by gravity. Galaxies, averaging an estimated 100 million stars, range in size from dwarfs with less than a hundred million stars, to the largest galaxies known super giants with one hundred trillion stars, each orbiting its galaxy's center of mass . Most of the mass in a typical galaxy is in the form of dark matter, with only a few percent of that mass visible in the form of stars and nebulae. Super massive black holes are a common feature at the centers of galaxies.

Galaxies are categorized according to their visual morphology as elliptical, spiral, or irregular. Many are thought to have super massive black holes at their centers.

It is estimated that there are roughly 200 billion galaxies in the observable universe. Most galaxies are 1000 to 100000 parsecs in diameter and are separated by distances on the order of millions of parsecs, or, mega parsecs.

For comparison, the Milky Way has a diameter of at least 26800 parsecs and is separated from the Andromeda Galaxy (with diameter of about 152000 ly ), its nearest large neighbor, by 780000 parsecs.

## 3 James Webb Space Telescope -JWST

JWST is able the detailed observations of distant and ancient galaxies. The telescope's improved capabilities enable scientists to study the structures, properties, and dynamics of these galaxies with remarkable precision. This revolutionizes our ability to unravel the mysteries of the early universe and gain insights into the processes that shaped cosmic evolution.

The JWST observations have revealed a staggering number of galaxies, surpassing earlier estimates. The JWST allows to observe distant objects that emit light at longer wavelengths. This enables the detection of ancient galaxies that have undergone significant redshift due to the expansion of the universe. At the same time, the JWST allows to observe galaxies forming the binaries and radiating the gravity waves. This is the new deal of JWST.

The telescope's enhanced resolution and sensitivity provide clearer and more detailed images of these galaxies, unveiling their structures and properties with unprecedented clarity. The discovery of thousands of galaxies in the GOODS-South region provides a wealth of data for studying cosmic evolution. By examining the properties and distribution of these galaxies, researchers can gain insights into how galaxies formed, evolved, and interacted in the early universe. This information helps piece together the puzzle of cosmic evolution and contributes our broader understanding of the universe's history.

## 4 Black holes

In 1916, Schwarzschild published the solution of the Einstein field equations (Schwarzschild, 1916) that was later understood to describe a black hole (Finkelstein, 1958; Kruskal, 1960) and in 1963 Kerr generalized the solution to rotating black holes (Kerr, 1963). The year 1970 was the starting point of the theoretical work leading to the understanding of black hole quasi normal modes (Vishveshwara, 1970; Press, 1971; Chandrasekhar et al., 1975) and in the 1990 higherorder post-Newtonian calculations (Blanchet et al., 1995) was performed and later the extensive analytical studies of relativistic two-body dynamics realized (Blanchet, 2014; Buonanno, et al. 1999). These advances, together with numerical relativity break through in the past decade (Pretorius, 2005; Campanelli et al., 2006; Baker et al. 2006). Numerous black hole candidates have now been identified through electromagnetic observations (Webster, 1972 et al.; Bolton, 1972; Casares, 2014). The black hole binary and their rotation and et al. mergers is open problem of the astrophysics and it is the integral part of the binary black hole physics.

The binary pulsar system PSR B1913+16 (also known as PSR J1915+1606) discovered by Hulse and Taylor (1975) and subsequent observations of its energy loss by Taylor and Weisberg (1982) demonstrated the existence of gravitational waves (Aasi et al., 2015)..

By the early 2000s, a set of initial detectors was completed, including TAMA 300 in Japan, GEO 600 in Germany, the Laser Interferometer Gravitational-Wave Observatory (LIGO) in the United States, and Virgo in Italy. In 2015, Advanced LIGO became the first of a significantly more sensitive network of advanced detectors (a second-generation interferometric gravitational wave detector) to begin observations (Acernese et al., 2015).

Taylor and Hulse, working at the Arecibo radio telescope, discovered the radio pulsar PSR B1913+16 in a binary, in 1974, and this is now considered as the best general relativistic laboratory (Taylor, Jr., 1993).

Pulsar PSR B1913+16 is the massive body of the binary system where each of the rotating pairs is 1.4 times the mass of the Sun. These neutron stars rotate around each other in an orbit not much larger than the Sun's diameter, with a period 7.8 hours. Every 59 ms , the pulsar emits a short signal that is so clear that the arrival time of a 5 -min string of a set of such signals can be resolved within $15 \mu \mathrm{~s}$.

A pulsar model based on strongly magnetized, rapidly spinning neutron stars was soon established as consistent with most of the known facts (Gold, 1968) its electrodynamic properties were studied theoretically (Goldreich, et al.1969) and shown to be plausibly capable of generating broadband radio noise detectable over interstellar distances. The binary pulsar PSR B1913+16 is now recognized as the harbinger of a new class of unusually short-period pulsars, with numerous important applications.

Because the velocities and gravitational energies in a high-mass binary pulsar system can be significantly relativistic, strong-field and radiative effects come into play. The binary pulsar PSR B1913+16 provides significant tests of gravitation beyond the weak-field, slow-motion limit (Damour et al., 1992; Taylor et al., 1992).

We do not here repeat the derivation of the Einstein quadrupole formula in the Schwinger gravity theory (Manoukian, 1990). We show that just in the framework of the Schwinger gravity theory it is easy to determine the spectral formula for emitted gravitons and the quantum energyloss formula of the binary system. The energy-loss formula is general, including black hole binary and it involves arbitrarily strong gravity.

Since the measurement of the motion of the black hole binaries goes on, we hope that sooner or later the confirmation of our formula will be established.

### 4.1 The spin 2 gravity in source theory

Source methods by Schwinger are in source theory adequate for the solution of the calculation of the spectral formula of gravitons and energy loss of binary. Source theory (Schwinger et al., 1976; Dittrich, 1978; Schwinger, 1976) was initially constructed to describe the particle physics situations occurring in high-energy physics experiments. However, it was found that the original formulation simplifies the calculations in the electrodynamics and gravity, where the interactions are mediated by photon and graviton respectively. The source theory of gravity forms the analogue of quantum electrodynamics because, while in QED the interaction is mediated by the photon, the gravitational interaction is mediated by the graviton (Schwinger, 1976). The basic formula in the source theory is the vacuum-to-vacuum amplitude (Schwinger, 1970):

$$
\begin{equation*}
\left\langle 0_{+} \mid 0_{-}\right\rangle=e^{\frac{i}{\hbar} W(S)}, \tag{1}
\end{equation*}
$$

where the minus and plus symbols refer to any time before and after region of space-time with action of sources. The exponential form is postulated to express the physically independent experimental arrangements, with result that the associated probability amplitudes multiply and the corresponding $W$ expressions add (Schwinger et al., 1976; Dittrich, 1978; Schwinger, 1976).

In the flat space-time, the field of gravitons is described by the amplitude (1) with the action ( $c=1$ in the following text) (Schwinger, 1970):

$$
\begin{equation*}
W(T)=4 \pi G \int(d x)\left(d x^{\prime}\right)\left[T^{\mu \nu}(x) D_{+}\left(x-x^{\prime}\right) T_{\mu \nu}\left(x^{\prime}\right)-\frac{1}{2} T(x) D_{+}\left(x-x^{\prime}\right) T\left(x^{\prime}\right)\right], \tag{2}
\end{equation*}
$$

where the dimensionality of $W(T)$ has dimension of the Planck constant $\hbar ; T_{\mu \nu}$ is the momentum and energy tensor that, for a particle trajectory $\mathbf{x}=\mathbf{x}(t)$, is defined by the equation (Cho, et al., 1976):

$$
\begin{equation*}
T^{\mu \nu}(x)=\frac{p^{\mu} p^{\nu}}{E} \delta(\mathbf{x}-\mathbf{x}(t)) \tag{3}
\end{equation*}
$$

where $p^{\mu}$ is the relativistic four-momentum of a particle with a rest mass $m$ and

$$
\begin{gather*}
p^{\mu}=(E, \mathbf{p})  \tag{4}\\
p^{\mu} p_{\mu}=-m^{2}, \tag{5}
\end{gather*}
$$

and the relativistic energy is defined by the known relation

$$
\begin{equation*}
E=\frac{m}{\sqrt{1-\mathbf{v}^{2}}} \tag{6}
\end{equation*}
$$

where $\mathbf{v}$ is the three-velocity of the moving particle.
Symbol $T(x)$ in formula (2) is defined as $T=g_{\mu \nu} T^{\mu \nu}$, and $D_{+}\left(x-x^{\prime}\right)$ is the graviton propagator whose explicit form will be determined later.

### 4.2 The graviton spectrum of the black hole binary

It may be easy to show that the probability of the persistence of vacuum is given by the following formula (Schwinger et al., 1976):

$$
\begin{equation*}
\left|\left\langle 0_{+} \mid 0_{-}\right\rangle\right|^{2}=\exp \left\{-\frac{2}{\hbar} \operatorname{Im} W\right\} \stackrel{d}{=} \exp \left\{-\int d t d \omega \frac{1}{\hbar \omega} P(\omega, t)\right\}, \tag{7}
\end{equation*}
$$

where the so-called power spectral function $P(\omega, t)$ has been introduced (Schwinger et al., 1976). For extraction of the spectral function from $\operatorname{Im} W$, it is necessary to know the explicit form of the graviton propagator $D_{+}\left(x-x^{\prime}\right)$. This propagator involves the graviton property of spreading with velocity $c$. It means that its mathematical form is identical with the photon propagator form. With regard to Schwinger et al. (1976) the $x$-representation of $D(k)$ in eq. (2) is as follows:

$$
\begin{equation*}
D_{+}\left(x-x^{\prime}\right)=\int \frac{(d k)}{(2 \pi)^{4}} e^{i k\left(x-x^{\prime}\right)} D(k), \tag{8}
\end{equation*}
$$

where

$$
\begin{equation*}
D(k)=\frac{1}{\left|\mathbf{k}^{2}\right|-\left(k^{0}\right)^{2}-i \epsilon} \tag{9}
\end{equation*}
$$

which gives

$$
\begin{equation*}
D_{+}\left(x-x^{\prime}\right)=\frac{i}{4 \pi^{2}} \int_{0}^{\infty} d \omega \frac{\sin \omega\left|\mathbf{x}-\mathbf{x}^{\prime}\right|}{\left|\mathbf{x}-\mathbf{x}^{\prime}\right|} e^{-i \omega\left|t-t^{\prime}\right|} . \tag{10}
\end{equation*}
$$

Now, using formulas (2), (7) and (10), we get the power spectral formula in the following form:

$$
\begin{gather*}
P(\omega, t)=4 \pi G \omega \int(d \mathbf{x})\left(d \mathbf{x}^{\prime}\right) d t^{\prime} \frac{\sin \omega\left|\mathbf{x}-\mathbf{x}^{\prime}\right|}{\left|\mathbf{x}-\mathbf{x}^{\prime}\right|} \cos \omega\left(t-t^{\prime}\right) \times \\
{\left[T^{\mu \nu}(\mathbf{x}, t) T_{\mu \nu}\left(\mathbf{x}^{\prime}, t^{\prime}\right)-\frac{1}{2} g_{\mu \nu} T^{\mu \nu}(\mathbf{x}, t) g_{\alpha \beta} T^{\alpha \beta}\left(\mathbf{x}^{\prime}, t^{\prime}\right)\right] .} \tag{11}
\end{gather*}
$$

### 4.3 The power spectral formula for the binary system

In the case of the binary system with masses $m_{1}$ and $m_{2}$, we suppose that they move in a uniform circular motion around their center of gravity in the $x y$ plane, with corresponding kinematic coordinates:

$$
\begin{gather*}
\mathbf{x}_{1}(t)=r_{1}\left(\mathbf{i} \cos \left(\omega_{0} t\right)+\mathbf{j} \sin \left(\omega_{0} t\right)\right)  \tag{12}\\
\mathbf{x}_{2}(t)=r_{2}\left(\mathbf{i} \cos \left(\omega_{0} t+\pi\right)+\mathbf{j} \sin \left(\omega_{0} t+\pi\right)\right) \tag{13}
\end{gather*}
$$

with

$$
\begin{equation*}
\mathbf{v}_{i}(t)=d \mathbf{x}_{i} / d t, \quad \omega_{0}=v_{i} / r_{i}, \quad v_{i}=\left|\mathbf{v}_{i}\right| \quad(i=1,2) \tag{14}
\end{equation*}
$$

For the tensor of energy and momentum of the binary we have:

$$
\begin{equation*}
T^{\mu \nu}(x)=\frac{p_{1}^{\mu} p_{1}^{\nu}}{E_{1}} \delta\left(\mathbf{x}-\mathbf{x}_{1}(t)\right)+\frac{p_{2}^{\mu} p_{2}^{\nu}}{E_{2}} \delta\left(\mathbf{x}-\mathbf{x}_{2}(t)\right) \tag{15}
\end{equation*}
$$

where we have omitted the tensor $t_{\mu \nu}^{G}$, which is associated with the mass-less, gravitational field distributed all over space and proportional to the gravitational constant $G$ (Cho, et al., 1976):

After insertion of eq. (15) into eq. (11), we get (Pardy, 1983):

$$
\begin{equation*}
P_{\text {total }}(\omega, t)=P_{1}(\omega, t)+P_{12}(\omega, t)+P_{2}(\omega, t) \tag{16}
\end{equation*}
$$

where $\left(t^{\prime}-t=\tau\right)$ :

$$
\begin{gather*}
P_{1}(\omega, t)=\frac{G \omega}{r_{1} \pi} \int_{-\infty}^{\infty} d \tau \frac{\sin \left[2 \omega r_{1} \sin \left(\omega_{0} \tau / 2\right)\right]}{\sin \left(\omega_{0} \tau / 2\right)} \cos \omega \tau \times \\
\left(E_{1}^{2}\left(\omega_{0}^{2} r_{1}^{2} \cos \omega_{0} \tau-1\right)^{2}-\frac{m_{1}^{4}}{2 E_{1}^{2}}\right),  \tag{17}\\
P_{2}(\omega, t)=\frac{G \omega}{r_{2} \pi} \int_{-\infty}^{\infty} d \tau \frac{\sin \left[2 \omega r_{2} \sin \left(\omega_{0} \tau / 2\right)\right]}{\sin \left(\omega_{0} \tau / 2\right)} \cos \omega \tau \times \\
\left(E_{2}^{2}\left(\omega_{0}^{2} r_{2}^{2} \cos \omega_{0} \tau-1\right)^{2}-\frac{m_{2}^{4}}{2 E_{2}^{2}}\right),  \tag{18}\\
P_{12}(\omega, t)=\frac{4 G \omega}{\pi} \int_{-\infty}^{\infty} d \tau \frac{\sin \omega\left[r_{1}^{2}+r_{2}^{2}+2 r_{1} r_{2} \cos \left(\omega_{0} \tau\right)\right]^{1 / 2}}{\left[r_{1}^{2}+r_{2}^{2}+2 r_{1} r_{2} \cos \left(\omega_{0} \tau\right)\right]^{1 / 2}} \cos \omega \tau \times \\
\left(E_{1} E_{2}\left(\omega_{0}^{2} r_{1} r_{2} \cos \omega_{0} \tau+1\right)^{2}-\frac{m_{1}^{2} m_{2}^{2}}{2 E_{1} E_{2}}\right) . \tag{19}
\end{gather*}
$$

### 4.4 The quantum energy loss of the binary

Using the following relations

$$
\begin{gather*}
\omega_{0} \tau=\varphi+2 \pi l, \quad \varphi \in(-\pi, \pi), \quad l=0, \pm 1, \pm 2, \ldots  \tag{20}\\
 \tag{21}\\
\sum_{l=-\infty}^{l=\infty} \cos 2 \pi l \frac{\omega}{\omega_{0}}=\sum_{l=-\infty}^{\infty} \omega_{0} \delta\left(\omega-\omega_{0} l\right)
\end{gather*}
$$

we get for $P_{i}(\omega, t)$, with $\omega$ being restricted to positive:

$$
\begin{equation*}
P_{i}(\omega, t)=\sum_{l=1}^{\infty} \delta\left(\omega-\omega_{0} l\right) P_{i l}(\omega, t) \tag{22}
\end{equation*}
$$

Using the definition of the Bessel function $J_{2 l}(z)$

$$
\begin{equation*}
J_{2 l}(z)=\frac{1}{2 \pi} \int_{-\pi}^{\pi} d \varphi \cos \left(z \sin \frac{\varphi}{2}\right) \cos l \varphi \tag{23}
\end{equation*}
$$

from which the derivatives and their integrals follow, we get for $P_{1 l}$ and $P_{2 l}$ the following formulas:

$$
\begin{gather*}
P_{i l}=\frac{2 G \omega}{r_{i}}\left(\left(E_{i}^{2}\left(v_{i}^{2}-1\right)-\frac{m_{i}^{4}}{2 E_{i}^{2}}\right) \int_{0}^{2 v_{i} l} d x J_{2 l}(x)+\right. \\
\left.4 E_{i}^{2}\left(v_{i}^{2}-1\right) v_{i}^{2} J_{2 l}^{\prime}\left(2 v_{i} l\right)+4 E_{i}^{2} v_{i}^{4} J_{2 l}^{\prime \prime \prime}\left(2 v_{i} l\right)\right), \quad i=1,2 \tag{24}
\end{gather*}
$$

Using $r_{2}=r_{1}+\epsilon$, where $\epsilon$ is supposed to be small in comparison with radii $r_{1}$ and $r_{2}$, we obtain

$$
\begin{equation*}
\left[r_{1}^{2}+r_{2}^{2}+2 r_{1} r_{2} \cos \varphi\right]^{1 / 2} \approx 2 a \cos \left(\frac{\varphi}{2}\right) \tag{25}
\end{equation*}
$$

with

$$
\begin{equation*}
a=r_{1}\left(1+\frac{\epsilon}{2 r_{1}}\right) . \tag{26}
\end{equation*}
$$

So, instead of eq. (19) we get:

$$
\begin{gather*}
P_{12}(\omega, t)=\frac{2 G \omega}{a \pi} \int_{-\infty}^{\infty} d \tau \frac{\sin \left[2 \omega a \cos \left(\omega_{0} \tau / 2\right)\right]}{\left.\cos \left(\omega_{0} \tau\right) / 2\right]} \cos \omega \tau \times \\
\left(E_{1} E_{2}\left(\omega_{0}^{2} r_{1} r_{2} \cos \omega_{0} \tau+1\right)^{2}-\frac{m_{1}^{2} m_{2}^{2}}{2 E_{1} E_{2}}\right) \tag{27}
\end{gather*}
$$

Now, we can approach the evaluation of the energy-loss formula for the binary from the power spectral formulae (24) and (27). The energy loss is defined by the relation

$$
-\frac{d U}{d t}=\int P(\omega) d \omega=
$$

$$
\begin{equation*}
\int d \omega \sum_{i, l} \delta\left(\omega-\omega_{0} l\right) P_{i l}+\int P_{12}(\omega) d \omega=-\frac{d}{d t}\left(U_{1}+U_{2}+U_{12}\right) \tag{28}
\end{equation*}
$$

From the book of formulas (Prudnikov et al., 1983) we have Kapteyn's formula

$$
\begin{equation*}
\sum_{l=1}^{\infty} \frac{J_{2 l}(2 l v)}{l^{2}}=\frac{v^{2}}{2} \tag{29}
\end{equation*}
$$

After differentiating the last relation with respect to $v$, we have

$$
\begin{equation*}
\sum_{l=1}^{\infty} l J_{2 l}^{\prime \prime \prime}(2 l v)=0 \tag{30}
\end{equation*}
$$

We learn other Kapteyn's formulas (Prudnikov et al., 1983):

$$
\begin{equation*}
\sum_{l=1}^{\infty} 2 l J_{2 l}^{\prime}(2 l v)=\frac{v}{\left(1-v^{2}\right)^{2}} \tag{31}
\end{equation*}
$$

and

$$
\begin{equation*}
\sum_{l=1}^{\infty} l \int_{0}^{2 l v} J_{2 l}(x) d x=\frac{v^{3}}{3\left(1-v^{2}\right)^{3}} \tag{32}
\end{equation*}
$$

So, after application of eqs. (30), (31) and (32) to eqs. (24) and (28), we get:

$$
\begin{equation*}
-\frac{d U_{i}}{d t}=\frac{G m_{i}^{2} v_{i}^{3} \omega_{0}}{3 r_{i}\left(1-v_{i}^{2}\right)^{3}}\left[13 v_{i}^{2}-15\right] \tag{33}
\end{equation*}
$$

Instead of using Kapteyn's formulas for the interference term, we will perform a direct evaluation of the energy loss of the interference term by the $\omega$-integration in (27) (Sokolov et al., 1983). So, after some elementary modification in the $\omega$-integral, we get:

$$
\begin{gather*}
-\frac{d U_{12}}{d t}=\int_{0}^{\infty} P(\omega) d \omega= \\
A \int_{-\infty}^{\infty} d \tau \int_{-\infty}^{\infty} d \omega \omega e^{-i \omega \tau} \sin \left[2 \omega a \cos \omega_{0} \tau\right]\left[\frac{B\left(C \cos \omega_{0} \tau+1\right)^{2}-D}{\cos \left(\omega_{0} \tau / 2\right)}\right] \tag{34}
\end{gather*}
$$

with

$$
\begin{equation*}
A=\frac{G}{a \pi}, \quad B=E_{1} E_{2}, \quad C=v_{1} v_{2}, \quad D=\frac{m_{1}^{2} m_{2}^{2}}{2 E_{1} E_{2}} \tag{35}
\end{equation*}
$$

Using the definition of the $\delta$-function and its derivative, we have, instead of eq. (34), with $v=a \omega_{0}:$

$$
\begin{gather*}
-\frac{d U_{12}}{d t}=A \omega_{0} \pi \int_{-\infty}^{\infty} d x \frac{\left[B(C \cos x+1)^{2}-D\right]}{\cos (x / 2)} \times \\
{\left[\delta^{\prime}(x-2 v \cos (x / 2))-\delta^{\prime}(x+2 v \cos (x / 2))\right]} \tag{36}
\end{gather*}
$$

According to the Schwinger article (Schwinger, 1998), we express the delta-function as follows:

$$
\begin{equation*}
\delta(x \pm 2 v \cos (x / 2))=\sum_{n=0}^{\infty} \frac{( \pm 2 v \cos (x / 2))^{n}}{n!}\left(\frac{d}{d x}\right)^{n} \delta(x) . \tag{37}
\end{equation*}
$$

Then

$$
\begin{equation*}
\delta^{\prime}(x \pm 2 v \cos (x / 2))=\sum_{n=0}^{\infty} \frac{( \pm 2 v \cos (x / 2))^{n}}{n!}\left(\frac{d}{d x}\right)^{n+1} \delta(x)= \tag{38}
\end{equation*}
$$

and it means that

$$
\begin{align*}
& \frac{\left[\delta^{\prime}(x+2 v \cos (x / 2))-\delta^{\prime}(x-2 v \cos (x / 2))\right]}{\cos (x / 2)}= \\
& (-2) \sum_{n=1}^{\infty} \frac{(2 v)^{2 n-1}(\cos (x / 2))^{2(n-1)}}{(2 n-1)!}\left(\frac{d}{d x}\right)^{2 n} \delta(x) \tag{39}
\end{align*}
$$

Now, we can write eq. (36) in the following form after some elementary operations

$$
\begin{align*}
& -\frac{d U_{12}}{d t}=A \omega_{0} \pi \int_{-\infty}^{\infty} d x\left(B(C \cos x+1)^{2}-D\right) \times \\
& (-2) \sum_{n=1}^{\infty} \frac{(2 v)^{2 n-1}(\cos (x / 2))^{2(n-1)}}{(2 n-1)!}\left(\frac{d}{d x}\right)^{2 n} \delta(x) \tag{40}
\end{align*}
$$

where $\left(B(C \cos x+1)^{2}-D\right)$ can be written as follows:

$$
\begin{gather*}
\left(B(C \cos x+1)^{2}-D\right)= \\
4 B C^{2}\left(\cos ^{4}(x / 2)+\left[4 C B-4 B C^{2}\right]\left(\cos ^{2}(x / 2)+\left[B C^{2}-2 C B+B-D\right] .\right.\right. \tag{41}
\end{gather*}
$$

After application of the per partes method, we get from eq. (40) the following mathematical object:

$$
\begin{gather*}
-\frac{d U_{12}}{d t}=(-2) A\left[4 B C^{2}\right] \omega_{0} \pi \int_{-\infty}^{\infty} d x \delta(x) \sum_{n=1}^{\infty}\left(\frac{d}{d x}\right)^{2 n}(2 v)^{2 n-1} \frac{(\cos (x / 2))^{2 n+2}}{(2 n-1)!}- \\
2 A\left[4 C B-4 B C^{2}\right] \omega_{0} \pi \int_{-\infty}^{\infty} d x \delta(x) \sum_{n=1}^{\infty}\left(\frac{d}{d x}\right)^{2 n}(2 v)^{2 n-1} \frac{(\cos (x / 2))^{2 n}}{(2 n-1)!}- \\
2 A\left[B C^{2}-2 C B+B-D\right] \omega_{0} \pi \int_{-\infty}^{\infty} d x \delta(x) \sum_{n=1}^{\infty}\left(\frac{d}{d x}\right)^{2 n}(2 v)^{2 n-1} \frac{(\cos (x / 2))^{2(n-1)}}{(2 n-1)!} . \tag{42}
\end{gather*}
$$

We get after some elementary operations $\int \delta f(x)=f(0)$

$$
\begin{align*}
J_{1} & =\left.\sum_{n=1}^{\infty}\left(\frac{d}{d x}\right)^{2 n}(2 v)^{2 n-1} \frac{(\cos (x / 2))^{2 n+2}}{(2 n-1)!}\right|_{x=0}=\sum_{n=0}^{\infty} f(n) v^{2 n}=F\left(v^{2}\right),  \tag{43}\\
J_{2} & =\left.\sum_{n=1}^{\infty}\left(\frac{d}{d x}\right)^{2 n}(2 v)^{2 n-1} \frac{(\cos (x / 2))^{2 n}}{(2 n-1)!}\right|_{x=0}=\sum_{n=0}^{\infty} g(n) v^{2 n}=G\left(v^{2}\right) \tag{44}
\end{align*}
$$

and

$$
\begin{equation*}
J_{3}=\left.\sum_{n=1}^{\infty}\left(\frac{d}{d x}\right)^{2 n}(2 v)^{2 n-1} \frac{(\cos (x / 2))^{2(n-1)}}{(2 n-1)!}\right|_{x=0}=\sum_{n=0}^{\infty} h(n) v^{2 n}=H\left(v^{2}\right) \tag{45}
\end{equation*}
$$

where $f, g, h, F, G, H$ are functions which must be determined
So we get instead of eq. (41) the following final form

$$
\begin{gather*}
-\frac{d U_{12}}{d t}=(-2) A\left[4 B C^{2}\right] \omega_{0} \pi G\left(v^{2}\right)-2 A\left[4 C B-4 B C^{2}\right] \omega_{0} \pi F\left(v^{2}\right)- \\
2 A\left[-2 C B+B C^{2}+B-D\right] \omega_{0} \pi H\left(v^{2}\right) \tag{46}
\end{gather*}
$$

Let us remark that we can use simple approximation in eq. (41) as follows: $(\cos (x / 2))^{2 n+2} \approx$ $(\cos (x / 2))^{2},(\cos (x / 2))^{2 n} \approx(\cos (x / 2))^{2},(\cos (x / 2))^{2(n-1)} \approx(\cos (x / 2))^{2}$. Then, after using the well-known formula

$$
\begin{equation*}
\left(\frac{d}{d x}\right)^{2 n} \cos ^{2}(x / 2)=\frac{1}{2} \cos (x+\pi n) \tag{47}
\end{equation*}
$$

and

$$
\begin{equation*}
\left.\frac{1}{2} \cos (x+\pi n)\right|_{x=0}=\frac{1}{2}(-1)^{n} . \tag{48}
\end{equation*}
$$

So, instead of eq. (46) se have:

$$
\begin{equation*}
-\frac{d U_{12}}{d t}=A \omega_{0} \pi\left\{2 B C+B C^{2}+B-D\right\} \sum_{n=1}^{\infty} \frac{(2 v)^{2 n-1}(-1)^{n}}{(2 n-1)!} \tag{49}
\end{equation*}
$$

## 5 Discussion and Summary

We have derived the spectral density of gravitons and the total quantum loss of energy of the black hole binary. The same formulas are valid for galaxy binaries which can be modelled as the two-body system. The energy loss is caused by the emission of gravitons during the motion of the two black hole binary around each other under their gravitational interaction. The energyloss formulas of the production of gravitons are derived here by the Schwinger method. Because the general relativity and theory of gravity do not necessarily contain the last valid words to be written about the nature of gravity and it is not, of course, a quantum theory (Taylor, 1993), they cannot give the answer on the production of gravitons and the quantum energy loss, respectively. So, this article is the original text that discusses the quantum energy loss caused by the production of gravitons by the black hole binary system and by the galaxy binary. It is evident that the production of gravitons by the binary system forms a specific physical situation, where a general relativity can be seriously confronted with the source theory of gravity.

This article is an extended version of an older article by the present author (Pardy, 1983), in which only the spectral formulas were derived. Here we have derived the quantum energy-loss formulas, with no specific assumption concerning the strength of the gravitational field. We
hope that future astrophysical observations will confirm the quantum version of the energy loss of the binary black hole and galaxy binary.

There is the fundamental problem concerning the maximal mass of the black hole and galaxy. The theory of the space-time with maximal acceleration constant was derived by author (Pardy 2018; Pardy, 2017). In this theory the maximal acceleration constant is the analogue of the maximal velocity in special theory of relativity. Maximal acceleration, determines the maximal black hole mass where the mass of the black hole is restricted by maximal acceleration of a body falling in the gravity field of the black hole.

The another question is what is the relation of our formulas to the results obtained by LIGO - the Laser Interferometer Gravitational-Wave Observatory. LIGO is the largest and most sensitive interferometer facility ever built. It has been periodically upgraded to increase its sensitivity. The most recent upgrade, Advanced LIGO (2015), detected for the first time the gravitational wave, with sensitivity far above the background noise. The event with number GW150914, was identified with the result of a merger of two black holes at a distance of about 400 Mpc from Earth LIGO, (215). Two additional, significant detections, GW151226 and GW170104, were reported later. We can say that at this time it is not clear if the LIGO results involves information on the spectrum of gravitons calculated in this chapter.

## References

Aasi, J. et al., (2015). Advanced Virgo: a second-generation interferometric gravitational wave detector, Classical Quantum Gravity 32, 074001.

Acernese, et al., F. (2015). Advanced Virgo: a 2nd generation interferometric gravitational wave detector, Class. Quantum Grav. 32, 024001.

Baker, J. G., Centrella, J., Choi, D. I., Koppitz, M. and van Blanchet, L. (2014). Gravitational radiation from post-Newtonian sources and inspiralling compact binaries, Living Rev. Relativity $17,2$.

Blanchet, L., Damour, L. T., Iyer, B. R., Will, C. M. and
Buonanno, A. and Damour, T. (1999). Effective one-body approach to general relativistic two-body dynamics, Phys. Rev. D 59, 084006.

Bolton, C. T. (1972). Is Cyg X-1 a neutron star?, Nature (London) 240, 124.
Campanelli, M., Lousto, C. O., Marronetti, P. and Zlochower, Y. (2006). Accurate evolutions of orbiting black-hole binaries without excision, Phys. Rev. Lett. 96, 111101.

Casares, J. and Jonker, P. G. (2014). Mass measurements of stellar and intermediate-mass black holes, Space Sci. Rev. 183, 223.

Damour, T. and Taylor, J. H. (1992). Strong field tests of relativistic gravity and binary pulsars, Phys. Rev. D 15 45(6) 1840-1868.

Dittrich, W. (1978). Source methods in quantum field theory, Fortschr. Phys. 26, 289.

Finkelstein, D. (1958). Past-future asymmetry of the gravitational field of a Point Particle, Phys. Rev. 110, 965.

Gold, T. (1968). Rotating neutron stars as the origin of the pulsating radio sources nature, Nature 218, 731.

Goldreich, P. and Julian, W. H. (1969). Pulsar electrodynamics, Astrophys. J. 157, 869.
Hulse, R. A. and Taylor, J. H. (1975). Discovery of a pulsar in a binary system, Astrophys. J. 195, L51.

Chandrasekhar, S. and Detweiler, S. L. (1975). The quasi-normal modes of the Schwarzschild black hole, Proc. R. Soc. A 344, 441.

Cho, C. F. and Harri Dass, N. D. (1976). Gravitational two-body problems, a source theory viewpoint, Ann. Phys. (NY) 90, 406.

Kerr, R. P. (1963). Gravitational field of a spinning mass as an example of algebraically special metrics, Phys. Rev. Lett. 11, 237.

Kruskal, M. D. (1960). Maximal extension of Schwarzschild metric, Phys. Rev. 119, 1743.
LIGO, (215). The Laser Interferometer Gravitational-Wave Observatory, The LIGO Scientific Collaboration, http://www.ligo.org.

Manoukian, E. B. (1990). A quantum viewpoint of gravitational radiation, Gen. Rel. Grav. 22, 501.

Pardy, M. (1983). The Synchrotron production of gravitons by the binary system, Gen. Rel. Grav. 15, No. 11, 1027.

Pardy, M. (2018). Thomas precession by uniform acceleration and gravity, Journal of High Energy Physics, Gravitation and Cosmology, 2018, 4, 132-142.

Pardy, M. From Lobachevsky-Beltrami-Fok space to Schwinger gravity and cosmology, (LAMBERT Academic Publishing, February 15, 2017)

Press, W. H. (1971). Long wave-trains of gravitational waves from a vibrating black hole, Astrophys. J. 170, L105.

Pretorius, F. (2005). Evolution of binary black hole space-times, Phys. Rev. Lett. 95, 121101.
Prudnikov, A. P., Bryčkov, Ju. A. and Maričev, O. I.
Integrals and series, - Special functions (Moscow, Nauka, 1983) (in Russian).
Schwarzschild, K. (1916). Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie, Sitzungsber. K. Preuss. Akad. Wiss. 1, 189; English translation: On the gravitational field of a mass point according to Einsteins theory, Gen. Relativ. Gravit. 35, 951 (2003).

Sokolov, A. A. and Ternov, I. M. The relativistic electron (Moscow, Nauka, 1983) (in Russian).

Schwinger, J. (1998). On radiation by Electrons in a Betatron, LNBL-39088, CBP Note-179, UC-414, Berkeley, CA 94720, January 13.

Schwinger, J. Tsai W. Y. and T. Erber, (1976). Classical and quantum theory of synergic synchrotron-Cherenkov radiation, Ann. Phys. (NY) 96, 303.

Schwinger, J. (1976). Gravitons and photons: The methodological unification of source theory, Gen. Rel. Grav. 7, 251.

Schwinger, J. Particles, sources and fields, Vol. 1 (Addison-Wesley Publ. Comp., Reading, Mass., 1970).

Taylor, J. H. and. Weisberg, J. M. (1982). A new test of general relativity: Gravitational radiation and the binary pulsar PSR B1913+16, Astrophys. J. 253, 908.

Taylor, Jr., J. H. Binary pulsars and relativistic gravity (Nobel Lecture, 1993).
Taylor, J. H., Wolszczan, A., Damour, T. and Weisberg, J. M. (1992). Nature 355132.
Vishveshwara, C. V. (1970). Scattering of gravitational radiation
by a Schwarzschild black hole, Nature (London) 227, 936.
Webster, B. L. and Murdin, P. (1972). Is Cygnus X-1-a spectroscopic binary with a heavy companion ?, Nature (London) 235, 37.

Weinberg, S. Gravitation and cosmology, (John Wiley and Sons, Inc., New York, 1972).

