Crystal spheres and a geometric concept of the world.

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Abstract: A geometric concept of the world (W) is considered where the manifold W is identified with a locally trivial fibre bundle $pr: W \to U$ of so–called *crystal spheres* over a manifold U called the *universal time*. For every point $p \in U$ $M^n = pr^{-1}(p)$ is a n–dimensional crystal sphere and close crystal spheres are called the *parallel universes*. There exists a *geometric black hole* on the smooth manifold M^n. Tensor fields, fibre bundles, operators (physical structures and equations) can be deformed towards the black hole into continuous and sectionally smooth those, further, they can be retracted together with the black hole into a small black ball to initiate the *Big Bang*.

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0. Introduction

The celestial spheres were the fundamental entities of the cosmological models developed by Plato, Eudoxus, Aristotle, Ptolemy and others, **[1]**. Our concept of the world can be considered as a modern interpretation of ideas of ancient greeks or, perhaps, of more old sources which we do not know.

Example. *A sphere bundle is a fiber bundle whose fiber is a n–sphere. Given a vector bundle E with a metric (such as the tangent bundle to a Riemannian manifold) one can construct the associated unit sphere bundle for which the fiber over a point x is the set of all unit vectors in* E_x. *When the vector bundle is the tangent bundle T(M), the unit sphere bundle is known as the unit tangent bundle, and is denoted UT(M).*

It is well known that a n–sphere is identified by the stereographic projection with $\mathbf{R}^n \cup \{\infty\}$ where $\{\infty\}$ is a singular point.

Definition *A n–dimensional, connected, simply connected, compact, closed, smooth manifold* M^n *is called a crystal sphere if there exists such a finite smooth triangulation on* M^n *which is coordinated with the smoothness structure of the manifold* M^n *i.e. every simplex (crystal) of the triangulation is an embedded smooth submanifold of* M^n *with a boundary.*

Theorem [2]. *A crystal sphere* M^n *is homeomorphic to the n–sphere.*

Further, we consider only one crystal sphere $M^n \subset W$ with a smooth triangulation considered above. We can fix some Riemannian metric g on the manifold M^n which defines the length of arc of a piecewise smooth curve and the continuous function $\rho(x;y)$ of the distance between two points $x,\ y \in M^n$. The topology defined by the function of distance (metric) ρ is the same as the topology of the manifold M^n, **[3]**.

For any *n*–simpex δ^n the diameter $d\left(\delta^n\right)$ is defined by the formula $d\left(\delta^n\right) = max\,\rho(x;y),\ \ x, y \in \delta^n$. The diameter of the triangulation is called the maximal value among the diameters of the *n*– simplexes. It seems that the diameter of the triangulation can be very small (*subatomic*).

In section 1, using a smooth triangulation above and the function of distance we consider an algorithm of extension of coordinate neighborhood (inner part of the *canonical polyhedron*) constructed in **[2], [4]**. The beginning of the algorithm we call the geometric *Big Bang*. The inner part of the canonical polyhedron is painted white and the boundary of the canonical polyhedron is painted black every step, the other part of the manifold which has not been still painted assumes to be grey (*three kinds of matter* from a physical point of view). A small closed neighborhood of the boundary of the canonical polyhedron we repaint black and call a geometric black hole, **[4]**.

In section 2, we consider deformation of tensor fields, fiber bundles and operators (physical structures and equations) towards the black hole. These deformations are continuous and sectionally smooth and they have a very simple constructions on a white neighborhood where a parameter $t(\gamma)$ of the deformations of structures can be considered as a local time along every piecewise smooth broken line γ. We have got only one black point $x_0 \in M^n$ at the end of all considered algorithms (other part of the manifold is white). Let $\overline{B}(x_0)$ be a small black closed ball with the center x_0. All the resulting parts of the deformed structures have been concentrated into $\overline{B}(x_0)$. We consider an inversion (*Big Bang*) painting inner part of $\overline{B}(x_0)$ white and other points of M^n grey and begin again the process above where the initial simplex is a subset of $\overline{B}(x_0)$. Thus, Big Bangs have a cyclical nature.

We remark that all the algorithms considered in the article are based on the mathematical methodology «step by step». From a physical point of view the processes must have explosive characters *i.e.* a big number of the steps of the algorithms must be produced almost simultaneously.

1. On algorithm of extension of coordinate neighborhood

1°. In this section, we consider some standart facts on a triangulation of a manifold.

Let M^n be a connected, compact, closed and smooth manifold of dimension n and C^n be a cell (coordinate neighborhood) on M^n. A standard simplex Δ^n of dimension n is the set of points $x=(x_1, x_2, ..., x_n)\in \boldsymbol{R}^n$ defined by conditions

$$0\le x_i\le 1,\ \ i=\overline{1,\ n},\ \ x_1+x_2+...+x_n\le 1.$$

We consider the interval of a straight line connected the center of some face of Δ^n and the vertex which is opposite to this face. It is clear that the center of Δ^n belongs to the interval. We can decompose Δ^n as a set of intervals which are parallel to that mentioned above. If the center of Δ^n is connected by intervals with points of some face of Δ^n then a subsimplex of Δ^n is obtained. All the faces of Δ^n considered, Δ^n is seen as a set of all such subsimplexes. Let $U(\Delta^n)$ be some open neighborhood of Δ^n in $\boldsymbol{R}^n$. A diffeomorphism $\varphi: U(\Delta^n)\to M^n\left(\delta^n=\varphi(\Delta^n)\right)$ is called a singular n–simplex on the manifold M^n. *Faces*, *edges*, *the center*, *vertexes* of the simplex δ^n are defined as the images of those of Δ^n with respect to φ.

The manifold M^n is triangulable, **[5]**. It means that for any l, $0\le l\le n$ such a finite set Φ^l of diffeomorphisms $\varphi: \Delta^l\to M^n$ is defined that

a) M^n is a disjunct union of images $\varphi\left(Int\Delta^l\right)$, $\varphi\in\Phi^l$;

b) if $\varphi\in\Phi^l$ then $\varphi\circ\varepsilon_i\in\Phi^{l-1}$ for every i where $\varepsilon_i: \Delta^{k-1}\to\Delta^k$ is the linear mapping transferring the vertexes $v_0,...,v_{k-1}$ of the simplex Δ^{k-1} in the vertexes $v_0,...,\hat{v}_i,...v_k$ of the simplex Δ^k.

We suppose that there exists a smooth finite triangulation on M^n which is coordinated with the smoothness structure of M^n and fix the triangulation. Such triangulations exist for manifolds of dimension 2 or 3.

2°. In this section, we consider an algorithm of extension of white coordinate neighborhood. It reminds us an extension of the universe from a physical point of view. We have got the decomposition described in the **Theorem 1** at the end where C^n is a white cell.

Let δ_0^n be some simplex of the fixed triangulation of the manifold M^n. We paint the inner part $Int\delta_0^n$ of the simplex δ_0^n white and the boundary $\partial\delta_0^n$ of δ_0^n black. There exist coordinates on $Int\delta_0^n$ given by diffeomorphism φ_0. A subsimplex $\delta_{01}^{n-1}\subset\delta_0^n$ is defined by a black face $\delta_{01}^{n-1}\subset\delta_0^n$ and the center c_0 of δ_0^n. We connect c_0 with the center d_0 of the face δ_{01}^{n-1} and decompose the subsimplex δ_{01}^n as a set of intervals which are parallel to the interval c_0d_0. The face δ_{01}^{n-1} is a face of some simplex δ_1^n that has not been painted. We draw an interval between d_0 and the vertex v_1 of the subsimplex δ_1^n which is opposite to the face δ_{01}^{n-1} then we decompose δ_1^n as a set of intervals which are parallel to the interval $d_0 v_1$. The set $\delta_{01}^n \cup \delta_1^n$ is a union of such broken lines every one from which consists of two intervals where the endpoint of the first interval coincides with the beginning of the second interval (in the face δ_{01}^{n-1}) the first interval belongs to δ_{01}^n and the second interval belongs to δ_1^n. We construct a homeomorphism (extension) φ_{01}^1: $Int\delta_{01}^n \to Int\left(\delta_{01}^n \bigcup \delta_1^n\right)$. Let us consider a point $x \in Int\delta_{01}^n$ and let x belong to a broken line consisting of two intervals the first interval is of a length of s_1 and the second interval is of a length of s_2 and let x be at a distance of s from the beginning of the first interval. Then we suppose that $\varphi_{01}^1(x)$ belongs to the same broken line at a distance of $\frac{s_1 + s_2}{s_1} \cdot s$ from the beginning of the first interval. It is clear that φ_{01}^1 is a homeomorphism giving coordinates on $Int\left(\delta_{01}^n \bigcup \delta_1^n\right)$. We paint points of $Int\left(\delta_{01}^n \bigcup \delta_1^n\right)$ white. Assuming the coordinates of points of white initial faces of subsimplex δ_{01}^n to be fixed we obtain correctly introduced coordinates on $Int\left(\delta_0^n \bigcup \delta_1^n\right)$. The set $\sigma_1 = \delta_0^n \bigcup \delta_1^n$ is called a *canonical polyhedron*. We paint faces of the boundary $\partial\sigma_1$ black.

We describe the contents of the successive step of the algorithm of extension of coordinate neighborhood. Let us have a canonical polyhedron σ_{k-1} with white inner points (they have introduced *white coordinates*) and the black boundary $\partial\sigma_{k-1}$. We look for such an n–simplex in σ_{k-1}, let it be δ_0^n that has such a black face, let it be δ_{01}^{n-1} that is simultaneously a face of some n–simplex, let it be δ_1^n, inner points of which are not painted. Then we apply the procedure described above to the pair δ_0^n, δ_1^n. As a result we have a polyhedron σ_k with one simplex more than σ_{k-1} has. Points of $Int\sigma_k$ are painted white and the boundary $\partial\sigma_k$ is painted black. The process is finished in the case when all the black faces of the last polyhedron border on the set of white points (the cell) from two sides.

After that all the points of the manifold M^n are painted black or white, otherwise we would have that $M^n = M_0^n \bigcup M_1^n$ (the points of M_0^n would be painted and those of M_1^n would be not) with M_0^n and M_1^n being unconnected, which would contradict of connectivity of M^n.

Thus, we have proved the following

Theorem 1. *Let M^n be a connected, compact, closed, smooth manifold of dimension n. Then $M^n = C^n \bigcup K^{n-1}, C^n \bigcap K^{n-1} = \varnothing$, where C^n is an n–dimensional cell and K^{n-1} is a union of some finite number of (n–1)–simplexes of the triangulation.*

3°. The main results of this section are based on the representation of C^{n} as a set of piecewise smooth broken lines connecting the initial point c_0 with all black points of the complex K^{n-1} . It reminds us the theory of strings in physics.

We consider the initial simplex δ_0^n of the triangulation and its center c_0. Drawing intervals between the point c_0 and points of all the faces of δ_0^n we obtain a decomposition of δ_0^n as a set of the intervals. In **2°** the homeomorphism $\Psi:\ Int\delta_0^n \to C^{n}$ was constructed and Ψ evidently maps every interval above on a piecewise smooth broken line γ in C^n. We denote $\tilde{M}^{n}=M^n \setminus\{c_0\}$. $\tilde{M}^{n}$ is a connected and simply connected manifold if M^{n} is that. Let $I = [0;1]$, we define a homotopy $F\colon \tilde{M}^{n}\times I\to \tilde{M}^{n}: (x;\, t) \mapsto y=F(x;t)$ in the following way

a) $F(z;\, t)=z$ for every point $z\in K^{n-1}$;

b) if a point x belongs to the broken line γ in C^n and the distance between x and its limit point $z\in K^{n-1}$ is $s(x)$ then $y=F(x;\, t)$ is on the same broken line γ at a distance of $(1-t)s(x)$ from the point z.

It is clear that $F(x;0)=x$, $F(x;1)=z$ and we have obtained the following

Theorem 2. *The spaces* $\tilde{M}^{n}$ and K^{n-1} *are homotopy–equivalent, in particular, the groups of singular homologies* $H_k\left(\tilde{M}^{n}\right)$ *and* $H_k\left(K^{n-1}\right)$ *are isomorphic for every k.*

Corollary 2.1. *The space* K^{n-1} *is connected and if* M^n *is simply connected then* K^{n-1} *is simply connected too.*

Remark 1. *The white coordinates are extended from the simplex* δ_0^n *in the simplex* δ_1^n *through the face* δ_{01}^{n-1} *hence* $Int\delta_{01}^{n-1}$ *has also the white coordinates. On the other hand there exist two linear structures (intervals, the center etc) on* δ_{01}^n *induced from* δ_0^n *and* δ_1^n *respectively. Further, we set that the linear structure of* δ_{01}^{n-1} *is the structure induced from* δ_0^n.

Remark 2. *In the process of getting of* C^n *in* **2°** *we can construct a maximal tree L connecting by intervals all the centers of the n–simplexes of the triangulation via the centers of some white faces.*

Conversely, if we have such a maximal tree L connecting by intervals all the centers of the n–simplexes of the triangulation via the centers of some faces (any from two possible centers of a face can be choosed) then we can extend white coordinates from any simplex δ_0^n *on the maximal cell* C^n *as it was shown in* **2°**. *Thus, the maximal tree L defines the maximal cell* C^3 *and white faces.*

4°. We can retract the complex K^{n-1} to a unique black point x_0. The set of piecewise smooth broken lines transforms in that every step of algorithms considered in **[2]**.

Definition 1. a) *A simplex* $\delta^k \in K^{n-1}\left(k = \overline{1, n-1}\right)$ *is called free if it has at least one free face* δ^{k-1} *i.e. such a face that it is not a face of any other k–simplex from* K^{n-1}.

b) *An edge* $\delta^1 = x_0 x_1$ *is called semi–isolated if it is not an edge of any simplex from* K^{n-1}*. A semi-isolated edge* δ^1 *is called isolated if it is free.*

Let us have a free simplex $\delta^k \in K^{n-1}$ *with some free face* δ^{k-1}*. We consider such a polyhedron* σ *that* σ *is the set of all n–simplexes having common point with* δ^{k-1}.

Theorem 3. *We can redistribute coordinates of white points of the polyhedron* δ *(retract the free simplex* δ^k*) i.e. construct the corresponding mapping* φ_σ *in such a way that the following conditions are fulfilled:*

a) *all the points of* $Int\sigma$ *are painted white i.e. have new white coordinates,*

b) *white coordinates of points of boundary faces of the polyhedron* σ *are not changed.*

c) φ_σ *maps broken lines having boundary black points on* δ^k *onto broken lines having boundary black points on the boundary of the polyhedron* σ.

Proof of a) and b) can be found in **[2]** (Proposition 3).

Proof of c) follows from *n*–dimensional version of proposition 3 considered in **[6]**.

Let σ_k be a canonical polyhedron at any step of the algorithm above. Points of $Int\sigma_k$ are painted white and the boundary $\partial\,\sigma_k$ is painted black, the other part of the manifold which has not been still painted assumes to be grey. In **2°** the homeomorphism Ψ: $Int\delta_0^n \to Int\sigma_k$ was constructed and Ψ evidently maps every interval from $Int\delta_0^n$ on a piecewise smooth broken line in $Int\sigma_k$. It is easy to see that any procedure considered in **[2]** brings to a transformation such a broken line into another broken line connecting the center c_0 of δ_0^n with some black point of the $\partial\,\sigma_k$ (by a analogy with c) of the theorem 3). At the end of all the algorithms considered in **[2]** we obtain a representation $M^n = C^n \cup \{x_0\}$ where C^n has white painting and x_0 is an unique black point in M^n *i.e.* we have a set piecewise smooth broken lines connecting c_0 and x_0.

2. Deformation of a tensor field and a fibre bundle towards a geometric black hole

1°. In this section, we consider the process of extension vector fields corresponding that of extension of the white coordinate neighborhood.

Let σ_k be a canonical polyhedron at any step of the algorithm from **1** and $L\left(M^n\right)$, $L\left(Int\sigma_k\right)$ be the principal fibre bundles of linear frames of the manifolds

M^n and $Int\sigma_k$. The diffeomorphism φ_0 (where $\delta_0^n = \varphi_0(\Delta^n)$) defines the coordinates (x_1, ..., x_n) in some neighborhood of the simplex δ_0^n and the corresponding vector fields $\frac{\partial}{\partial x_1},\dots,\frac{\partial}{\partial x_n}$ on this neighborhood (a local cross–section of $L(M^n)$). Similarly, the diffeomorphism φ_1 (where $\delta_1^n = \varphi_1(\Delta^n)$) defines the coordinates (y_1, ..., y_n) in some neighborhood of the simplex δ_1^n and the vector fields $\frac{\partial}{\partial y_1},\dots,\frac{\partial}{\partial y_n}$ on this neighborhood. We have assumed that the white face δ_{01}^{n-1} has the equation: $y_1=0$ (it can always be obtained by corresponding linear change of variables in $\boldsymbol{R}^n \supset \Delta^n$). The vector fields $X_i = \frac{\partial}{\partial x_i}, \frac{\partial}{\partial y_j}, \quad i, j = \overline{1,n},$ are defined on the face δ_{01}^{n-1} therefore for any point $x \in \delta_{01}^{n-1}$ we have $X_i = \sum_{j=1}^{n} f_{ij}(x)\frac{\partial}{\partial y_i}$ where the functions $f_{ij}(x)$ are smooth. We decompose δ_1^n as a union of the intervals having the following equations: $y_1=t$, $y_2=c_2$, $y_3=c_3$,…, $y_n=c_n$, where 0, c_2,…, c_n are the coordinates of the beginning y_0 of the corresponding interval. For any point $y \in \delta_1^n$ we assume $f_{ij}(y) = f_{ij}(y_0)$ where $y_0 \in \delta_{01}^{n-1}$ is the beginning of the interval where the point y is situated. The vector fields $X_i, \quad i = \overline{1,n},$ are defined on δ_1^n by the formula $X_i = \sum_{j=1}^{n} f_{ij}(y)\frac{\partial}{\partial y_i}$. It is obvious that the constructed vector fields $X_i, \quad i = \overline{1,n},$ are continuous on $\delta_0^n \cup \delta_1^n$ and smooth in any point $x \in \delta_0^n \cup \delta_1^n, x \notin \delta_{01}^{n-1}$.

For the process of the extension of a coordinate neighborhood (**1**, **2**°) we can consider the process of the extension of the vector fields X_1,…, X_n. If these fields are defined on a polyhedron σ_{k-1} and in order to get a polyhedron σ_k we use simplexes δ_0^n, δ_1^n then we apply the procedure described above to obtain the vector fields on σ_k. As a result we obtain correctly defined vector fields X_1,…, X_n on $Int\sigma_k$ *i.e.* a cross–section of $L(Int\sigma_k)$.

So, we come to the following

Proposition 4. *There exists a continuous cross–section of* $L(Int\sigma_k)$: $x \to (X_1,\dots, X_n)_x$, $x \in Int\sigma_k$. *If a point* $x \in Int\sigma_k$ *does not belong* to the subsimplexes of the *triangulation then the cross–section above is smooth at the point x.*

We consider a tensor of type (r, s) on $\boldsymbol{R}^n$:

$$K^0 = \sum k^{\lambda_1 \dots \lambda_r}_{\mu_1 \dots \mu_s}(0)\, e_{\lambda_1} \otimes \dots \otimes e_{\lambda_r} \otimes e^{\mu_1} \otimes \dots \otimes e^{\mu_s},$$

where $e_1,\dots,e_n$ is the standard basic of $\boldsymbol{R}^n$ and $e^1,\dots,e^n$ is the dual basis of $\boldsymbol{R}^n*$.

A tensor field of type (r, s) is defined on $Int\sigma_k$:

$$K^0 = \sum k^{\lambda_1..\lambda_r}_{\mu_1..\mu_s}(0)\, X_{\lambda_1} \otimes ... \otimes X_{\lambda_r} \otimes X^{\mu_1} \otimes ... \otimes X^{\mu_s} \qquad (1)$$

Since the functions $k^{\lambda_1..\lambda_r}_{\mu_1..\mu_s}$ are constant on $Int\sigma_k$ we obtain that the tensor field K^0 is O–deformable on $Int\sigma_k$ *i.e.* some G–structure on $Int\sigma_k$ is defined by K^0 (see [**7**], [**8**]). If the cross–section $(X_1,\ldots, X_n)_x$ is smooth at a point $x\in Int\sigma_k$ then the tensor field K^0 is also smooth at the point.

2°. We define a geometric black hole as a small closed neighborhood of the black boundary of the canonical polyhedron. Then we consider deformations of tensor fields and operators towards the geometric black hole.

For any point $z\in \partial\sigma_k$ we can consider the closed geodesic ball $\overline{B}(z,\varepsilon)$ of a small radius $\varepsilon > 0$. Let $Tb(\partial\sigma_k, \varepsilon) = \bigcup\limits_{z\in\partial\sigma_k} \overline{B}(z,\varepsilon) = GBH(\varepsilon)$.

Definition 2. *We call the set $GBH(\varepsilon)$ a geometric black hole of radius $\varepsilon>0$ of the manifold M^n if $\sigma_k \setminus GBH(\varepsilon)$ is a cell (it is true for some small ε). We paint the points of $GBH(\varepsilon)$ black.*

Any piecewise smooth broken line γ considered in **1**, **4**° can be represented as $\gamma=\gamma_0 \cup \gamma_1$ where $\gamma_1 = \gamma \cap GBH(\varepsilon)$, $\gamma_0 = \gamma \setminus \gamma_1$. The points of γ_0 are painted white and the points of γ_1 are painted black. Let the segment γ_0 have a length s_0 and the segment γ_1 have a length s_1 then (s_0+s_1) is a length of the broken line γ from c_0 to $z\in \partial\sigma_k$.

Let $K(x)$, $x\in M^n$, be a tensor field of type (r, s) and $K^0=K(c_0)$ where c_0 is the center of the initial simplex δ^n_0 of the triangulation of M^n. Also, deformations of structures were considered in [**9**]. So, we construct a deformation $\overline{K}(x)$ of the tensor field $K(x)$ on the manifold M^n.

1) If a point $z\in M^n \setminus Int\sigma_k$ then $\overline{K}(z) = K(z)$.

2) If a point $x\in \sigma_k \setminus GBH(\varepsilon)$ then $\overline{K} = K^0 = K(c_0)$ where K^0 is defined by the formula (1).

3) We assume that $K(x) = \sum k^{\lambda_1..\lambda_r}_{\mu_1..\mu_s}(x)\, X_1 \otimes ... \otimes X_{\lambda_r} \otimes X^{\mu_1} \otimes ... \otimes X^{\mu_s}$, $x \in Int\sigma_k$, where $X_1,\ldots, X_r$ are the vector fields from the proposition 4, a point x belongs a broken line γ and $s(x)$ is the distance from x to c_0 along the broken line γ. For any point $y \in \gamma_1$ we define the tensor field

$$\overline{K}(y) = \sum \bar{k}^{\lambda_1..\lambda_r}_{\mu_1..\mu_s}(y)\, X_1 \otimes ... \otimes X_{\lambda_r} \otimes X^{\mu_1} \otimes ... \otimes X^{\mu_s}$$

in the following way: $\bar{k}^{\lambda_1..\lambda_r}_{\mu_1..\mu_s}(y) = k^{\lambda_1..\lambda_r}_{\mu_1..\mu_s}(x)$ where $s(x) = \dfrac{s(y)-s_0}{s_1}(s_0+s_1)$, $s(y)$ is the distance from y to c_0 along the broken line γ.

It is easy to see that the constructed tensor field $\overline{K}$ is continuous and sectionally smooth, $\overline{K}$ is not smooth on the boundary of *GBH*(ε) and in the points of $Int\sigma_k$ where the cross–section $(X_1,\ldots, X_n)_x$ is not smooth.

Let *L* be some operator defined on the algebra (or some subalgebra) of all the tensor fields on the manifold M^n and $L(K)=K_1$ for a tensor field *K*.

Definition 3. *An operator* $\overline{L}$ *is called a deformation of L towards GBH*(ε) if *it is defined by condition* $\overline{L}(K)= \overline{K}_1$.

3°. Some standarts facts about fibre bundles are considered. We follow **[10], [11].**

A fiber bundle (E, π, M^n, F) consists of manifolds (spaces) E, M^n, F and a smooth (continuous) mapping $\pi : E \to M^n$, furthemore each $x \in M^n$ has an open neighborhood U such that $E_{|u} \cong \pi^{-1}(U)$ is diffeomorphic (homeomorphic) to $U \times F$ via a fiber respecting diffeomorphism (homeomorphism):

$$\begin{tikzcd} E_{|u} \arrow[r, leftrightarrow, "\psi"] \arrow[d, "\pi"'] & U \times F \arrow[d, "pr_1"] \\ U \arrow[r, leftrightarrow] & U \end{tikzcd}$$

E is called the total space, M^n is called the base space, π is called the projection, F is called standard fiber, (U,ψ) is called a fiber chart.

A collection of fiber charts (U_α,ψ_α), such that $\{U_\alpha\}$ is an open cover of M^n, is called a fiber bundle atlas. If we fix such an atlas, then $\psi_\alpha \circ \psi_\beta^{-1}(x,a) = (x, \psi_{\alpha\beta}(x,a))$, where $\psi_{\alpha\beta} : (U_\alpha \times U_\beta) \times F \to F$ is smooth (continuous) and $\psi_{\alpha\beta}(x,\ldots)$ is a diffeomorphism (homeomorphism) of F for each $x \in U_{\alpha\beta} := U_\alpha \bigcap U_\beta$. Thus, we may consider the mappings $\psi_{\alpha\beta} : U_{\alpha\beta} \to G(F)$ with values in the group $G(F)$, $G(F)=Diff(F)$ is the group of all diffeomorphisms of F or $G(F)=Homeo(F)$ is the group of all homeomorphisms of F. Mappings $\psi_{\alpha\beta}$ are called the transition functions of the bundle. They satisfy the cocycle conditions: $\psi_{\alpha\beta}(x) \circ \psi_{\beta\gamma}(x) = \psi_{\alpha\gamma}(x)$ for $x \in U_{\alpha\beta\gamma}$ and $\psi_{\alpha\alpha}(x) = Id_F$ for $x \in U_\alpha$. The collection $\{\psi_{\alpha\beta}\}$ is called a cocycle of transition functions.

Given an open cover $\{U_\alpha\}$ of manifold M^n and cocycle of transition functions we may construct a fiber bundle (E, π, M^n, F).

Principal fiber bundles and vector bundles are the most important cases of fibre bundles.

4°. In this section, we consider deformation of fiber bundles towards the geometric black hole.

If $\Psi : Int\sigma_0^n \to Int\sigma_k, W = \sigma_k \setminus GBH(\varepsilon)$ and $W_0 = \Psi^{-1}(W)$ then $W_0 \subset Int\delta_0^n$. We consider any piecewise smooth broken line $\gamma = \gamma_0 \cup \gamma_1$ from **2°**. If $\gamma_{01} = \Psi^{-1}(\gamma_0)$ and $\gamma_{02} = \gamma_0 \setminus \gamma_{01}$ then $\gamma = \gamma_{01} \cup \gamma_{02} \cup \gamma_1$. We define a homeomorphism $\overline{\Psi} : M^n \to M^n$ by the following conditions:

a) $\overline{\Psi}_{|W_0} = \Psi_{|W_0}$ i.e. $\overline{\Psi}(\gamma_{01}) = \gamma_0$ and $\overline{\Psi}(W_0) = W$;

b) $\overline{\Psi}$ maps every segment $\gamma_{02} \cup \gamma_1$ on the segment γ_1 by the length as it was shown above;

c) $\overline{\Psi}(z) = z$ for every $z \in M^n \setminus Int\sigma_k$.

It is evident that $\overline{\Psi}$ is a sectionally–smooth homeomorphism.

Let (E, π, M^n, F) be a smooth fibre bundle with a collection fibre charts (U_α, Ψ_α). We can choose such a triangulation, let it be initial one, that $W_0 \subset U_0$. We define $\overline{U}_\alpha = \overline{\Psi}(U_\alpha)$ and $\overline{\Psi}_{\alpha\beta}(x) = \Psi_{\alpha\beta}(\overline{\Psi}^{-1}(x))$.

The open cover $\{\overline{U}_\alpha\}$of the manifold M^n and the cocycle $\{\overline{\Psi}_{\alpha\beta}\}$ defines a continuons and sectionally–smooth fiber bundle $(\overline{E}, \overline{\pi}, M^n, F)$.

Since $\overline{U}_0 = \overline{\Psi}(U_0) \supset W$ it follows that the fiber bundle $(\overline{E}, \overline{\pi}, M^n, F)$ is trivial over W i.e.

Difinition 4. *The fiber bundle* $(\overline{E}, \overline{\pi}, M^n, F)$ *is called a deformation of the fibre bundle* (E, π, M^n, F) *towards the* $GBH(\varepsilon)$.

Such characteristics of (E, π, M^n, F) as connections, curvatures *etc* play an important role in the gauge theory, **[11].**

Problem. *It seems to be interesting to consider good defined deformations of the characteristics above towards the* $GBH(\varepsilon)$*i.e. to obtain some similar characteristics of* $(\overline{E}, \overline{\pi}, M^n, F)$.

Remark 3. *At the end of all the algorithms considered in this article and in* **[2]** *we have got a representation* $M^n = C^n \cup \{x_0\}$ *where* C^n *has white painting and* x_0 *is a black point i.e.* $GBH(\varepsilon) = \overline{B}(x_0, \varepsilon)$ *and the resulting parts of the deformed structures are concentrated into* $\overline{B}(x_0, \varepsilon)$. *We consider an inversion called Big Bang painting* $Int\overline{B}(x_0, \varepsilon)$ *white and begin again the processes of extension of coordinate neighborhood and deformations of structures where the initial simplex* δ_0^n *is a subset of* $\overline{B}(x_0, \varepsilon)$.

The set $M^n \setminus \delta_0^n$ *is painted grey after this inversion.*

Thus, Big Bangs have a cyclycal nature.

Conclusion

We consider a crystal sphere as a geometric model of an universe where the world is identified with a fibre bundle of crystal spheres. The following mathematical notions are considered which are close to those studied in physics.

1) Extension of white coordinate neighborhood – extension of the universe.
2) Three paintings – three kinds of matter.
3) The set of piecewise smooth broken lines – strings.
4) A parameter of deformations along a line – a local time along the line.
5) Geometric black hole – black holes (It seems that black holes observed in astronomy are presentations of one big black object).
6) Deformations of tensor fields, operators, fibre bundle towards the geometric black hole – corresponding situations in physics.
7) Geometric Big Bang – Big Bang.

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