THE ANOMALOUS MAGNETIC MOMENT

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Abstract

The anomalous magnetic moment of electron is calculated in the framework of the Schwinger source method from the assumption the electron and is immersed in the magnetic field. The magnetic field causes the modification of the Green function of the charged particle and therefore the modification of the vacuum-to-vacuum amplitude. The derived value of the anomalous magnetic moment of electron is in excelent agreement with experiment. The muon magnetic moment is discussed at the experimental and methodological basis. This article is written in the form of the pedagogical simplicity.

1 Introduction

When motion of electron in applied electromagnetic field is considered, deviations from the primitive electromagnetic interaction can appear. This effect is caused by the existence of the subsequent interaction which is not present in case of the primitive electromagnetic interactions. In such a way such interpretation imply modifications of the effective electromagnetic coupling. This will be made explicit in later considerations. We can draw an experimental consequence of this fact by a simple modification of algorithm for calculation of the charged particle propagation function. We use here the Schwinger source method of quantum field theory. The standard calculation of the magnetic moment of electron can be seen, for instance, in the classical textbooks (Itzykson et al., 1980)

We have seen in (Schwinger, 1973; 1989) that the vacuum amplitude for noninteracting particles that represents the exchange of one photon and one electron is as follows:

$$\langle 0_+|0_-\rangle^{J\eta} = 1 + ... + i\int (d\xi)(d\xi')J_1^{\mu}(\xi)D_+(\xi-\xi')J_{2\mu}(\xi') \quad \times$$

$$i\int(dx)(dx')\eta_1(x)\gamma^0G_+(x-x')\eta_2(x')+... \tag{1}$$

We arrange eq. (1) in such a way that it will involve only the term with the two-particle exchange. Then, for the vacuum amplitude we have:

$$\langle 0_+|0_-\rangle=\int(d\xi)(d\xi')(dx)(dx') \quad \times$$

$$J_1^\mu(\xi)\eta_1(x)\gamma^0D_+(\xi-\xi')G_+(x-x')iJ_{2\mu}(\xi')\eta_2(x'). \tag{2}$$

It should be inserted into this amplitude the following effective sources - eq. 3-11.66 - (Schwinger, 1970; 2018) with g = 2

$$iJ^\mu(\xi)\eta(x)|_{eff}=eq\left[\delta(\xi-x)\gamma^\mu\psi(x)-if^\mu(\xi-x)\eta(x)\right]. \tag{3}$$

However, we simplyfy them by the following way

$$iJ^\mu(\xi)\eta(x)|_{eff}=eq\delta(\xi-x)\gamma^\mu\psi(x), \tag{4}$$

because in our situation it is possible to pick out the contributions that does not involve photons emitted or absorbed by the sources.

With regard to eq. (4) we write for the emission source

$$iJ_2^\mu(\xi)\eta_2(x)|_{eff\ emiss}=eq\delta(\xi-x)\gamma^\mu\eta_2(x) \tag{5}$$

and for the absorption one

$$iJ_1^\mu(\xi)\eta_1(x)\gamma^0|_{eff\ abs}=\psi_1(x)\gamma^0\gamma^\mu\delta(x-\xi). \tag{6}$$

Eqs. (5) and (6) reads in the momentum representation as

$$iJ_2^\mu(k)\eta_2(p)|_{eff\ emiss}=eq\gamma^\mu\psi_2(P); \quad P=k+p, \tag{7}$$

$$iJ_1^\mu(-k)\eta_1(-p)\gamma^0|_{eff\ abs}=\psi_1(-P)\gamma^0\gamma^\mu; \quad P=k+p. \tag{8}$$

After insertion of eqs. (7) and (8) into the momentum representation of the vacuum amplitude (2) we get

$$\langle 0_+|0_-\rangle=-e^2\int d\omega_p d\omega_k\psi_1(-P)\gamma^0\gamma^\mu(m-\gamma p)\gamma_\mu\psi_2(P) \quad =$$

$$-e^2\int dM^2d\omega_P\psi_1(-P)\gamma^0\gamma^\mu F(P)\gamma_\mu\psi_2(P) \tag{9}$$

with

$$F(P)=(2\pi)^3\int d\omega_p d\omega_k\delta(P-p-k)(m-\gamma p) \quad =$$

$$\frac{1}{(4\pi)^2}\left(1-\frac{m^2}{M^2}\right)^{1/2}\left(m-\frac{M^2+m^2}{2M^2}\gamma P\right). \tag{10}$$

In case of the nonexistence of the electromagnetic field the vacuum amplitude for the two-particle exchange in the x-representation follows from eqs. (2), (5) and (6) in the following form:

$$\langle 0_+|0_-\rangle = e^2\int (dx)(dx')\psi_1(x)\gamma^0\gamma^\mu D_+(x-x')G_+(x-x')\gamma_\mu\psi_2(x'). \tag{11}$$

2 The anomalous magnetic moment of electron

Now, the question arises, how this amplitude must be modified in order to involve the presence of a weak and homogenous electromagnetic field. We will suppose that the change of such physical beckground is hidden in the Green function of electron, or, the modification of the amplitude is based on the transformation

$$G_+(x,x') \to G_+^A(x,x'). \tag{12}$$

Then,

$$\langle 0_+|0_-\rangle \to e^2\int (dx)(dx')\psi_1(x)\gamma^0\gamma^\mu D_+(x-x')G_+^A(x-x')\gamma_\mu\psi_2(x'). \tag{13}$$

In case of a free particle we have:

$$x^0 > x'^0: \quad D_+(x-x')G_+(x-x') = -\int dM^2 d\omega_P e^{iP(x-x')}F(P) \tag{14}$$

and after space-time extrapolation

$$D_+(x-x')G_+(x-x') \quad =$$

$$\frac{i}{(4\pi)^2}\int_{m^2}^{\infty}\left(1-\frac{m^2}{M^2}\right)^{1/2}\left(m-\frac{M^2+m^2}{2M^2}\gamma(\frac{1}{i})\partial\right)\Delta_+(x-x',M^2)+C.T., \tag{15}$$

where $C.T.$ are so called contact terms (Schwinger, 1973; 2018).

For electromagnetic field situation we have obviously

$$D_+(x-x')G_+(x-x') \to D_+(x-x')G_+^A(x-x') \tag{16}$$

and the determination of G_+^A wil be the program of the next text.

The Green function of the charged particle with spin 1/2 and mass m (electron) is determined by the symbolic equation:

$$(\gamma\Pi + m)\,G_+^A = 1 \tag{17}$$

with

$$\Pi = p - eqA. \tag{18}$$

Let us put

$$G_+^A = (m - \gamma\Pi)\,\Delta_+^A. \tag{19}$$

Then,

$$(m + \gamma\Pi)\,G_+^A = \left(m^2 - (\gamma\Pi)^2\right)\Delta_+^A = 1. \tag{20}$$

However,

$$-(\gamma\Pi)^2 = -\frac{1}{2}\{\gamma^\mu, \gamma^\nu\}\,\Pi_\mu\Pi_\nu - \frac{1}{2}\{\gamma^\mu, \gamma^\nu\}\,\frac{1}{2}[\Pi_\mu, \Pi_\nu] \quad =$$

$$g^{\mu\nu}\Pi_\mu\Pi_\nu + i\sigma^{\mu\nu}\frac{1}{2}ieqF_{\mu\nu}, \tag{21}$$

where we have used relations

$$-\frac{1}{2}\{\gamma^\mu, \gamma^\nu\} = g^{\mu\nu} \tag{22}$$

$$[\Pi_\mu, \Pi_\nu] = ieqF_{\mu\nu} \tag{23}$$

$$\sigma \stackrel{d}{=} \frac{1}{2}\{\gamma^\mu, \gamma^\nu\}. \tag{24}$$

If we further intruduce symbolic expression

$$\sigma F \stackrel{d}{=} \frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu}, \tag{25}$$

then instead of eq. (20) we write

$$\left[\Pi^2 - eq\sigma F + m^2\right]\Delta_+^A = 1. \tag{26}$$

If F is a homogenous electromagnetic field, the additional term in (26) ($-eq\sigma F$) is a constant matrix that can be grouped with m^2. In such a way we use the ansatz

$$\Delta_+^A(x, x') = e^{ieq\varphi(x,x')}\Delta_+(x - x', m^2 - eq\sigma F), \tag{27}$$

where

$$\varphi(x, x') = \int_{x'}^{x} d\xi^\mu A_\mu(\xi) \tag{28}$$

denoting a straight line path integral. Obviously

$$(-i\partial_\mu - eqA_\mu(x))\, e^{ie\varphi(x,x')} \quad = $$

$$e^{ie\varphi(x,x')}\left(-i\partial_\mu + eq\frac{1}{2}F_{\mu\nu}(x-x')^\nu\right), \tag{29}$$

which expresses the gauge transformation that produces the vector potential

$$A'^\mu(x) = -\frac{1}{2}F_{\mu\nu}(x-x')^\nu. \tag{30}$$

Another relation similar to eq. (29) is obtained by interchanging x and x' while reversing the sign of $F_{\mu\nu}$.

$$e^{ie\varphi(x,x')}\left(-i\partial'^T_\mu - eqA_\mu(x')\right) \quad = $$

$$\left(-i\partial'^T_\mu - eq\frac{1}{2}F_{\mu\nu}(x'-x)^\nu\right)e^{ie\varphi(x,x')}, \tag{31}$$

where we have wrote ∂^T as an indication of differentiation to the left with an associated minus sign. Obviously

$$G^A_+ = \frac{1}{2}\left[(m-\gamma\Pi)\Delta^A_+ + \Delta^A_+(m-\gamma\Pi)\right], \tag{32}$$

or,

$$G^A_+ = e^{ie\varphi(x,x')}\int \frac{(dp)}{(2\pi)^4}e^{ip(x-x')}(m-\gamma p)\frac{1}{p^2+m^2-eq\sigma F-i\varepsilon}. \tag{33}$$

The structure of eq. (33) shows that the Green function G^A_+ is produced by the phase factor

$$\Phi = e^{ieq\varphi(x,x')} \tag{34}$$

and replacing m^2 by $m^2 - eq\sigma F$ and thereby M^2 by $M^2 - eq\sigma F$, while positioning γ^μ at the extremities of all products in a symmetrical way. For combination $D_+G^A_+$ we have:

$$D_+(x-x')G^A_+(x-x') = \Phi\frac{i}{(4\pi)^2}\int_{m^2}^\infty dM^2\frac{M^2-m^2}{M^2-eq\sigma F} \quad \times$$

$$\left[m - \left(1-\frac{1}{2}\,\frac{M^2-m^2}{M^2-eq\sigma F}\right)\gamma(\frac{1}{i})\partial\right]\Delta_+(x-x', M^2-eq\sigma F) + C.T.. \tag{35}$$

Since we consider only the situation with weak fields, we can replace (35) by the equivalent version

$$D_+(x-x')G^A_+(x-x') = \Phi\frac{i}{(4\pi)^2}\int_{m^2}^\infty dM^2\left(1-\frac{m^2}{M^2}\right) \quad \times$$

$$\left(m - \frac{M^2+m^2}{2M^2}\gamma(\frac{1}{i})\partial\right)\Delta_+(x-x', M^2 - eq\sigma F) \quad +$$

$$\Phi\frac{i}{(4\pi)^2}\int_{m^2}^{\infty}\frac{dM^2}{M^2}\left(1-\frac{m^2}{M^2}\right)eq\sigma F\left(m-\frac{m^2}{M^2}\right)\gamma(\frac{1}{i})\partial\Delta_+(x-x', M^2). \tag{36}$$

Now, we involve into eq. (36) the γ^μ-factors appearing in eq. (13). We have:

$$\gamma^\mu\sigma_{\lambda\nu}\gamma_\mu = 0. \tag{37}$$

This also implies

$$\gamma^\mu\Delta_+(x-x', M^2 - eq\sigma F)\gamma_\mu = -4\Delta_+(x-x', M^2) \tag{38}$$

in case of the linear limit. The other combinations are

$$\gamma^\mu\gamma\partial\sigma F\gamma_\mu = -2\gamma\partial\sigma F \tag{39}$$

and

$$\gamma^\mu\partial\sigma\Delta_+(x-x', M^2 - eq\sigma F)\gamma_\mu = -2\gamma\partial\Delta_+(x-x', M^2 - eq\sigma F) \quad +$$

$$4\gamma\partial\Delta_+(x-x', M^2). \tag{40}$$

We shall also restore the $M^2 - eq\sigma F$ combination, in accordance with

$$\Delta_+(x-x', M^2) \quad =$$

$$\Delta_+(x-x', M^2 - eq\sigma F) + eq\sigma F\frac{d}{dM^2}\Delta_+(x-x', M^2), \tag{41}$$

which can be followed by partial integration on the variable M^2. There are no contributions at the integration boundaries, m^2 and ∞. With regard to the equation (39), (40) and (41) we have:

$$\gamma^\mu D_+(x-x')G_+^A(x-x')\gamma^\nu \quad =$$

$$-\frac{i}{(4\pi)^2}\int_{m^2}^{\infty}dM^2\left(1-\frac{m^2}{M^2}\right)\left(4m+\frac{M^2+m^2}{M^2}\gamma\Pi\right)\Delta_+(x-x', M^2) \quad +$$

$$\frac{i}{(4\pi)^2}\int_{m^2}^{\infty}dM^2\frac{2m^2}{M^4}\left(2m+\frac{M^2+m^2}{M^2}\gamma\Pi\right)eq\sigma F\Delta_+(x-x', M^2) + C.T.. \tag{42}$$

The formal matrix transcription

$$\Delta_+^A(M^2) = \frac{1}{-(\gamma\Pi)^2 + M^2 - i\varepsilon} \tag{43}$$

enables to write eq. (42) in the following form:

$$\gamma^\mu D_+(x-x')G_+^A(x-x')\gamma_\mu \quad =$$

$$\frac{i}{(4\pi)^2}\int_{m^2}^{\infty}\frac{dM}{M}\left(1-\frac{m^2}{M^2}\right)\left[\frac{(M-m)^2-2mM}{\gamma\Pi+M-i\varepsilon}+\frac{(M+m)^2+2mM}{\gamma\Pi-M+i\varepsilon}\right] \quad -$$

$$\frac{i}{(4\pi)^2}\int_{m^2}^{\infty}\frac{dM}{M}\frac{2m^2}{M^2}eq\sigma F\left[\frac{(1-m/M)^2}{\gamma\Pi+M-i\varepsilon}+\frac{(1+m/M)^2}{\gamma\Pi-M+i\varepsilon}\right]+C.T. \tag{44}$$

with performed symmetrized matrix multiplication.

Now, let us approach the determination of the contact term. The physical requirements that determine this term apply in the absence of the electromagnetic field. The practice is that the derived matrix element $M(\gamma p)$ must have no singularity in $\gamma\Pi=-m$. It leads to (Schwinger, 1973; 1989; Dittrich, 1978)

$$\frac{1}{\gamma\Pi+M-i\varepsilon}\rightarrow\frac{(\gamma\Pi+m)^2}{(M-m)^2}\frac{1}{\gamma\Pi+M-i\varepsilon} \tag{45}$$

$$\frac{1}{\gamma\Pi-M+i\varepsilon}\rightarrow\frac{(\gamma\Pi+m)^2}{(M+m)^2}\frac{1}{\gamma\Pi-M+i\varepsilon}. \tag{46}$$

Then, using (45) and (46), we have instead of first term of eq. (44)

$$\frac{i}{(4\pi)^2}(\gamma\Pi+m)^2\int_{m^2}^{\infty}\frac{dM}{M}\left(1-\frac{m^2}{M^2}\right)\left[\frac{1-\frac{2mM}{(M-m)^2}}{\gamma\Pi+M-i\varepsilon}+\frac{1+\frac{2mM}{(M+m)^2}}{\gamma\Pi-M+i\varepsilon}\right]. \tag{47}$$

The additional action term is now

$$-\frac{1}{2}\int(dx)(dx')\psi(x)\gamma^0 M(x,x',F)\psi(x'), \tag{48}$$

where

$$M(F)=-(\gamma\Pi+m)^2\frac{\alpha}{(4\pi)}\int_{m}^{\infty}\frac{dM}{M}\left(1-\frac{m^2}{M^2}\right) \quad \times$$

$$\left[\frac{1-\frac{2mM}{(M-m)^2}}{\gamma\Pi+M-i\varepsilon}+\frac{1+\frac{2mM}{(M+m)^2}}{\gamma\Pi-M+i\varepsilon}\right] \quad +$$

$$\frac{\alpha}{2\pi}\int_{m}^{\infty}\frac{dM}{M}\frac{m^2}{M^2}eq\sigma F\left[\frac{(1-m/M)^2}{\gamma\Pi+M-i\varepsilon}+\frac{(1+m/M)^2}{\gamma\Pi-M+i\varepsilon}\right]. \tag{49}$$

This supplements the initial action expression

$$W=-\frac{1}{2}\int(dx)\psi(x)\gamma^0(\gamma\Pi+m)\psi(x). \tag{50}$$

In the situation of the weak field limit and far from its sources, where

$$(\gamma\Pi + m)\psi = 0 \tag{51}$$

we have

$$M(F) \to -\frac{\alpha}{2\pi}\frac{1}{2m}eq\sigma F \int_m^\infty \frac{dM}{M}\frac{m^2}{M^2}\left(\frac{2m}{M}\right)^2 = -\frac{\alpha}{2\pi}\frac{1}{2m}eq\sigma F. \tag{52}$$

The resulting action under these circumstances is after combining of eqs. (48), (50) and (51)

$$W \to -\frac{1}{2}\int (dx)\psi(x)\gamma^0\left(\gamma\Pi + m - \frac{\alpha}{2\pi}\frac{eq}{2m}\sigma F\right)\psi(x) \tag{53}$$

and it involves the additional magneytic moment of $\alpha/2\pi$ magnetons. In terms of the g-factor defined for instance by Schwinger (1973; 2018), we have

$$g = 2\left(1 + \frac{\alpha}{2\pi}\right). \tag{54}$$

With the fine structure constant

$$\alpha = \frac{1}{137,036} \tag{55}$$

we have

$$\frac{\alpha}{2\pi} = 0,00226141..., \tag{56}$$

which is in remarkable accord with those measured for electron

$$\frac{1}{2}g = 1,0011596... \quad \text{(el., exp)} \tag{57}$$

and the muon

$$\frac{1}{2}g = 1,001166... \quad \text{(muon, exp.)} \tag{58}$$

Further discussion on the magnetic moment calculation is for instance in the Schwinger book (Schwinger, 1989; 2018).

Equation (54) is the famous Schwinger correction (Schwinger, 1949). Historically, Schwinger derived this result for the electron, rather than the muon, to explain the measurement of the hyperfine splittings in gallium atoms by Foley and Kusch (Kusch et al., 1947). However, we see in eq. (54), the one-loop QED contribution to the anomalous magnetic moment does not depend on the mass of the fermion and is, therefore, the same for muons and electrons.

3 Discussion

We have considered magnetic moment of electron in the framework Schwinger source theory methods. The magnetic moment of the Lee model was calculated by author at the different article with the interesting results. (Pardy, 1979). We have seen that the anomalous magnetic moment of electron was not the consequence of the internal structure of electron but it was the result of the interaction of electron with electromagnetic field.

The magnetic moment investigation can be extended to further physical objects defined as Nobelian problems. Namely, the amomalous magnetic moment of proton and antiproton, neutron and antineutron, neutrino and antineutrino, omega-meson and omega-antimeson and so on. And, the anomalou mgnetic moments of all chemical elements, and, 30.000.000 organic compounds. So, the goals of the particlle physics of anomalous magnetic moment are great.

The magnetic moment of the muon was not calculated by Schwinger, so it is interesting to disusse the intellectual situation concerning the magnetic moment of muon. The magnetic moment of muon (Jegerlehner, 2008; 2017) was investigated by many experiments. The muon $g-2$ experiments at Fermilab in the US and at J-PARC in Japan have been reached a four times better precision of 16 $\times 10^{-11}$ (from 0.54 ppm (parts per million) to 0.14 ppm). This has triggered a lot of new research activities. The main motivation is the minimal deviation between standard theory and experiment.

Standard Model match perfectly all experimental information. The very high precision experiments are competing with searches for new physics at the high energy frontier set by the Large Hadron Collider at CERN.

There has been remarkable progress in the calculation of the higher order corrections of $g-2$. Aoyama, Hayakawa, Kinoshita and Nio managed to evaluate the five-loop QED correction, which includes about 13 000 diagrams thereby reducing the uncertainty of the QED part which has been dominated by the missing α^5 correction.

The corresponding contributions to the electron $g-2$ together with the extremely precise determination of $g-2$ by Gabrielse et al. allows one to determine a more precise value of the fine structure constant α, which in turn affect the numbers predicted for $g-2$.

Also more precise lepton mass ratios recommended by the CODATA group are slightly affecting the predictions. To the weak interaction contribution the uncertainty could be reduced mainly by the fact that after the discovery of the Higgs particle by ATLAS and CMS at the Large Hadron Collider at CERN, the last relevant missing Standard Model parameter could be determined with remarkable precision.

The largest uncertainties in the SM prediction come from the leading hadronic contributions, or, the hadronic vacuum polarization and the hadronic light-by-light scattering insertions. The hadronic vacuum polarization at α^5, evaluated in terms of electron-positron annihilation data via a dispersion relation has been improved substantially mainly with new data from initial state radiation approach that the U factory DAFNE at Frascati with the KLOE detector and at the B factory at SLAC with the BaBar detector. Lately also new results from BEPC-II at Beijing with the BES-III detector and from VEPP-

2000 at Novosibirsk with the CMD-3 and SND detectors contributed to further reduce the uncertainties.

On the theory side the τ-decay spectra versus electron-positron annihilation data which should essentially agree after an isospin rotation has been resolved by including missing $\gamma - \varrho^0$ mixing effects. Besides the NLO vacuum polarization new the NNLO amounting to 12×10^{-11} has been calculated by Kurz et al. recently.

The most challenging problem remains the hadronic light-by-light contribution of α^3. Unlike the hadronic vacuum polarization which is a one scale problem, the hadronic light-by-light scattering involves three different scales and there are many different hadronic channels contributing. Quite recently, a new approach has been worked out by Colangelo, Hoferichter, Procura and Stoffer, and Pauk and Vanderhaeghen with hadronic light-by-light scattering data in conjunction with dispersion relations (Jegerlehner, 2017).

In spite of the fact that data for a complete evaluation are largely missing there is definitely progress possible with exploiting existing data for $\gamma\gamma \to \pi^-\pi^+, \pi^0\pi^0$ in particular, where new data from Belle are of good quality, which allows one to get more solid evaluations than existing ones. For the singly tagged pion transition form factor there have been new useful data from BaBar and Belle which cover a much larger energy range now (Jegerlehner, 2017).

So, we have seen that the magnetic moment of the mu-meson represented in future by the source theory methods will be evidently the gigantic goal for the Schwinger theorists.

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