The Equivalence Principle, Holographic Principle and Quantum-Gravitational Corrections for Regge Calculus

Alexander Shalyt-Margolin*
Research Institute for Nuclear Problems,
Belarusian State University
Minsk 220040, Belarus

This paper presents a study of quantum-gravitational corrections to General Relativity (GR) when the latter is considered in a discrete pattern of the Regge Calculus. These corrections have been found explicitly in the case when the space-time foam a la Wheeler at the Planck scale comprises Planck (quantum) black holes, whereas the physical system per se, where GR is considered, represents a black hole in stationary state (i.e. in the absorption and radiation processes absence) or after of minimal accretion, and in a more general case,if it satisfies the **Hooft-Susskind Holographic Principle**. It is shown that these corrections generate the contributions changing the plane geometry of the corresponding simplices in the Regge Calculus and that such changes are most significant at high (Planck) energies.

PACS numbers: 11.10.-z,11.15.Ha,12.38.Bx
Keywords: Regge Calculus,holographic principle,quantum-gravitational corrections

I. INTRODUCTION

It is well known that, within the scope of a perturbation theory, quantum gravity is an unrenormalizable theory. Actually, this is associated with the fact that, on going to the progressively higher energies E close to the Planck energy $E \approx E_p$, the arbitrary metric $g_{\mu\nu}$ even locally could not be considered in the form

$$g_{\mu\nu} = \eta_{\mu\nu} + o_{\mu\nu}, \tag{1}$$

where $\eta_{\mu\nu}$ - Minkowski metric and $o_{\mu\nu}$ – its certain small increment.
As the perturbation theory methods in a quantum approach to gravity are disabled, to obtain quantum gravity, the researchers actively use the approaches beyond the scope of a perturbation theory, in particular, the discrete methods independent of the background space-time such as Regge Calculus [1] – [4],Causal Dynamical Triangulations (CDT) [5]–[10], and so on. However, in all the above-mentioned discrete methods from the start it is expected that elementary components of the corresponding discretizations of the initial space (more generally, manifolds) possess a plane geometry, that connives the validity of the Einstein Strong Equivalence Principle (SEP) at all energy scales. But the situation may be different. For example, in [11] and [12] it has been demonstrated that SEP may be violated at high energies if space-time foam at Planck scales comprises quantum (Planck) black holes [13]. Consequently, there exists the problem of taking into consideration the quantum-gravitational corrections generated by this quantum black holes for all quantities, in particular for simplices - elementary discretization components in the Regge Calculus.
This paper presents a study of quantum-gravitational corrections to General Relativity

*Electronic address: a.shalyt@mail.ru

(GR) [14]–[16] when the latter is considered in a discrete pattern of the Regge Calculus [4]. These corrections have been found explicitly in the case when the space-time foam a la Wheeler at Planck's scale comprises the Planck (quantu8m) black holes, whereas the physical system per se, where GR is considered, is either a black hole in stationary state (i.e. in the absorption and radiation processes absence) or after of minimal accretion, and in a more general case,if it satisfies the **Hooft-Susskind Holographic Principle**. It is shown that the corrections generate the contributions changing a plane geometry of the corresponding simplices in the Regge Calculus and that these changes are most significant at high (Planck) energies.

II. THE EQUIVALENCE PRINCIPLE AND QUANTUM BLACK HOLES AS FOUNDATION OF SPACE-TIME FOAM

The "geometry" of space-time foam unknown to present day prevents correct generalization of the well-known Einstein Equivalence Principle (EP) to high (Planck) energies. At the same time, researchers are actively engaged in studies of this Principle and of its possible violations.
In particular, in [17],[18] it has been demonstrated that in a field of a large (i. e, classical) four-dimensional Schwarzschild black hole with the metric

$$ds^2 = \left(1 - \frac{2MG}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 \tag{2}$$

(where M is the mass of this black hole and horizon radius is $r_{BH} = 2MG$), the EP is violated for an observer distant from the black hole event horizon.
The principal result of the works [17],[18] is based on the inference that, provided EP is valid, for a distant observer in a field of the above-mentioned black hole the equality to the Unruh temperature given by [19] is the case

$$T_{Unruh} = \frac{\hbar a}{2\pi}, \tag{3}$$

where $a = |\mathbf{a}|$ is the corresponding acceleration,
with a temperature conditioned by Hawking radiation [20] and given by formula [17] as follows:

$$T_{H,r} = \frac{\hbar}{8\pi GM\sqrt{1 - \frac{r_{BH}}{r}}} , \tag{4}$$

where r - distance between an observer and the event horizon ($r > r_{BH}$); $c = k_B = 1$.
In [17] it is shown that an observer, positioned at the fixed distance $r > r_{BH}$ from the above-mentioned black holes and measuring Hawking temperature with the value $T_{H,r}$, experiences the local acceleration

$$a_{BH,r} = \frac{1}{\sqrt{1 - \frac{r_{BH}}{r}}} \left(\frac{r_{BH}}{2r^2}\right) . \tag{5}$$

Another observer in the Einstein elevator, moving with acceleration through Minkowskian space-time, will measure the same acceleration toward the floor of the elevator, thermal radiation with the Unruh temperature given by formula (3). As shown in [17], a_{BH} is coincident with the quantity a from the formula in (3). Then substituting the acceleration

$a = a_{BH}$ from formula (5) into formula (3), we can obtain a formula for $T_{Unruh,r}$ in this case [18]:

$$T_{Unruh,r} = \frac{\hbar}{2\pi\sqrt{1-\frac{r_{BH}}{r}}}\left(\frac{r_{BH}}{2r^2}\right) \quad . \tag{6}$$

If EP is valid, the quantities $T_{Unruh,r}$ from formula (6) and $T_{H,r}$ in (4) should be coincident for $r > r_{BH}$ to a high degree of accuracy. However, we see that this is not true and for $r > r_{BH}$ we have $T_{H,r} > T_{Unruh,r}$ and similarly for $r \gg r_{BH}$,i.e. for distant observer, $T_{H,r} \gg T_{Unruh,r}$.
All the foregoing results are valid in the low-energy domain $E \ll E_p$. The problem arises, what happens in the high-energy (quantum-gravitational) domain $E \approx E_p$, i.e. in the region, where the space-time foam exists [21]–[28].In [24]–[28] the space-time foam has been studied in the assumption that it comprises micro-black holes (or quantum black holes **qbh** by the terminology of [13]).
As shown in [11], the principal results from [17],[18] may be generalized to **qbh** with due regard for quantum-gravitational corrections [29] – [33]. This is briefly explained as follows.
Without loss of generality, it is considered that at Planck's energies the Generalized Uncertainty Principle (GUP) is satisfied [34]–[37] according to [34]

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' l_p^2 \frac{\Delta p}{\hbar}, \tag{7}$$

where α' is a constant on the order of 1. Evidently, this formula (7) initially leads to the minimal length $l_{min} = \widetilde{\ell}$ on the order of the Planck length $\widetilde{\ell} \doteq 2\sqrt{\alpha'} l_p$.
Then, in conformity with the principal result from [38], radiation of **qbh** will persist so long as its radius r_{qbh} were not minimal and equal to $\widetilde{\ell}$

$$r_{qbh,min} = l_{min} = \widetilde{\ell}. \tag{8}$$

The final stage of this radiation is represented by a minimal nonradiative Planck's remnant [38],[13]) that is called the extremal **qbh**.
It is supposed that the **qbh** under study is not extremal (it is radiative), with the radius $r = r_{qbh} > r_{qbh,min}$ on the order of the Planck one $r_{qbh} \propto l_p$.
According to the present-day knowledge, a semi-classical region starts between 5 and 20 times the Planck scale [39]. Then, despite the fact that this **qbh** is itself in the quantum gravity region, the scales of $r \gg r_{qbh}$ (corresponding to a distant observer) determine the region, where a semi-classical approximation is valid.
This means that, on substitution of **qbh** with the mass m_{qbh} for a large (classical) black hole with the mass M, in the case under study the results, substantiated when an observer uses the standard Unruh-Dewitt detector in radiation measurement for coupled to a massless scalar field [40]–[42], are valid with the corresponding quantum corrections [29] – [33]. This quantum corrections in the four-dimensional Schwarzschild classical black hole case are vanishingly small. Nevertheless, for **qbh** they are already high.
In [11] it is demonstrated that, if $r \gg r_{qbh}$ (semiclassical approach), then $T_{H,r,q} \gg T_{U,r,q}$,where $T_{H,r,q}$ is the temperature Schwarzschild **qbh** but with regard to the corresponding quantum corrections (for example, [29],[30],[31],[32],[33]). Similarly, $T_{U,r,q}$ is the Unruh temperature with the corresponding quantum corrections.
In this way the principal result from [17],[18] may be generalized to **qbh** comprising the space-time foam [24]–[28]. As canonical QFT is a local theory defined in space-time with plane geometry [43]–[45], validity of EP is implied. Because of this, the main objective

of [11] was to show that, when space-time foam comprises the above-mentioned **qbh**,the applicability boundary of QFT falls within energies considerably lower than the Planck energy $E \ll E_p$.
It should be noted that Albert Einstein formulated his strong Equivalence Principle only for classical objects against a classical space-time background. This is explained by the fact that in the period, when General Relativity was created (1915,1916) and this principle was formulated, a quantum theory was at the stage of conception and its main postulates were unknown. Because of this, formulation of the principal result from [17],[18] is not quite exact. Strong EP, in its initial formulation, is still valid and the principal result from [17],[18] should be as follows:
generalization of the strong Equivalence Principle for the case of a semiclassical approximation in a black hole gravitational field becomes invalid.

III. REGGE CALCULUS AND QUANTUM-GRAVITATIONAL CORRECTIONS TO IT

Now it is assumed that space-time foam at Planck scales comprises **qbh** and our objective is to calculate the quantum-gravitational corrections **qgc**in this case. Specifically, we are interested in calculation of **qgc** for the case of quantum black hole using the Regge Calculus.
Let us consider the Einstein gravitational theory for d-dimensional space-time manifold $\mathcal{M}$ in terms of the lattice discretized Regge description [1],[3].
In the standard Regge Calculus it is assumed the simplicial decomposition of any space-time d-dimensional manifold $\mathcal{M}$ [3],[4]. This means that in such a case the elementary building blocks represent simplices of the dimension d. A point represents 0-simplex, an edge–1-simplex, a triangle–2-simplex, a tetrahedron–3-simplex,..., accordingly d-simplex has $d+1$ vertices and $d(d+1)/2$ edges connecting them. All angles of such a construction, named the simplicial complex, are unambiguously defined; it consists of numerous "glued together" simplices, which in pairs are not intersecting or have a common edge. The lattice model, where "the relative position of points on the lattice is thus completely specified by an incidence matrix (it tells which point is next to which) and the edge lengths, and this in turn induces a metric structure on the piecewise linear space" (p.831 in [3]),is the case. It is assumed that each d-simplex has a plane geometry and contains sub-simplices of smaller dimension.
Then combined gravitational action in the Euclidean regime, that includes the cosmological term as well, by this approach used in the Regge formalism (formula (89) in [3]) may be given as follows:

$$I_{\rm latt}(l^2) \;=\; \lambda_0 \sum_{\text{simplices s}} V_s^{(d)} \;-\; k \sum_{\text{hinges h}} \delta(h)\, V_h^{(d-2)}, \tag{9}$$

where the second term

$$I_R(l^2) \;=\; -\,k \sum_{\text{hinges h}} \delta(h)\, V_h^{(d-2)} \tag{10}$$

is the Euclidean lattice action for pure gravity (formulae (formula (86) in [3]), λ_0– cosmological constant value, $V_s^{(d)}$ – d-simplex volume (centered on s),$V_h^{(d-2)}$ – $d-2$-simplex volume (centered on h), and $k = 1/(8\pi G)$. And $\delta(h)$ – deficit angle at h (formulae (62) in [3] and (6.13) in [4])

$$\delta(h) \;=\; 2\pi \;-\; \sum_{s\supset h} \theta(s,h) \tag{11}$$

for the sum extends over all simplices s meeting on h. The $\delta(h)$ is a measure of the curvature at h [2],[3], [4].
Let us assume that space-time dimension $d=4$. With the use of the Regge Calculus, it is supposed that all the involved simplices and sub-simplices are of plane geometry [2]–[4]. But, due to the results from the previous section, in particular those generated by **qbh**. It is assumed that any **qbh** is radiative until it becomes **qbh**, with the minimal radius $r=r_{min}$ [38].
We use the results from [30] and, for convenience, the notation of this paper. Then GUP may be given as follows ([30],formula (16)):

$$(\delta X)(\delta P) \geq \frac{\hbar}{2}\left(1+\frac{\alpha^2 l_p^2}{\hbar^2}(\delta P)^2\right). \qquad (12)$$

with the following minimal length l_{min}

$$(\delta X)_0 = \alpha l_p \propto l_p. \qquad (13)$$

Compared to (7), here the symbols of $\Delta x \mapsto \delta X, \Delta p \mapsto \delta P$ are changed.
The radius and mass of a minimal Schwarzschild black hole are given as [30] (formula (20))

$$r_h = (\delta X)_0 = \sqrt{\frac{e}{2}}\alpha l_p, \quad M_0 = \frac{\alpha\sqrt{e}}{2\sqrt{2}} M_{Pl}, \qquad (14)$$

where e is the natural logarithms base and the following condition is satisfied [30]

$$\frac{\alpha^2 l_p^2 \exp(\frac{2\alpha^2 l_p^2}{\hbar^2}\langle P\rangle^2)}{2(\delta X)^2} \leq \frac{1}{e}. \qquad (15)$$

Let's first consider the case of a black hole's minimal accretion.
In [46],[47] a minimal increase in the area of a black hole absorbing a classical particle of the energy E has been calculated, and the size R is given by $(\Delta A)_0 \simeq 4l_p^2(\ln 2)ER$. In a quantum pattern we have $R\sim 2\delta X$ and $E\sim \delta P$.
Based on this result, [30] presents a derivation of a new explicit expression for $(\Delta A)_0$ that may be considered as a quantum-gravitational correction **qgc** to the arbitrary black hole horizon area [30] (formula (27)):

$$(\Delta A)_0 \approx 4l_p^2 \ln 2 \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\frac{A_0}{A}\right)\right), \qquad (16)$$

where A is the black hole horizon area of the given black hole and $A_0=4\pi(\delta X)_0^2$ is the black hole horizon area of a minimal quantum black hole from formula (14). Here the expression $W\left(-\frac{1}{e}\frac{A_0}{A}\right)$ in the right-hand side of (16) represents a value at the pint $-\frac{1}{e}\frac{A_0}{A}$ of the Lambert function $W(u)$ satisfying the equation (formulae (1.5) in [48] and (9) in [30])

$$W(u)\,e^{W(u)} = u. \qquad (17)$$

In what follows, we assume that the left and the right sides in the equation (16) are, to a high accuracy, coincident and hence, instead of an approximate equality, we use $(\Delta A)_0 = 4l_p^2\ln 2\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\frac{A_0}{A}\right)\right)$.
It is clear that for a large (classical) Schwarzschild black hole, when A is the horizon area, the value of A_0/A is very small and very close to zero. Because of this, a value of the

expression $W(\frac{1}{e}\frac{A_0}{A})$ is close to $W(0)$. It is easily seen that $W(0)=0$ presents an explicit solution of the equation (17). Then in the right side we have (16)

$$\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\frac{A_0}{A}\right)\right)\approx 1, \tag{18}$$

and hence

$$\left(\Delta A\right)_0 \approx 4l_p^2\ln 2. \tag{19}$$

For a small Schwarzschild black hole with the horizon area A, The quantity A_0/A is already significant and, on condition $A\rightarrow A_0$,it is close to 1 when the given black hole is close to a minimal quantum black hole (minimal **qbh**).
Obviously, the quantum-gravitational correction **qgc** (16) presents a *deformation* (or more exactly, the *quantum deformation* of a classical black-holes theory from the viewpoint of the paper [49], with the deformation parameter A_0/A.
However, we have

$$\frac{A_0}{A}=\frac{4\pi r_h^2}{4\pi R^2(A)}=\frac{l_{min}^2}{R^2(A)}, \tag{20}$$

where $r_h=l_{min}$ is the horizon radius of minimal **qbh** from formula (14) and $R(A)$ is the horizon radius of the given black hole with the black hole horizon area A. It should be noted that this deformation parameter

$$l_{min}^2/R^2(A)\doteq\alpha_{R(A)} \tag{21}$$

has been introduced by the author in his earlier works [50]–[52], where he studied deformation of quantum mechanics at Planck scales in terms of the deformed quantum mechanical density matrix. In the Schwarzschild black hole case $\alpha_{R(A)}=l_{min}^2\mathcal{K}$ – product of the minimal surface area l_{min}^2 by the Gaussian curvature $\mathcal{K}=1/R^2(A)$ of the black-hole horizon surface [53].
So, if the initial closed manifold $\mathcal{M}$ is a black hole with the horizon area $A(\mathcal{M})$, all the foregoing calculations relating to "quantum corrections of the area" (in particular formula (16) for this manifold are valid).
Obviously, a value of the deformation parameter $\alpha_{R(A)}$ for the manifolds $\mathcal{M}$ of large linear sizes, with the surface area $A(\mathcal{M})$ adequately given by formula (16), equals $A_0/A(\mathcal{M})=1/N_{A(\mathcal{M})}, N_{A(\mathcal{M})}\gg 1$ and for manifolds $\mathcal{M}$ of linear sizes close to Planck scales we have $A_0/A(\mathcal{M})=1/N_{A(\mathcal{M})}, N_{A(\mathcal{M})}\gtrsim 1$.
For Schwarzschild **bh** with the event horizon area $A(\mathcal{M})$ after above minimal absorbtion in virtue (16) we obtain Schwarzschild **bh** with the event horizon area $A(\mathcal{M})+\Delta A(\mathcal{M})_0$ and radius $R(A(\mathcal{M})+\Delta A(\mathcal{M})_0)$.
Arbitraries Linear Elements (**LE**) and Areas in first black hole in this transition transformates into corresponding quantities in second black hole as

$$LE\mapsto LE\times\frac{R(A(\mathcal{M})+\Delta A(\mathcal{M})_0)}{R(A(\mathcal{M})},$$
$$Area\mapsto Area\times\frac{R^2(A(\mathcal{M})+\Delta A(\mathcal{M})_0)}{R^2(A(\mathcal{M}))}=Area\times\frac{A(\mathcal{M})+\Delta A(\mathcal{M})_0}{A(\mathcal{M})} \tag{22}$$

Then from (10) for $d = 4$ we have

$$V_h^{(2)} \mapsto V_{h,q}^{(2)} = V_h^{(2)} \frac{A(\mathcal{M}) + \Delta A(\mathcal{M})_0}{A(\mathcal{M})};$$

$$\begin{aligned} I_R(l^2) &= -k \sum_{\text{hinges h}} \delta(h)\, V_h^{(2)} \mapsto I_R(l^2, q) \doteq -k \sum_{\text{hinges h}} \delta(h)\, V_{h,q}^{(2)} = \\ &= -k \sum_{\text{hinges h}} \delta(h)_q\, V_h^{(2)}, \delta(h)_q = \delta(h) \frac{A(\mathcal{M}) + \Delta A(\mathcal{M})_0}{A(\mathcal{M})} \end{aligned} \tag{23}$$

Here $\delta(h)_q$ is **"quantum deformation"** of a curvature measure $\delta(h)$ from formula (11).

I.Semiclassical Picture
In this case above contributions are small due to smallness of the quantity $A_0/A(\mathcal{M}) = 1/N_{A(\mathcal{M})}, N_{A(\mathcal{M})} \gg 1$ for any Schwarzschild **bh** which may be studied in a semiclassical approximation,i.e. for large enough **bh**. This result is independent of the available energies and then

$$\delta(h)_q \approx \delta(h) \tag{24}$$

II.Quantum-gravitational Consideration
In this case size of **bh** (i.e. of the initial manifold $\mathcal{M}$ a close to Planck sizes. Then there is no way for consideration of such $\mathcal{M}$ in a classical pattern.
Then there are significant quantum gravity corrections

$$\delta(h)_q \neq \delta(h) \tag{25}$$

These quantum-gravitational corrections make contributions into all other quantities in a discrete representation of gravity using the Regge Calculus, specifically, in the gravity field equations in vacuum and to the statistical sum of a theory.
Remark 3.1 Most often a black hole is considered within the scope of the above-mentioned accretion process. When a black hole is studied in the stationary state (i.e. without absorption and emission of matter), the foregoing procedure for calculation of **qgc** is radically simplified.
From the formula for temperature of a black hole with regard to **qgc** within the scope of GUP $T_{\mathrm{H},q}$ ((23) in [30])

$$T_{\mathrm{H},q} = \frac{1}{8\pi M l_p^2} \exp\left(-\frac{1}{2} W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right), \tag{26}$$

one can easily find a mass of this black hole, with regard to **qgc**, denoted by M_q

$$M_q = M \exp\left(\frac{1}{2} W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right). \tag{27}$$

As, with due regard for **qgc**, a Schwarzschild black hole remains the same, a radius of the initial black hole $R(A)$ and its radius with regard to the corrections of $R(A_q)$ are proportional to M and M_q, respectively, and have the same proportionality factor $2G$.
Then from formula (27) it follows that the area of a black hole, with regard to **qgc** A_q , takes the form

$$A_q = A \exp\left(W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right). \tag{28}$$

This suggests that the term $\exp\left(W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)$ in the right side of the last formula should be simply added as a factor for each component $\delta(h)$ to measure the curvature at the hinge h in formula (9). Further analysis follows the foregoing analysis after **Remark 4.1**: for large black holes this factor is very close to 1 and it makes a contribution only in the case of small black holes at the energies $E \approx E_p$.
At the same time, these calculations are also valid in the case when $\mathcal{M}$ satisfies the **Hooft-Susskind Holographic Principle** [54]–[56] assuming that all information about a physical system on $\mathcal{M}$ is found at its boundary with the area $A(\mathcal{M})$.
Really, in this case $\mathcal{M}$ is a holographic screen and by virtue of results in [57] a physical system on $\mathcal{M}$ is equivalent to black hole with event horizon coinciding with the surface of this holographic screen,i.e. with the $A(\mathcal{M})$. Thus,in this case all the above methods and formulae remain valid.

IV. CONCLUSION

Let us formulate the obtained results.

1) In conditions, when a physical system considered for the manifold $\mathcal{M}$ is either a black hole in stationary state (i.e. in the absorption and radiation processes absence) or after of minimal accretion, and in a more general case,if it satisfies the **Hooft-Susskind Holographic Principle**, the following arguments are valid:

1.a) for the manifold $\mathcal{M}_{\mathcal{L}_{\mathcal{M}}}$ with large linear sizes $\mathcal{L}_{\mathcal{M}}$, much larger than the Planck's $\mathcal{L}_{\mathcal{M}} \gg l_p$ (i.e. the manifold under classical consideration), **qgc** are small and calculable in Regge Calculus for 2-simplices with any edge lengths or for the curvature measure $\delta(h)$ at any hinge h;

1.b)on the contrary, for the manifold $\mathcal{M}$ with linear sizes $l_{\mathcal{M}}$ commensurable with the Planck scales $l_{\mathcal{M}} \gtrsim l_p$ (i.e. the manifold having **only** quantum consideration), **qgc** are also calculable and significant both in Regge Calculus for each 2-simplex with any edge lengths or for the curvature measure $\delta(h)$ at any hinge h;

2) Also, the obtained results imply:
in the canonical Regge Calculus [1] – [4] it is assumed that all simplices of the corresponding manifold $\mathcal{M}$ have a plane geometry, implicitly pointing to validity of the Einstein Strong Equivalence Principle (SEP) that, as noted above, in particular cases is not true. In fact, the foregoing quantum-gravitational corrections **qgc** *represent corrections to a plane geometry in this case.*

3) Hypothesis
In a gravitational field of the **quantum black hole** all the foregoing results are valid within the scope of the Regge Calculus for *any physical system* considered on the manifold $\mathcal{M}$.

Commentary

A greatly expanded version of this work, including the necessary calculations for the corresponding partition function, was submitted to the journal NPCS.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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