A Linear Equation of Motion

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Abstract

In classical mechanics, this paper presents a linear equation of motion, which can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.

Linear Equation of Motion

If we consider two particles A and B of mass m_a and m_b respectively, then the linear equation of motion, is given by:

$$m_a m_b \left[\frac{(\mathbf{r}_a - \mathbf{r}_b)}{|\mathbf{r}_a - \mathbf{r}_b|} \cdot (\mathbf{v}_a - \mathbf{v}_b) \right] = m_a m_b \left[\frac{(\mathbf{r}_a - \mathbf{r}_b)}{|\mathbf{r}_a - \mathbf{r}_b|} \cdot \int \left(\frac{\mathbf{F}_a}{m_a} - \frac{\mathbf{F}_b}{m_b} \right) dt \right]$$

where $\mathbf{r}_a$ and $\mathbf{r}_b$ are the positions of particles A and B, $\mathbf{v}_a$ and $\mathbf{v}_b$ are the velocities of particles A and B, and $\mathbf{F}_a$ and $\mathbf{F}_b$ are the net forces acting on particles A and B.

This linear equation of motion can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces. In addition, this linear equation of motion is invariant under transformations between reference frames.

Conservation of Linear Momentum

A system of particles forms a system of biparticles. For example, the system of particles A, B, C and D forms the system of biparticles AB, AC, AD, BC, BD and CD.

The total linear momentum of a system of biparticles, is given by:

$$\sum_i \sum_{j>i} m_i m_j \left[\frac{(\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_i - \mathbf{r}_j|} \cdot (\mathbf{v}_i - \mathbf{v}_j) - \frac{(\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_i - \mathbf{r}_j|} \cdot \int \left(\frac{\mathbf{F}_i}{m_i} - \frac{\mathbf{F}_j}{m_j} \right) dt \right] = 0$$

where m_i and m_j are the masses of the i-th and j-th particles, $\mathbf{r}_i$ and $\mathbf{r}_j$ are the positions of the i-th and j-th particles, $\mathbf{v}_i$ and $\mathbf{v}_j$ are the velocities of the i-th and j-th particles, and $\mathbf{F}_i$ and $\mathbf{F}_j$ are the net forces acting on the i-th and j-th particles.

Consequently, from the above equation it follows that the total linear momentum of a system of biparticles is always in equilibrium.

On the other hand, the above equation would be valid even if Newton's third law were false.

General Equation of Motion

The linear equation of motion can be obtained from the following general equation of motion:

$$\sum_i \sum_{j>i} m_i m_j \left[\frac{(\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_i - \mathbf{r}_j|} \cdot (\mathbf{r}_i - \mathbf{r}_j) - \frac{(\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_i - \mathbf{r}_j|} \cdot \int\!\!\int \left(\frac{\mathbf{F}_i}{m_i} - \frac{\mathbf{F}_j}{m_j} \right) dt\, dt \right] = 0$$

where m_i and m_j are the masses of the i-th and j-th particles, $\mathbf{r}_i$ and $\mathbf{r}_j$ are the positions of the i-th and j-th particles, and $\mathbf{F}_i$ and $\mathbf{F}_j$ are the net forces acting on the i-th and j-th particles.