Maximal Mass Within Sphere,Repulsion,Black Holes and Some Implications

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Abstract

This paper presents one of the approaches to solution of the problem of the *repulsion* origin in gravity. The approach is based on the property of compactness characteristic for a self-gravitating object in General Relativity. Here we understand "compactness" as estimation of the upper boundary for such an object in a static two-dimensional sphere. Repulsion originates when this boundary is violated. The main hypothesis is formulated in the form of the principle – principle of maximal mass within a two-dimensional static sphere. It is demonstrated that the principle is true for Schwarzschild black holes on absorption of the matter in the process of accretion, both in the classical case and with due regard for quantum-gravitational corrections. The results have been extended to black holes with the Schwarzschild-de Sitter metric in the early Universe. The applicability of the principle suggested is analyzed for the early and for the present Universe.

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1 Introduction

By its nature, gravity represents the attracting force, as it had been indicated in the Newtonian formulation and is still accepted in General Relativity (GR) [1]–[4]. But it is well known that cosmology involves *repulsion*

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as well [5]–[7]. A similar pattern is taken for the initial Universe expansion due to negative sign of the pressure in the energy-momentum tensor of the Einstein equation, opposite to a sign of the vacuum energy density. It is important to understand the possibility for origination of the repulsion phenomenon in Gravity. Currently, possible *repulsion* in gravity is extensively studied from different points of view (for example,[8]–[10]).
This paper suggests one of the approaches to solving the *repulsion* origin problem in gravity. The approach is based on the property of compactness characteristic for the self-gravitating object in GR. Here "compactness" is understood as an estimate of the upper boundary for the mass of this object in a static two-dimensional sphere.
The paper is structured as follows. In the next section the key assumption, called the **principle**, is formulated and it is shown that Schwarzschild black holes completely satisfy this **principle** both in the classical discussion and with regard to the quantum-gravitational corrections. In Section 3 the results of Section 2 are generalized to the primordial black holes **pbh** with the Schwarzschild-de Sitter metric in the early Universe. Section 4 presents an analysis of the **principle** applications to the early and the present Universe. The conclusion presents the relevant problems.

2 Maximal Mass Principle within Two-Dimensional Static Sphere and Schwarzschild Black Holes

Let us recall the Buchdahl Theorem **BT** [11] stating that
the mass $\mathcal{M}$ of a spherically symmetric self-gravitating material object with the radius $\mathcal{R}$, the interior of which may be taken within the scope of General Relativity as a perfect fluid, satisfies the condition

$$\mathcal{M} \leq \frac{4}{9}\frac{\mathcal{R}c^2}{G}. \qquad (1)$$

But we know that for a Schwarzschild black hole, with the same mass and radius, the following relation [12] is true:

$$\mathcal{M} = \frac{1}{2}\frac{\mathcal{R}c^2}{G}. \qquad (2)$$

In this way we have contradiction between values of the dimensionless coefficient $\mathcal{C} = 4/9$ in the right side of (1) and $\mathcal{C} = 1/2$ in the right side of (2). In [13],[14] this contradiction has been studied to show that (section 2.2 in [14]):

2.1 BT was proven for an incompressible fluid, with an infinite speed of sound $c_s = \infty$, and this is in contradiction with causality as the speed of sound c_s should always be lower than the speed of light c, i.e. should satisfy the condition $c_s \leq c$. Moreover, the **BT**-bound (1) in the case of an incompressible fluid violates the **Dominant Energy Condition** (**DEC**) in General Relativity [15]:

$$\mathbf{DEC}: \quad \rho \geq |p_{\text{rad}}|, \quad \rho \geq |p_{\text{tan}}|, \tag{3}$$

where ρ is the matter density and $p_{\text{rad}}, p_{\text{tan}}$ are radial and tangential pressures, respectively;

2.2 for strongly anisotropic materials, maximum compactness grows monotonically with the longitudinal wave speed and in this case an elastic matter can exceed Buchdahl's boundary and reach the black hole compactness $\mathcal{C} = 1/2$ continuously. However, in this case some of the energy conditions **DEC** [15] in General Relativity are violated or the interior of this fluid contains ad-hoc thin shells, or again the speed of sound within the medium exceeds the speed of light $c_s > c$. Besides, as shown in [16], if the matter satisfies (**DEC**), with nonnegative radial and tangential pressures $p_{\text{rad}} \geq 0, p_{\text{tan}} \geq 0$, we have $\mathcal{C} \lesssim 0.4815$;

2.3 as noted in [14]) for elastic balls, within the scope of the causality condition and given the radial pressure, the condition (1) is the case but its upper boundary $\mathcal{C} = 4/9$ is unattainable.

Thus, in all the cases mentioned above the value of $1/2$ is limiting for $\mathcal{C}$.

Let us formulate an assumption for **BT**, calling it the

Maximal (or Limiting) Mass Principle within Sphere - PMM:

the *mass $\mathcal{M}$ of a self-gravitating material object within a two-dimensional static uncharged sphere $\mathcal{S}_{\mathcal{R}}$, with the radius $\mathcal{R}$, satisfies the condition*

$$\mathcal{M} \leq \frac{1}{2}\frac{\mathcal{R}c^2}{G}. \tag{4}$$

When the condition of (4) is violated, specifically when within the sphere $\mathcal{S}_{\mathcal{R}}$ at some moment we have the inequality

$$\mathcal{M}' = \mathcal{M} + \mathbf{m} > \frac{1}{2}\frac{\mathcal{R}c^2}{G}, \tag{5}$$

a part of the mass $\mathcal{M}'$ is **forced** *beyond the boundary $\mathcal{S}_{\mathcal{R}}$ and two different outcomes are possible:*

PMM.a

The initial radius $\mathcal{R}$ of the sphere $\mathcal{S}_{\mathcal{R}}$ increases by the magnitude offering satisfaction of the condition (4) for the mass $\mathcal{M}' = \mathcal{M} + \mathbf{m}$ also of the self-gravitating object, contained within a new static sphere $\mathcal{S}'_{\mathcal{R}'}$ with a new radius $\mathcal{R}'$, to satisfy the same condition (4)

$$\mathcal{M}' \leq \frac{1}{2}\frac{\mathcal{R}'c^2}{G}. \tag{6}$$

PMM.b

The process becomes dynamic for a long period of time with the involvement of the parameters for the positively determined radial sphere $u \doteq d\mathcal{R}(t)/dt$ and of the corresponding acceleration $d^2\mathcal{R}(t)/dt^2$.

Gravity as an attractive force is the case only when the formula of (4) is valid. If in some instance the condition of (4) in $\mathcal{S}_{\mathcal{R}}$ is violated and we have the formula of (5)instead, then

for **PMM.a***, gravity in $\mathcal{S}_{\mathcal{R}}$ becomes the repulsive force, extending $\mathcal{S}_{\mathcal{R}}$ to a new sphere, $\mathcal{S}_{\mathcal{R}'} \supset \mathcal{S}'_{\mathcal{R}}$, for the interior of which the validity of (4) is restored and its attractivity is retained;*

for **PMM.b***, attractivity of gravity is replaced by repulsivity, i.e. gravity becomes a repulsive force.*

Remark 2.1.

2.1.1. In **PMM** *we use the word "principle" rather than "hypothesis" as usually the latter is associated with a proof of some statement in the canonical paradigm. In case at hand the paradigm is extended because in some instants gravity from the attractive force changes to the repulsive force;*
2.1.2. *Obviously, in this pattern (i.e. within the scope of the* **PMM** *validity), it is assumed that an object with the mass $\mathcal{M}$ is self-gravitating only if the formula (4)is the case. But at instants of time, when the condition of (4) is violated and we have the formula of (5), the object ceases to be self-gravitating.*

For a Schwarzschild black hole considered within the canonical theory of gravitation, i.e. in General Relativity (**GR**) [1],[12],[4], the validity of (4) at the equality of the left and right sides is doubtless.
Let us consider the formed Schwarzschild black hole with the metric [1],[12]

$$ds^2 = \left(1 - \frac{2MG}{r}\right) dt^2 - \left(1 - \frac{2MG}{r}\right)^{-1} dr^2 - r^2 d\Omega^2, \tag{7}$$

where normalization for the speed of light is taken $c = 1$.
Due to **GR** and black holes theory [1],[12], the radius $R \equiv \mathcal{R}_{BH}$ of a black hole (7) and its mass $M \equiv \mathcal{M}_{BH}$ exactly satisfy (4) for the case of equal left and right sides by substitution

$$\mathcal{R} \mapsto \mathcal{R}_{BH}, \mathcal{M} \mapsto \mathcal{M}_{BH}, c = 1. \tag{8}$$

So, during the formation of a Schwarzschild black hole, due to the validity of **GR**, there arises an object, for which the formula of (4)is evidently fulfilled if in it the left and the right side are equal (i.e. in the limiting case) and hence **PMM** is valid. Provided a Schwarzschild black hole is further in the stationary state (without the processes of absorption and emission), this pattern remains unaltered.
But at accretion of the mass $\mathbf{m}$ on a black hole, the formula of (4) with substitution in (8 becomes invalid, (5)is the case and a new Schwarzschild black hole is formed, having the following mass and radius:

$$\mathcal{M}'_{BH} = \mathcal{M}_{BH} + \mathbf{m} = \frac{1}{2}\frac{(\mathcal{R}_{BH} + \Delta\mathcal{R}_{\mathbf{m}})}{G},$$
$$\mathcal{R}'_{BH} = \mathcal{R}_{BH} + \Delta\mathcal{R}_{\mathbf{m}} = 2G(\mathcal{M}_{BH} + \mathbf{m}) = 2G\mathcal{M}'_{BH}, \tag{9}$$

where

$$\Delta\mathcal{R}_{\rm m} = 2G\mathbf{m}. \qquad (10)$$

The last two formulae are equivalent to (5),(6), with the substitution $(\mathcal{M}_{BH}+\mathbf{m}) \rightarrow \mathcal{M}'_{BH}, \mathcal{R}' \rightarrow \mathcal{R}'_{BH}$ to choose the equality sign in (6), and on the normalization $c = 1$.
Consequently, the process of accretion satisfies all the requirements of **PMM.a** in the case of the equality sign in (4), because in fact the process of the additional mass absorption $\mathbf{m}$ may be represented as **forcing** of this mass outward of the initial black hole and the formation of a new black hole, with the mass and the radius $\mathcal{M}'_{BH}, \mathcal{R}'_{BH}$, respectively.
Let us briefly recall the formulae required for the interior solution in the case of a Schwarzschild black hole with the metric (7). Then within $\mathcal{S}_{\mathcal{R}}$, i.e. within the black hole, the matter energy-momentum tensor takes the form corresponding to the perfect fluid

$$T_{\mu\nu} = \rho u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu), \qquad (11)$$

where ρ and p – corresponding density and pressure; u^μ is the four-velocity [1].
The mass of a black hole $\mathcal{M}$ may be given similarly to the Newtonian gravity (formula (6.2.10) in [1]):

$$\mathcal{M}(\mathcal{R}) = \int_0^{\mathcal{R}} \rho(r) d\mathcal{V} = 4\pi \int_0^{\mathcal{R}} \rho(r) r^2 dr. \qquad (12)$$

It is important that for the interior of **BH** the formula (12) is incorrect due to the fact that in **GR** in the right-hand side the proper volume element $\sqrt{g^3}d^3x$ should be added as a factor (formula (B.2.17) in [1]). Then the total proper mass within a Schwarzschild black hole takes the following form ((6.2.11) from [1]):

$$\mathcal{M}(\mathcal{R})_p = \pi \int_0^{\mathcal{R}} \rho(r) r^2 [1 - \frac{2m(r)}{r}]^{-1/2} dr, \qquad (13)$$

where ((6.2.8) from [1])

$$m(r) = 4\pi \int_0^r \rho(r') r'^2 dr' \qquad (14)$$

and the difference $E_B = \mathcal{M}(\mathcal{R})_p - \mathcal{M}(\mathcal{R})$ is *the gravitational binding energy.* As seen, all the above formulae (12)–(14) remain valid when a black hole absorbs the matter with the mass $\mathbf{m}$ and we make the substitutions in these formulae

$$\mathcal{M}(\mathcal{R}) \mapsto \mathcal{M}'(\mathcal{R}'), \mathcal{R} \mapsto \mathcal{R}', \mathcal{M}(\mathcal{R})_p \mapsto \mathcal{M}'(\mathcal{R}')_p. \tag{15}$$

So, the process of accretion for a black hole (absorption of the matter by a black hole) results in the formation of a new Schwarzschild black hole with the mass and the radius $\mathcal{M}'(\mathcal{R}'), \mathcal{R}'$ from formula (15). But according to the well-known **No hair theorem** (pp.875–877 in [17]):
all stationary black hole solutions of the Einstein–Maxwell equations for gravitation and electromagnetism in general relativity can be completely characterized by only three independent externally-observable classical parameters: mass $\mathbf{M}$, *electric charge* $\mathbf{Q}$, *and angular momentum* $\mathbf{J}$.
An immediate consequence of the **No hair theorem** is the fact that all Schwarzschild black holes (i.e. $\mathbf{Q} = 0, \mathbf{J} = 0$)having the same mass $\mathbf{M}$ are physically equivalent.
Therefore, the black hole with the mass $\mathcal{M}'(theradius\mathcal{R}')$ that originated due to absorption of the matter with the mass $\mathbf{m}$ by a black hole having the mass $\mathcal{M}(\mathcal{R})$ is equivalent to (indistinguishable from) a black hole of the same mass $\mathcal{M}'(\mathcal{R}')$ resultant from a stellar collapse [1]. All the formulae for the black hole formed as a result of the collapse are valid in this case, in particular, the equation of hydro-static equilibrium *Tolman-Oppenheimer-Volkoff equation* (formula (6.2.19) in [1]):

$$\frac{dp}{dr} = -(p + \rho)\frac{m(r) + 4\pi r^3 p}{r[r - 2m(r)]}. \tag{16}$$

Remark 2.2
In this case we ignore the Hawking evaporation process of black holes [1],[12] as it is clear that the process leads to a decrease of the black hole mass, whereas a Schwarzschild black hole remains the Schwarzschild one, and hence (4) is valid.
Conclusion 2.3 *In such a way a Schwarzschild black hole with the initial mass and the initial radius* $\mathcal{M}$ *and* $\mathcal{R}$*,respectively, in the process of accretion (matter absorption) completely satisfies* **PMM.a***, with the equality sign*

in (4). This is due to the fact that, after the process is finished, this hole remains the Schwarzschild black hole, yet with the new mass $\mathcal{M}'$ and new radius $\mathcal{R}'$. GR is valid for this hole both before the beginning and after finishing of this process, the process per se being considered as forcing out of the additional mass into a sphere of greater radius that is in line with General Relativity.

However, all the calculations in [1],[12] are valid in a semi-classical approximation, i.e. for black holes with great radius and mass. It is interesting to find how looks the above-mentioned pattern at high energies with significant quantum gravitational corrections (**qgc**).

Specifically, for the energies on the order of Plank's energies (quantum gravity scales) $E \simeq E_p$, the Heisenberg Uncertainty Principle (**HUP**) [18]

$$(\delta X)\,(\delta P) \geq \frac{\hbar}{2}, \tag{17}$$

may be replaced by the Generalized Uncertainty Principle (**GUP**) [19]

$$(\delta X)\,(\delta P) \geq \frac{\hbar}{2}\left\langle \exp\left(\frac{\alpha^2 l_p^2}{\hbar^2}P^2\right)\right\rangle, \tag{18}$$

which, on retention of the leading term, gives the first-order GUP [20]–[28]:

$$(\delta X)\,(\delta P) \geq \frac{\hbar}{2}\left(1 + \frac{\alpha^2 l_p^2}{\hbar^2}\,(\delta P)^2\right). \tag{19}$$

Then there is a possibility for existence of Planck's Schwarzschild black hole, and accordingly of a Schwarzschild sphere (further referred to as "minimal") with the minimal mass M_0 and the minimal radius r_{min} (formula (20) in [19])that is a theoretical minimal length r_{min}:

$$r_{min} = l_{min} = (\delta X)_0 = \sqrt{\frac{e}{2}}\alpha l_p, \quad M_0 = \frac{\alpha\sqrt{e}}{2\sqrt{2}}m_p, \tag{20}$$

where α - model-dependent parameters on the order of 1, e - base of natural logarithms, and $r_{min} \propto l_p, M_0 \propto m_p$.

In this case, due to GUP (18), the physics becomes nonlocal and the position

of any point is determined accurate to l_{min}. It is impossible to ignore this nonlocality at the energies close to the Planck energy $E \approx E_p$, i.e. at the scales $l \propto l_p$ (equivalently we have $l \propto r_{min} = l_{min}$).
Using the terminology from [29], we will call black holes with the event horizon radii $r \propto l_p$ the quantum black holes (**qbh** rather than micro black holes.
Actually, [19] presents calculated values of the mass $\mathcal{M}$ and the radius $\mathcal{R}$ for Schwarzschild BH with regard to the quantum-gravitational corrections within the scope of GUP (18).
With the use of the normalization $G = l_p^2$ adopted in [19], temperature of a Schwarzschild black hole having the mass $\mathcal{M}$ (the radius $\mathcal{R}$) [12] in a semi-classical approximation takes the form

$$T_{\mathrm{H}} = \frac{1}{8\pi G\mathcal{M}}. \tag{21}$$

Within the scope of GUP (18),the temperature T_{H} with regard to (**qgc**) is of the form ((23) in [19]))

$$ratherthanT_{\mathrm{H},q} = \frac{1}{8\pi \mathcal{M}G} \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2\right)\right), \tag{22}$$

where $W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2\right)$ – value at the corresponding point of the Lambert W-function $W(u)$ satisfying the equation (formulae (1.5) in [30] and (9) in [19])

$$W(u)\, e^{W(u)} = u. \tag{23}$$

$W(u)$ is the multifunction for complex variable $u = x+yi$. However, for real $u = x, -1/e \leq u < 0$,$W(u)$ is the single-valued continuous function having two branches denoted by $W_0(u)$ and $W_{-1}(u)$, and for real $u = x, u \geq 0$ there is only one branch $W_0(u)$ [30].
It is clear that, for a great black hole having large mass $\mathcal{M}$ and great event horizon area $\mathcal{A}$, the deformation parameter $\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2$ is vanishingly small and close to zero. Then a value of $W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2\right)$ Is also close to $W(0)$. As seen,

$W(0) = 0$ is an obvious solution for the equation (23). We have

$$\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2\right)\right) \approx 1. \tag{24}$$

So, a black hole with great mass $M \gg m_p$ necessitates no consideration of **qgc**.
But in the case of small black holes we have

$$\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2\right)\right) > 1. \tag{25}$$

In formulae above it is assumed that $\mathcal{M} > M_0$, i.e. the black hole under study is not minimal (20).
We can rewrite the formula of (22) as follows:

$$T_{\mathrm{H},q} = \frac{1}{8\pi \mathcal{M}_q G}, \mathcal{M}_q = \mathcal{M}\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2\right)\right);$$
$$\mathcal{R}_q = 2\mathcal{M}_q G = \mathcal{R}\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2\right)\right), \tag{26}$$

where $\mathcal{M}_q$ and $\mathcal{R}_q$ are respectively the initial black0-hole mass and event horizon radius considering **qgc** caused by GUP (18).
Taking in account these **qgc**, a mass and a radius of the initial Schwarzschild black hole, absorbing the matter with the mass $\mathbf{m}$, will change in the following way:

$$\mathcal{M}_q = \mathcal{M}\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2\right)\right) \mapsto \mathcal{M}'_q = \mathcal{M}'\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}'}\right)^2\right)\right),$$
$$\mathcal{R}_q = \mathcal{R}\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}}\right)^2\right)\right) \mapsto \mathcal{R}'_q = \mathcal{R}'\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{\mathcal{M}'}\right)^2\right)\right),$$
$$,(27)$$

where $\mathcal{M}' = \mathcal{M} + \mathbf{m}, \mathcal{R}' = \mathcal{R} + 2G\mathbf{m}/c^2$.
Let us make sure that, within the constant factor c^2/G, the right-hand side

has the equality

$$\frac{\mathcal{M}}{\mathcal{R}} = \frac{\mathcal{M}_q}{\mathcal{R}_q} = \frac{\mathcal{M}'}{\mathcal{R}'} = \frac{\mathcal{M}'_q}{\mathcal{R}'_q} = \frac{1}{2}. \tag{28}$$

As directly follows from (28), **Conclusion 2.3** is valid at high (Planck's) energies within the scope of **GUP** on the substitution in **PMM**

$$\mathcal{M} \mapsto \mathcal{M}_q, \mathcal{R} \mapsto \mathcal{R}_q, \mathcal{M}' \mapsto \mathcal{M}'_q, \mathcal{R}' \mapsto \mathcal{R}'_q. \tag{29}$$

Remark 2.4
It follows from the formulae that, due to (25), substitution of (29) is most actual at high energies, when $\mathcal{M}, \mathcal{M}'$ and $\mathcal{R}, \mathcal{R}'$ Are close to $M_0, r_{min} = l_{min}$, respectively. Otherwise, when $\mathcal{M} \gg M_0, \mathcal{R} \gg r_{min}$, substitution in formula (29) is insignificant as it is clear that, because of (24), all exponents in the right side of (29) are close to 1, and we have $\mathcal{M} \approx \mathcal{M}_q, \mathcal{M}' \approx \mathcal{M}'_q, \mathbf{m} \approx \mathbf{m}_q, ...$

3 PMM and Primordial Black Holes with the Schwarzschild-de Sitter Metric in the Early Universe

At the same time, Schwarzschild black holes with the metric (7) in real physics (cosmology, astrophysics) are idealized objects. As noted in (p.324,[12]): "Spherically symmetric accretion onto a Schwarzschild black hole is probably only of academic interest as a testing for theoretical ideas. It is of little relevance for interpretations of the observations data. More realistic is the situation where a black hole moves with respect to the interstellar gas..." Nevertheless, black holes just of this type may arise and may be realistic in the early Universe. In this case they are primordial black holes (**pbh**). Most common mechanism for the formation of **pbh** is the high-density gravitation matter collapse generated by cosmological perturbations arising, e.g.,

in the process of inflation (not necessarily) in the early Universe [31]. But the idea about the formation of **pbh** has been suggested much earlier than the first inflation models, specifically in [32] and independently in [33] or [34].
During studies of the early Universe the Schwarzschild metric (7) for **pbh** is replaced by the Schwarzschild-de Sitter (SdS) metric [35] that is associated with Schwarzschild black holes with small mass M in the early Universe, in particular in pre-inflation epoch

$$ds^2 = -f(\tilde{r})dt^2 + \frac{d\tilde{r}^2}{f(\tilde{r})} + \tilde{r}^2 d\Omega^2 \qquad (30)$$

where $f(\tilde{r}) = 1 - 2GM/\tilde{r} - \Lambda\tilde{r}^2/3 = 1 - 2GM/\tilde{r} - \tilde{r}^2/L^2, L = \sqrt{3/\Lambda} = H_0^{-1}$, M - black hole mass, Λ – cosmological constant, and $L = H_0^{-1}$ is the Hubble radius.
In general, such a black hole may have two different horizons corresponding to two different zeros $f(\tilde{r})$: event horizon of a black hole and cosmological horizon. This is just so in the case under study when a value of M is small [36],[37]. In the general case of $L \gg GM$, for the event horizon radius of a black hole having the metric (30), $r_{\rm H}$ takes the following form (formula (9) in [38]):

$$r_{\rm H} \simeq 2GM \left[1 + \left(\frac{r_M}{L}\right)^2\right], where\ r_M = 2MG. \qquad (31)$$

Then, due to the assumption concerning the initial smallness of Λ, we have $L \gg r_M$. In this case, to a high accuracy, the condition $r_{\rm H} = r_M$ is fulfilled, i.e. for the considered (SdS) BH we can use the formulae, given in the previous section for a Schwarzschild BH, to a great accuracy.
Thus, in this case for **pbh**, with the Schwarzschild-de Sitter (SdS) metric (30) and with small radii, **Conclusion 2.3** is valid and in **PMM.a**, due to **Remark 2.4**,**qgc** must be taken into consideration. Provided these **pbh** were formed in the early Universe at very high energies close to the Planck's, without loss of generality, such black holes may be considered as **qbh**.
Remark 3.1.
Note that, because Λ is very small, the condition $L \gg GM$ and hence the formula of (31) are obviously valid not only for black hole with the mass $M \propto m_p$ but also for a much greater range of masses, i.e. for black holes

with the mass $M \gg m_p$, taking into account the condition $L \gg GM$. In fact we obtain ordinary Schwarzschild black holes considered in the first part of Section 2, which do not require consideration of **qgc** due to formula (24).
But the problem arises, how high is the probability that **pbh** with Schwarzschild-de Sitter **SdS** metric (30) arise in the pre-inflation epoch. This problem has been studied in [35] without due regard for **qgc**. Let us demonstrate that consideration of **qgc** in this case makes the probability of arising **pbh** higher.
To this end in cosmology, in particular inflationary, the metric (30) is conveniently described in terms of the conformal time η [35]:

$$ds^2 = a^2(\eta)\Bigg\{ - d\eta^2 + \left(1 + \frac{\mu^3\eta^3}{r^3}\right)^{4/3} \left[\left(\frac{1-\mu^3\eta^3/r^3}{1+\mu^3\eta^3/r^3}\right)^2 dr^2 + r^2 d\Omega^2\right]\Bigg\}, \tag{32}$$

where $\mu = (GMH_0/2)^{1/3}$, H_0 – de Sitter-Hubble parameter and scale factor, a – conformal time function η:

$$a(\eta) = -1/(H_0\eta), \eta < 0. \tag{33}$$

Here r satisfies the condition $r_0 < r < \infty$ and a value of $r_0 = -\mu\eta$ in the reference frame of (32) conforms to singularity of the back hole.
Due to (31), μ may be given as

$$\mu = (r_M H_0/4)^{1/3}, \tag{34}$$

where $\tilde{r} = r_M$ is the radius of a black hole with the SdS Schwarzschild-de Sitter metric (30).
In the conventional consideration it is assumed, similar to [35], that in (34) we have $\mu = const$. Then, if in formula (34) $r_M : r_M \mapsto \widetilde{r_M}$ is "shifted", $H_0 : H_0 \mapsto \widetilde{H}_0$ is adequately "shifted" too, and we have

$$\mu = (r_M H_0/4)^{1/3} = (\widetilde{r_M}\widetilde{H}_0/4)^{1/3}, \widetilde{H}_0 = \frac{r_M}{\widetilde{r_M}} H_0 = const. \tag{35}$$

Specifically,in the case $\mu = const$, in (35) substitution of $r_M \mapsto r_{M_q}$ for $r_M = \mathcal{R}, r_{M_q} = \mathcal{R}_q$, formula (27),results in substitution of $H_0 \to H_{0,q}$ to meet the condition

$$\mu = (r_M H_0/4)^{1/3} = (r_{M_q} H_{0,q}/4)^{1/3}. \tag{36}$$

From the last formula it follows that

$$H_{0,q} = H_0 \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right). \qquad (37)$$

Similar to [35], it is assumed that in pre-inflation period non-relativistic particles with the mass $m < M_p$ are dominant (Section 3 in [35]). For convenience, let us denote the Schwarzschild radius r_M by R_S.
When denoting, in analogy with [35], by $N(R,t)$ the number of particles in a *comoving* ball with the physical radius $R = R(t)$ and the volume V_R at time t, in the case under study this number (formula (3.9) in [35]) will have **qgc** $N(R,t) \mapsto N(R,t)_q$

$$(\langle N(R,t)\rangle = \frac{m_p^2 H^2 R^3}{2m}) \mapsto (\langle N(R,t)_q\rangle = \frac{m_p^2 H_q^2 R^3}{2m}). \qquad (38)$$

Here the first part of the last formula agrees with formula (3.9) in [35], whereas H, H_q in this case are in agreement with $H_0, H_{0,q}$. And from (37) it follows that

$$\langle N(R,t)_q\rangle = \langle N(R,t)\rangle \exp\left(-W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right). \qquad (39)$$

According to (26), it is necessary to replace the Schwarzschild radius R_S by $R_{S,q} = R_S \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)$.
Then from the general formula $N(R_S,t) = \langle N(R_S,t)\rangle + \delta N(R_S,t)$, used because of the replacement of $R_S \mapsto R_{S,q}$, we obtain an analog of (3.12) from [35]

$$\delta N > \delta N_{\rm cr,q} \doteq \frac{m_p^2 R_{S,q}}{2m} - \langle N(R_S,t)_q\rangle = \frac{m_p^2 R_{S,q}}{2m}[1-(HR_S)^2] =$$
$$= \frac{m_p^2 R_S}{2m}[1-(HR_S)^2]\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) = \delta N_{\rm cr}\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right). \qquad (40)$$

In the last formula in square brackets we should have $(H_q R_{S,q})^2$ instead of $(HR_S)^2$ but, as we consider the case $\mu = const$, these quantities are

coincident.
It should be noted that here the following condition is used:

$$HR_S < 1, \tag{41}$$

i.e. Schwarzschild radius R_S less than Hubble radius, $R_S < R_H = 1/H$.
As we have $\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) < 1$, then

$$\delta N_{\mathrm{cr,q}} < \delta N_{\mathrm{cr}}. \tag{42}$$

Considering that for the formation of a Schwarzschild black hole with the radius R_S it is required that, due to statistical fluctuations, the number of particles $N(R_S, t)$ with the mass m within the black hole volume $V_{R_S} = 4/3\pi R_S^3$ be in agreement with the condition [35]

$$N(R_S, t) > R_S M_p^2/(2m), \tag{43}$$

which, according to **qgs** in the formula of (26), may be replaced by

$$N(R_{S,q}, t) > R_{S,q} M_p^2/(2m) = \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) R_S M_p^2/(2m). \tag{44}$$

As follows from these expressions, with regard to **qgc** for the formation of **pbh** in the pre-inflation period, the number of the corresponding particles may be lower than for a black hole without such regard, leading to a higher probability of the formation.
Such a conclusion may be made by comparison of this probability in a semi-classical consideration (formula (3.13) in [35])

$$P\big(\delta N(R_S, t) > \delta N_{\mathrm{cr}}(R_S, t)\big) = \int_{\delta N_{\mathrm{cr}}}^{\infty} d(\delta N) P(\delta N) \tag{45}$$

and with due regard for **qgc**

$$P\big(\delta N(R_{S,q}, t) > \delta N_{\mathrm{cr}}(R_{S,q}, t)\big) = \int_{\delta N_{\mathrm{cr,q}}}^{\infty} d(\delta N) P(\delta N). \tag{46}$$

Considering that in the last two integrals the integrands take positive values and are the same, whereas the integration domain in the second integral is wider due to (42), we have

$$\int_{\delta N_{\rm cr,q}}^{\infty} d(\delta N)P(\delta N) =$$

$$= \int_{\delta N_{\rm cr,q}}^{\delta N_{\rm cr}} d(\delta N)P(\delta N) + \int_{\delta N_{\rm cr}}^{\infty} d(\delta N)P(\delta N) > \int_{\delta N_{\rm cr}}^{\infty} d(\delta N)P(\delta N). \quad (47)$$

As follows from the last three formulae, in the case under study the probability that the above-mentioned **pbh** will be formed is higher with due regard for **qgc**.

It is interesting to find which changes should be expected in the pattern studied if the parameter μ ceases to be constant and is shifted with regard to **qgc** of the black hole mass $M \mapsto M_q$ (26): $(\mu = (GMH_0/2)^{1/3}) \mapsto (\mu_q = (GM_qH_0/2)^{1/3})$.

Note that in this case the general formula form Section 3 in [35] are also valid but for this pattern in formula (40) there is substitution of $HR_S \mapsto HR_{S,q}$:

$$\delta N > \delta N_{\rm cr,q} \doteq \frac{m_p^2 R_{S,q}}{2m} - \langle N(R_S,t)_q \rangle = \frac{m_p^2 R_{S,q}}{2m}[1-(HR_{S,q})^2] =$$

$$= \frac{m_p^2 R_S \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)}{2m}[1 - H^2R_S^2\exp\left(W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)].(48)$$

To understand variations in the probability of **pbh** arising as compared to the case when **qgc** are neglected in the consideration, we compare the last expression with the corresponding quantity $\delta N_{\rm cr} = \frac{m_p^2 R_S}{2m}[1-(HR_S)^2]$.

Dividing the last expression and the right side (48) by the same positive number $\frac{m_p^2 R_S}{2m}$ and subtracting the second number from the first, we can obtain

$$\delta N_{\rm cr} - \delta N_{\rm cr,q} \sim [1 - H^2R_S^2 + H^2R_S^2\exp\left(\frac{3}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) -$$

$$- \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)] \quad (49)$$

with a positive proportionality factor.
To have a positive quantity in the right side (49), fulfillment of the following inequality is required:

$$1-\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) > R_S^2H^2[1-\exp\left(\frac{3}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)].(50)$$

As from formula (23) it follows that $W(u)<0$ for $u<0$, we have $1-\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)>0, 1-\exp\left(\frac{3}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)>0$, from where it follows that (50) is equivalent to the inequality

$$(HR_S)^2 < \frac{1-\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)}{1-\exp\left(\frac{3}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)}=$$

$$=\frac{1}{1+\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)+\exp\left(W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)} \qquad (51)$$

or

$$HR_S < \frac{1}{\sqrt{1+\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)+\exp\left(W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)}}. \qquad (52)$$

We need that in the case under study $\mu \neq const$ the probability of **pbh** arising with regard to **qgc** be higher than the same probability but without due regard for **qgc**. It is sufficient to replace the condition $HR_S<1$ in formula (41) by the condition in formula (52).
Note that, due to smallness of R_S, $\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right), \exp\left(W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)$ are also small and in the right side (52) the quantity is close to 1, i.e. the shorter the Schwarzschild radius of **pbh**,the greater consideration of **qgc** increases the probability of **pbh** arising.

4 PMM,Early and Present Universe

Now let us realize that for the metric (7) (or (30)) and for the small radius $\mathcal{R}_{\mathcal{M}}$ of the sphere $\mathcal{S}_{\mathcal{R}_{\mathcal{M}}}$ the condition (4) in **PMM** is *from the start* violated, i.e. *initially* for the time $t = 0$, instead of (4), we had (5), or

$$\mathcal{M}' = \mathcal{M} + \mathbf{m} > \mathcal{M} = \frac{\mathcal{R}_{\mathcal{M}}\mathbf{c^2}}{\mathbf{2G}}. \tag{53}$$

The following aspects should be particularly emphasized.

1. Provided $\mathcal{M}, \mathcal{R}_{\mathcal{M}}$ represent the mass and the radius of a black hole, respectively, and $\mathbf{m}$ – mass of the matter absorbed by this black hole on accretion, it is connived that $\mathbf{m} < \mathcal{M}$, whereas in the vast majority of cases – $\mathbf{m} \ll \mathcal{M}$.
Besides, as on accretion of the matter for a black hole this black hole remains unchanged, the condition (4) in the case of equality is unaltered for a new black hole and we have $\mathcal{M}' = \mathcal{R}_{\mathcal{M}'}c^2/(2G)$. This means that in (6) the equality is always the case

$$\mathcal{R}' = \mathcal{R}_{\mathcal{M}'} = \mathcal{R}_{\mathcal{M}} + 2G\mathbf{m}/c^2. \tag{54}$$

2. However, this is not true in the general case when there is no consideration for a black hole and the accretion process on this black hole, in particular when formula (5) (or equivalently (53)) is valid from the very beginning. It is clear that in this case, according to point **2.1.2.** of the **Remark 2.1.**, the system is not self-gravitating and we initially consider the pattern of the matter forcing-out beyond the sphere $\mathcal{S}_{\mathcal{R}_{\mathcal{M}}}$, i.e. the case with **PMM.b**.
If (4) is violated, specifically if

$$\mathcal{M}' > \frac{\mathcal{R}_{\mathcal{M}}c^2}{2G}, \tag{55}$$

then the mean density $\rho_{\mathcal{M}'}$ of the sphere interior $\mathcal{S}_{\mathcal{R}}$ with the mass $\mathcal{M}'$ should satisfy the condition

$$\rho_{\mathcal{M}'} > \frac{3c^2}{8\pi\mathcal{R}_{\mathcal{M}}^2 G},$$
$$or\ \rho_{\mathcal{M}'} = \kappa\frac{3c^2}{8\pi\mathcal{R}_{\mathcal{M}}^2 G}, \kappa > 1. \tag{56}$$

Obviously, it is impossible to take such scenario of the early Universe for explanation of its initial expansion.
Assuming this scenario for the very beginning of the Universe origination, in this case we denote $\mathcal{R}_{\mathcal{M}} \doteq \mathcal{R}(0)$ as $\mathcal{R}_{origin}$ (or equivalently $\mathcal{R}_{source}$).
Within the scope of a perfect fluid model, in cosmology [7] an equation for such liquid takes the form

$$p[\rho(t)] = \omega[\rho(t)]\rho(t). \tag{57}$$

It is assumed that a value of $\rho(0) = \rho_{\mathcal{M}'}$ is associated with the vacuum. As from the start we use the pattern of **PMM.b**,repulsion is the case and hence the initial pressure is negative. Then, without loss of generality, it is believed that

$$p[\rho(0)] = \omega[\rho(0)]\rho(0), \omega[\rho(0)] \doteq \omega_0 = -1. \tag{58}$$

Provided in the early Universe in the process of the initial expansion we have the scenario of **PMM.b**, for the dynamic quantity $\mathcal{R}(t)$ at small times $t \geq 0$ in the point $t = 0$ the following condition must be fulfilled: $\mathcal{R}(0) = \mathcal{R}_{origin}, \rho_{\mathcal{M}'} = \rho(0) \doteq \rho_{vac}$.
In this case the expression (56) may be written as

$$\rho(0) = \kappa \frac{3c^2}{8\pi \mathcal{R}(0)^2 G} = \kappa \frac{3c^2}{8\pi \mathcal{R}^2_{origin} G}, \kappa > 1, \tag{59}$$

where κ–dimensionless parameter.
With the normalization $c = \hbar = 1, G = l_p^2 = m_p^{-2}$ used in [5]–[7], (59) we can rewrite the expression, where the left side is given in the well-known form (formulae (3.34) in [5] and (12.1) in [6])

$$\frac{8\pi}{3} \frac{\rho(0)}{m_p^2} = \frac{\kappa}{\mathcal{R}^2_{origin}}. \tag{60}$$

We can see that

$$\frac{\kappa}{\mathcal{R}^2_{origin}} \neq H^2_{vac} = H^2_{dS} = H_0. \tag{61}$$

Indeed, since in the early Universe the typical size of a two-dimensional sphere is Planckian or close to the Planck's [39]–[44],i.e. $\mathcal{R}_{origin} \propto l_p$, from formula $\kappa c^2/\mathcal{R}^2_{origin} = H^2_{vac} = H^2_{dS}$, that at $c = 1$ is equivalent to the condition $\kappa/\mathcal{R}^2_{origin} = H^2_{vac} = H^2_{dS}$, for the quantity $\mathcal{R}_{origin} \propto l_p$ the proportionality factor is κ

$$H_0 \propto \frac{c}{\mathcal{R}_{origin}} = \frac{3 \cdot 10^5 km \cdot s^{-1}}{l_p} \approx \frac{c}{\mathcal{R}_{origin}} = \frac{3 \cdot 10^5 km \cdot s^{-1}}{10^{-33} cm} \approx 10^{43} s^{-1}. \quad (62)$$

Still, it is known that $H_{vac} = H_{dS} = H_0$ is a very small quantity and, according to modern estimates, we have

$$\begin{aligned} H_0 &\approx (1,5 - 2,5) \cdot 10^{-18} s^{-1}, \\ t_{H_0} &\approx 5 \cdot 10^{17} s. \end{aligned} \quad (63)$$

Assuming that $\mathcal{R}_{origin}$ takes a real value in the early Universe, in particular $\mathcal{R}_{origin} \propto l_p$, in (60) the values of $\rho(0) = \rho_{vac}$ and H_0 (formula (62)) are enormous, deviating drastically from the experimental data. The same problem is observed with tremendous discrepancy between the vacuum energy density (cosmological constant) $\Lambda, \rho_{vac} \doteq \rho_\Lambda$,m calculated by the canonical quantum field theory [45],[46] and its experimental value [47].
Now we consider the present Universe with the characteristic radius of the (Metagalactic) luminous horizon:

$$\mathcal{R}_{**} = c t_{H_0} \approx 4.4 \cdot 10^{28} cm = 4.4 \cdot 10^{26} m = 4.4 \cdot 10^{23} km. \quad (64)$$

As the corresponding sphere $\mathcal{S}_{\mathcal{R}_{**}}$ with the radius $\mathcal{R}_{**}$ at the present time period is not static, expanding continuously, we can use **PMM** from Section 2 only in the case of repulsion, i.e. we have formula (5) in the pattern **PMM.b**. Let us verify an extent of violation of the condition (4) in the present Universe for the radius $\mathcal{R}_{**}$.
As known, the mean density ρ_{Univ} of the total energy in the present Universe is approaching the critical density $\rho_c = \frac{3H^2}{8\pi G}$

$$\rho_{Univ} \approx 9.9 \cdot 10^{-27} kg \cdot m^{-3}. \quad (65)$$

Then the total mass $\mathcal{M}_{\mathcal{R}_{**},total}$ contained within $\mathcal{S}_{\mathcal{R}_{**}}$ is equal to

$$\mathcal{M}_{\mathcal{R}_{**},total} = \rho_{Univ} \frac{4\pi}{3} \mathcal{R}^3_{**} \approx 9.9 \cdot 10^{-27} \frac{kg}{m^3} \cdot 3.566 \cdot 10^{80} m^3 \approx 3.53 \cdot 10^{54} kg. \quad (66)$$

On the other hand, the Schwarzschild mass $\mathcal{M}_{\mathcal{R}_{**},Sch}$ contained in the sphere $\mathcal{S}_{\mathcal{R}_{**}}$ with the radius $\mathcal{R}_{**}$, i.e. the mass satisfying (4) (for $\mathcal{R} = \mathcal{R}_{**}$ in the case of the equality), equals

$$\mathcal{M}_{\mathcal{R}_{**},Sch} = \frac{\mathcal{R}_{**}c^2}{2G} \approx \frac{4.4 \cdot 10^{23} km \cdot 9 \cdot 10^{10} km^2/s^2}{2 \cdot 6,67 \cdot 10^{-20} km^3 \cdot s^{-2} \cdot kg^{-1}} \approx 2.969 \cdot 10^{53} kg, \quad (67)$$

where the Newton constant $G = 6,67430 \cdot 10^{-11} m^3 s^{-2} kg^{-1} = 6,67430 \cdot 10^{-20} km^3 s^{-2} kg^{-1}$. In this way from (66),(67) it follows that

$$\mathcal{M}_{\mathcal{R}_{**},total} \gg \mathcal{M}_{\mathcal{R}_{**},Sch}. \quad (68)$$

In this case the condition (4) is greatly violated. In fact we obtain the pattern of **PMM.b** with the difference that initially the sphere was not static $\mathcal{S}_{\mathcal{R}_{**}}$.
But, if the rate of variations of the radius $d\mathcal{R}_{**}(t)/dt$ is sufficiently low, variations of the sphere $\mathcal{S}_{\mathcal{R}_{**}}$ are rather slow–to a high accuracy the sphere may be considered static for a long period of time.
Nevertheless, the ordinary (baryonic) matter makes 0.049 of the whole contents of the Universe and for the corresponding mass $\mathcal{M}_{\mathcal{R}_{**},baryonic}$ we get

$$\mathcal{M}_{\mathcal{R}_{**},baryonic} \approx 3.53 \cdot 0.049 \cdot 10^{54} kg \approx 1.7 \cdot 10^{53} kg. \quad (69)$$

Comparison of this number with $\mathcal{M}_{\mathcal{R}_{**},Sch}$ demonstrates that there is no violation of (4) in the case of ordinary (baryonic) matter.
But, when the dark matter forming 0.268 of the Universe contents is added to baryonic matter, the corresponding mass $\mathcal{M}_{\mathcal{R}_{**},matter}$ is equal to

$$\mathcal{M}_{\mathcal{R}_{**},matter} \approx 3.53 \cdot 0.317 \cdot 10^{54} kg \approx 1.119 \cdot 10^{54} kg. \quad (70)$$

Since $\mathcal{M}_{\mathcal{R}_{**},matter} > \mathcal{M}_{\mathcal{R}_{**},Sch}$, in this case repulsion also arises and we have the pattern **PMM.b**.
Let us return to formula (56) for $\mathcal{R}_{\mathcal{M}} = \mathcal{R}_{**}, \mathcal{M}' = \mathcal{M}_{\mathcal{R}_{**},total}$. As directly follows from (68), we can write (56) as

$$\rho_{\mathcal{M}_{\mathcal{R}_{**},total}} = \kappa \frac{3c^2}{8\pi \mathcal{R}_{**}^2 G}, \kappa \gg 1. \quad (71)$$

It should be noted that in the general case $\rho \propto a^{-3(1+\omega)}$ the second line of formula (56) immediately gives

$$\kappa^{1/2}\mathcal{R}(t) \propto a^{\frac{3}{2}(1+\omega)}(t), \kappa > 1, \omega > -1. \tag{72}$$

The parameter κ is a dynamic quantity, i.e. $\kappa = \kappa(t)$. From (71) it follows that at the present epoch it is rather high $\kappa \gg 1$.

5 Final Comments and Conclusion

PMM and its violation offer the possibility to introduce repulsive forces into gravity. If **PMM** is valid, we should consider three important problems:

5.1. Correct integrity of **PMM** with General Relativity;

5.2. Obvious relation of **PMM** to cosmological models, specifically to inflation models;

5.3. **PMM** and the Dark Universe Problem (Dark Matter+Dark Energy).

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this work.

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