Radiation of a moving charge in rotating dielectric medium

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Abstract

Abstract. The power spectral formula of the radiation of an electron moving in a rotating dielectric disc is derived. We suppose the index of refraction is constant during the rotation. This is in accord with the Fermi dielectric rotating disc for the determination of the light polarization gyration. While the well-known Čerenkov effect, transition effect, the Čerenkov-synchrotron effect due to the motion of particles in magnetic field are experimentally confirmed, the new phenomenon - the radiation due to a charge motion in rotating dielectric medium and the Čerenkov-synchrotron radiation due to the superluminal motion of particle in the rotating dielectric medium is still in the state of the preparation of experiment.

Key words: Rotation, Čerenkov radiation, synchrotron radiation, dielectric medium, oscillator, spectral formula.

1 Introduction

To consider the radiation by charges moving in the rotating dielectric medium (disc), it is suitable to describe such system from the viewpoint of the Lagrange dynamics and then to investigate the special case including the Čerenkov-synchrotron radiation. So, we define the non-inertial mechanical systems by the differential equations following from the Lagrange formulation of the mechanical systems in the non-inertial systems. We follow the Landau et al. monograph (Landau et al. 1965) and author article (Pardy, 2007).

Let be the Lagrange function of a point particle in the inertial system as follows:

$$L_0 = \frac{m\mathbf{v}_0^2}{2} - U \tag{1}$$

with the following equation of motion

$$m\frac{d\mathbf{v}_0}{dt} = -\frac{\partial U}{\partial \mathbf{r}}, \tag{2}$$

where the quantities with index 0 correspond to the inertial system.

The Lagrange equations in the non-inertial system is of the same for as in the inertial one, or

$$\frac{d}{dt}\frac{\partial L}{\partial \mathbf{v}} = \frac{\partial L}{\partial \mathbf{r}}. \tag{3}$$

However, the Lagrange function in the non-inertial system is not the same as in eq. (1), because of its adequate transformation.

Let us first consider the system K' moving relatively to the system K with the velocity $\mathbf{V}(t)$. If we denote the velocity of a particle with the regard to system K' as $\mathbf{v}'$, then evidently

$$\mathbf{v}_0 = \mathbf{v}' + \mathbf{V}(t). \tag{4}$$

After insertion of eq. (4) into eq. (1), we get

$$L_0' = \frac{m\mathbf{v}'^2}{2} + m\mathbf{v}'\mathbf{V} + \frac{m}{2}\mathbf{V}^2 - U. \tag{5}$$

The function $\mathbf{V}^2$ is the function of time only as it can be expresed as to total derivation of time of some new function and it means that the term with this function in the Lagrange function can be removed from the Lagrangian. We also have:

$$m\mathbf{v}'\mathbf{V}(t) = m\mathbf{V}\frac{d\mathbf{r}'}{dt} = \frac{d}{dt}(m\mathbf{r}'\mathbf{V}(t)) - m\mathbf{r}'\frac{d\mathbf{V}}{dt}. \tag{6}$$

After inserting the last formula into the Lagrange function and after removing the total time derivation we get

$$L' = \frac{mv'^2}{2} - m\mathbf{W}(t)\mathbf{r}' - U, \tag{7}$$

where $\mathbf{W} = d\mathbf{V}/dt$ is the the acceleration of the linear motion of the system K'.

The Lagrange equations following from the Lagrangian (7) are as follows:

$$m\frac{d\mathbf{v}'}{dt} = -\frac{\partial U}{\partial \mathbf{r}'} - m\mathbf{W}(t). \tag{8}$$

We see that after acceleration of the system K' the new force $-m\mathbf{W}$ appears. This force is invisible as every force and it is fictitious because it is not generated by the internal properties of some body.

In case of the situation when the system K' rotates with the angle velocity $\mathbf{\Omega}$ with regard to the system K, then radius vectors $\mathbf{r}$ and $\mathbf{r}'$ are identical and (Landau et al., 1987; Pardy, 2007)

$$\mathbf{v}' = \mathbf{v} + \mathbf{\Omega} \times \mathbf{r}. \tag{9}$$

The Lagrange function for this situation is

$$L = \frac{mv^2}{2} - m\mathbf{W}(t)\mathbf{r} - U + m\mathbf{v}(\mathbf{\Omega} \times \mathbf{r}) + \frac{m}{2}(\mathbf{\Omega} \times \mathbf{r})^2. \tag{10}$$

The corresponding Lagrange equation for Lagrange function is as follows (Landau et al., 1987; Pardy, 2007):

$$m\frac{d\mathbf{v}}{dt} = -\frac{\partial U}{\partial \mathbf{r}} - m\mathbf{W} + m\mathbf{r} \times \dot{\mathbf{\Omega}} + 2m(\mathbf{v} \times \mathbf{\Omega}) + m\mathbf{\Omega} \times (\mathbf{r} \times \mathbf{\Omega}). \tag{11}$$

We observe three so called inertial forces. The force $m\mathbf{r} \times \dot{\mathbf{\Omega}}$ is connected with the nonuniform rotation of the system K' and the forces $2m\mathbf{v} \times \mathbf{\Omega}$ and $m\mathbf{\Omega} \times \mathbf{r} \times \mathbf{\Omega}$ correspond to the uniform rotation. The force $2m\mathbf{v} \times \mathbf{\Omega}$ is so called the Coriolis force and it depends on the velocity of a particle. The force $m\mathbf{\Omega} \times (\mathbf{r} \times \mathbf{\Omega})$ is called the centrifugal force. It is perpendicular to the rotation axes and the magnitude of it is $m\varrho\Omega^2$, where ϱ is the distance of the particle from the rotation axes.

Equation (11) can be applied to many special cases.

2 Oscillator in the rotating plane

We get from eq. (11) equations for harmonic oscillator in the rotating system:

$$\ddot{x} + \omega^2 x = 2\Omega_z \dot{y} \tag{12}$$

$$\ddot{y} + \omega^2 y = -2\Omega_z \dot{x}. \tag{13}$$

Let us introduce $\xi = x + iy$. Then instead of equations (12-13), we have:

$$\ddot{\xi} + 2i\Omega_z \dot{\xi} + \omega^2 \xi = 0. \tag{14}$$

The solution of the last equation is for $\Omega_z \ll \omega$

$$\xi = e^{-i\Omega_z t}(A_1 e^{i\omega t} + A_2 e^{-i\omega t}), \tag{15}$$

which is in the x-y representation as

$$x + iy = e^{-i\Omega_z t}(x_0(t) + iy_0(t)), \tag{16}$$

where $x_0(t), y_0(t)$ describes the trajectory of oscillator without of the rotation of system. This case is identical with the so called Foucault pendulum and it evidently gives no substantial contribution to the synchrotron radiation by oscillator moving in the dielectric rotating disc. So, let us consider the second limiting case, namely the situation with $\Omega_z \gg \omega$.

Then, instead of eq. (14), we have:

$$\ddot{\xi} + 2i\Omega_z\dot{\xi} = 0. \tag{17}$$

By the separation of variables, we get the solution in the form:

$$\xi = Ae^{-i2\Omega_z t} = x + iy, \tag{18}$$

or,

$$x = A\cos(-2i\Omega_z t), \; y = A\sin(-2i\Omega_z t) \qquad (19),$$

The charged particle (electron) which moves according to the last parametric equations evidently produces the synchrotron radiation in the presence of the rotating medium with the index of refraction. The produced radiation is so called synergic synchrotron-Čerenkov radiation in case that the velocity of the particle in dielectric medium is greater than the velocity of light in this medium. So, We are prepared to determine the spectral formula for such situation. Let us remember the basic steps leading to the general spectral formula in the framework of the Schwinger quantum field theory.

3 The quantum field theory formulation of the problem of radiation

The basic formula which we use is the vacuum to vacuum amplitude (Schwinger, 1970; ibid., 1976):

$$< 0_+|0_- >= e^{\frac{i}{\hbar}W(S)}, \tag{20}$$

where the minus and plus tags on the vacuum symbol are causal labels, referring to any time before and after the space-time region where sources are manipulated. The exponential form is introduced to account for the existence of physically independent experimental arrangements, which has a simple consequence that the associated probability amplitudes multiply and the corresponding W expressions add (Schwinger, 1970; ibid., 1976).

The electromagnetic field is described by the amplitude, given in eq. (20), with the action

$$W(J) = \frac{1}{2c^2}\int\int (dx)(dx')J^\mu(x)D_{+\mu\nu}(x-x')J^\nu(x'), \tag{21}$$

where the dimensionality of $W(J)$ is the same as the dimensionality of the Planck constant $\hbar$. J_μ is four-current density. The symbol $D_{+\mu\nu}(x-x')$ is the photon propagator, and its explicit form will be determined later.

It is easy to show that the probability of the persistence of the vacuum is given by the following formula (Schwinger et al., 1976):

$$| < 0_+|0_- > |^2 = \exp\Big\{-\frac{2}{\hbar}\mathrm{Im}\,(W)\Big\} \stackrel{d}{=} \exp\Big\{-\int\int dtd\omega\frac{P(\omega,t)}{\hbar\omega}\Big\}, \tag{22}$$

where we have introduced the so-called "power spectral function" (Schwinger et al., 1976) $P(\omega, t)$. In order to extract this spectral function from $\mathrm{Im}\,(W)$, it is necessary to know the explicit form of the photon propagator, $D_{+\mu\nu}(x - x')$.

The electromagnetic field is described by the four-potential $A^\mu(\phi, \mathbf{A})$, and it is generated by the four-current density $J^\mu(c\varrho, \mathbf{J})$ according to the differential equation (Schwinger et al., 1976):

$$\left(\Delta - \frac{\mu\varepsilon}{c^2}\frac{\partial^2}{\partial t^2}\right)A^\mu = \frac{\mu}{c}\left(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu\right)J_\nu \tag{23}$$

with the corresponding Green function $D_{+\mu\nu}$:

$$D_+^{\mu\nu} = \frac{\mu}{c}\left(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu\right)D_+(x-x'), \tag{24}$$

where $\eta^\mu \equiv (1, \mathbf{0})$, μ is the magnetic permeability of the dielectric medium with dielectric constant ε, c is the velocity of light in vacuum, n is the index of refraction of this medium, and $D_+(x - x')$ was derived by Schwinger et al. (Schwinger, 1976) in the following form:

$$D_+(x-x') = \frac{i}{4\pi^2 c}\int_0^\infty d\omega\frac{\sin\left[\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|\right]}{|\mathbf{x}-\mathbf{x}'|}e^{-i\omega|t-t'|}. \tag{25}$$

Using equations (21), (22), (24), and (25), we obtain the following expression (Schwinger et al., 1976) for the power spectral formula:

$$P(\omega,t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\int\int\int d\mathbf{x}d\mathbf{x}'dt'\frac{\sin\left[\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|\right]}{|\mathbf{x}-\mathbf{x}'|}\cos[\omega(t-t')]\times$$

$$\times\left\{\varrho(\mathbf{x},t)\varrho(\mathbf{x}',t') - \frac{n^2}{c^2}\mathbf{J}(\mathbf{x},t)\cdot\mathbf{J}(\mathbf{x}',t')\right\}. \tag{26}$$

Now, we are prepared to apply the last formula to the case of an electron moving in a magnetron.

4 The radiation of an electron in a rotating disk

In this Section, we will determine, in the spirit of Schwinger at al. article (Schwinger et al., 1976), the synergic photon production initiated by the motion of an electron in a rotating disc. This process is the synergic Čerenkov radiation. The process includes the effect of the medium, which is represented by the phenomenological index of refraction n and magnetic permeability μ, and it is well-known that these phenomenological constants depend on the external magnetic field. We consider here, that n and μ do not depend on rotation velocity.

First, we write for the charge density ϱ and for the current density, $\mathbf{J}$, the equations concerning only the circular motion of the electron and then we will show how to apply the derived formulas for the circular motion of an electron in a rotating disk. We write for the circular motion (Schwinger et al., 1976):

$$\varrho(\mathbf{x}, t) = e\delta(\mathbf{x} - \mathbf{R}(t)), \qquad \mathbf{J}(\mathbf{x}, t) = e\mathbf{v}(t)\delta(\mathbf{x} - \mathbf{R}(t)) \tag{27}$$

with

$$\mathbf{R}(t) = R(\mathbf{i}\cos(\omega_0 t) + \mathbf{j}\sin(\omega_0 t)), \tag{28}$$

where we will later identify $A = R$, and $-\Omega_z = \omega_0$ in order to get harmony with eq. (19).

In this specific case, we have:

$$\mathbf{v}(t) = d\mathbf{R}/dt, \quad \omega_0 = v/R, \quad \beta = v/c, \quad v = |\mathbf{v}|. \tag{29}$$

After insertion of eq. (27) into eq. (26), we get

$$P(\omega, t) = \sum_{l=1}^{\infty} \delta(\omega - l\omega_0) P_l(\omega, t) \tag{30}$$

with

$$P_l(\omega, t) = \frac{e^2}{4\pi^2 n^2} \frac{\omega\mu\omega_0}{v} \left(2n^2\beta^2 J'_{2l}(2ln\beta) - (1 - n^2\beta^2) \int_0^{2ln\beta} dx J_{2l}(x) \right) \tag{31}$$

where during the derivation of eq. (31), we have used the relations:

$$t' - t = \tau, \quad dt' = d\tau \tag{32}$$

$$|\mathbf{R}(t + \tau) - \mathbf{R}(t)| = 2R \left| \sin \frac{1}{2}\omega_0 \tau \right| \tag{33}$$

$$\mathbf{v}(t) \cdot \mathbf{v}(t + \tau) = v^2 \cos\omega_0 \tau \tag{34}$$

$$\omega_0 \tau = \varphi + 2\pi l, \qquad \varphi \in (-\pi, \pi),\ l = 0,\ \pm 1,\ \pm 2,\ .... \tag{35}$$

Let us remark that formula (31) is for $n = 1$ and $\mu = 1$ identical with formula derived in monograph by Sokolov, et al. (1983).

5 Conclusion and perspectives

We have derived the power spectral formula of the synergic Čerenkov-synchrotron radiation of an electron moving in a rotating dielectric disc. The Grenoble accelerator produces the synchrotron radiation by motion of a charged particle along the circle with radius R. Here, the radius can be determined from the corresponding angular velocity $\omega_0 = v/R$ where $v$is the initial velocity of an electron in the dielectric medium.

Fermi used the dielectric rotating disc to prove the gyration of the polarization plane of light. The formula derived by Fermi involves also the index of refraction, which is considered to be constant during the rotation (Landau et al., 1989). We suppose here that the index of refraction is not changed during the rotation. Such assumption enables to obtain simple formulas for the Čerenkov-synchrotron radiation. On the other hand, in case of the Faraday disc (Jackson, 1998), which is metal rotating disc, the physical parameters (density of free electrons, and so on) of the disc depend on its rotation velocity and on the local position of the elementary (infinitesimal) volume in the disc. It leads to new effects such as the potential difference between the axis of rotation and the edge of the disc, and so on.

The similar situation is in case of the rotating graphene disc. Such reality enables to observe new physical effects, such as the rotation Hall effect, or the quantum fractional Hall effect, the formation of the Onsager quantum vortexes and so on, forming in a such a way the Nobelian experimental situation.

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