Two Approaches to Measurability Conception and Quantum Theory

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Abstract

In the present article in terms of **measurability** conception, introduced in the previous papers of the author, the quantum theory is studied. Within the framework of this conception several examples are investigated in Schrodinger picture, analogs of Fourier transformations are constructed. It is shown how to produce **measurable** the analog of Heisenberg picture. At the end of the article the received results are used for substantiation, of other (more overall) definition of **measurability** conception, which, on the one hand, isn't grounded on Heisenberg Uncertainty Principle and its generalization, being the fundamental provisions of the previous papers of the author, and, on the other hand, it is equally suitable either for non-relativistic, or for relativistic cases.

1 Introduction.

The present work continues the previous papers, published by the author on the issue under research. [1]–[11]. The main idea and target of these works is to construct a correct quantum theory and gravity in terms of the variations (increments) dependent on the existent energies.

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It is clear that such a theory should not involve infinitesimal space-time variations

$$dt, dx_i, i = 1, ..., 3. \tag{1}$$

The main instruments specified in the above mentioned articles was **measurability** conception, introduced in [2].
Within the framework of the conception the theory becomes discrete, but at low energy levels E distant from the plank energies ones $E \ll E_p$ it becomes very close to the initial theory in continuous space-time paradigm.
In the present work quantum mechanics is studied in terms of **measurability** notion. In Sections 2,3 there are a short presentation, specializing and some supplementation of the previous results received by the author in relation to the non-relativistic and relativistic quantum theories.
Hereinafter in Section 4 **measurable** an analog of the Wave function and the Schrodinger Equation is considered, as well as main differential operators, appearing in quantum mechanics, in particular, the Laplace operator. **Measurable** analogs of the **momentum projection operator** and **momentum angular projection operator** are studied.
In terms of **measurability** concept, analogs of Fourier transformations are constructed. It is shown how to produce the **measurable** analog of the Heisenberg picture.
At the end of the article, in Section 5, the received results are used for substantiation, of other (more overall) definition of **measurability** conception, which, on the one hand, isn't grounded on Heisenberg Uncertainty Principle and its generalization, being the fundamental provisions of the previous papers of the author, and, on the other hand, it is equally suitable either for non-relativistic, or for relativistic cases.

2 Measurability Conception

2.1 Primary Measurability in Non-relativistic Case. Brief Reminding

In this Subsection we briefly recall the principal assumptions [2], that underlie further research.

According to **Definition I.** from [2] we call as **primarily measurable variation** any small variation (increment) $\widetilde{\Delta} x_i$ of any spatial coordinate x_i of the arbitrary point $x_i, i = 1, ..., 3$ in some space-time system R if it may be realized in the form of the uncertainty (standard deviation) Δx_i when this coordinate is measured within the scope of Heisenberg's Uncertainty Principle (HUP)[13],[14].
Similarly, we call any small variation (increment) $\widetilde{\Delta} x_0 = \widetilde{\Delta} t_0$ by **primarily measurable variation** in the value of time if it may be realized in the form of the uncertainty (standard deviation) $\Delta x_0 = \Delta t$ for pair "time-energy" (t, E) when time is measured within the scope of Heisenberg's Uncertainty Principle (HUP) too.
Next we introduce the following assumption:
Supposition II. There is the minimal length l_{min} as *a minimal measurement unit* for all **primarily measurable variations** having the dimension of length, whereas the minimal time $t_{min} = l_{min}/c$ as *a minimal measurement unit* for all quantities or **primarily measurable variations (increments)** having the dimension of time, where c is the speed of light.
According to HUP l_{min} and t_{min} lead to P_{max} and E_{max}. For definiteness, we consider that E_{max} and P_{max} are the quantities on the order of the Planck quantities, then l_{min} and t_{min} are also on the order of Planck quantities $l_{min} \propto l_P$, $t_{min} \propto t_P$. **Definition I.** and **Supposition II.** are quite natural in the sense that there are no physical principles with which they are inconsistent.
The combination of **Definition I.** and **Supposition II.** will be called the **Principle of Bounded Primarily Measurable Space-Time Variations (Increments)** or for short **Principle of Bounded Space-Time Variations (Increments)** with abbreviation (PBSTV).

As the minimal unit of measurement l_{min} is available for all the **primarily measurable variations** ΔL having the dimensions of length, the "Integrality Condition" (IC) is the case

$$\Delta L = N_{\Delta L} l_{min}, \tag{2}$$

where $N_{\Delta L} > 0$ is an integer number.
In a like manner the same "Integrality Condition" (IC) is the case for all the **primarily measurable variations** Δt having the dimensions of time. And similar to Equation (2), we get the for any time Δt:

$$\Delta t \equiv \Delta t(N_t) = N_{\Delta t} t_{min}, \tag{3}$$

where similarly $N_{\Delta t} > 0$ is an integer number too.

Definition 1(**Primary or Elementary Measurability.**)
(1) *In accordance with the PBSTV let us define the quantity having the dimensions of length or time as* **primarily (or elementarily) measurable***, when it satisfies the relation Equation (2) (and respectively Equation (3)).*
(2)*Let us define any physical quantity* **primarily (or elementarily) measurable***, when its value is consistent with points (1) of this Definition.*

Here HUP is given for the nonrelativistic case. In the next subsection we consider the relativistic case for low energies $E \ll E_P$ and show that for this case **Definition 1** (**Primary Measurability**) keeps its meaning. Further everywhere for convenience, we denote the minimal length $l_{min} \neq 0$ by ℓ and $t_{min} \neq 0$ by $\tau = \ell/c$.

2.2 Primary Measurability in Relativistic Case

In the Relativistic case HUP has the distinctive features ([16],Introduction). As known, in the relativistic case for **low energies** $E \ll E_P$, when the total energy of a particle with the mass m and with the momentum p equals [17]:

$$E = \sqrt{p^2c^2 + m^2c^4}, \tag{4}$$

a minimal value for Δx in general case takes the form ([16],formula(1.3))

$$\Delta q \approx \frac{c\hbar}{E} = \frac{\hbar}{\sqrt{p^2 + m^2c^2}}. \tag{5}$$

Nothing in this case prevents existing minimal length $\ell \neq 0$, and time $\tau = \ell/c$ and execution of the conditions (2) and (3). Particularly, in the equation (2) for $\Delta L = \Delta q$ due to the fact that $E \ll E_P$, we result in the following:

$$\Delta q = N_{\Delta q}\ell; N_{\Delta q} \gg 1. \tag{6}$$

The formula (5) can be rewritten as follows:

$$E \approx \frac{c\hbar}{N_{\Delta q}\ell} \tag{7}$$

And due to the fact that the integral number $N_{\Delta q} \gg 1$, in general, energy E may vary almost continuously, similar as in the canonical theory with $\ell = 0$. The similar equation (7) in this case can be applied for the momentum p from the right side of (5) as well. Obviously, p changes almost continuously. The analogue of (7) equation is easily to produce in the Ultrarelativistic case ($E \approx p$) and in a rest frame of a particle ($E \approx mc^2$). It is absolutely obvious that at **low energies** due to the abovementioned equations we receive almost continuous picture.
Therefore in relativistic case, at least at **low energies** $E \ll E_P$,**Definition 1** (**Primary Measurability**) of the previous subsection keeps its meaning, however, within the framework of the **Uncertainty Principle for Relativistic System** ([16],Introduction).

3 Generalized Measurability

3.1 Generalized Measurability and Generalized Uncertainty Principle

Basic results of this Subsection are contained in [2] and [15].
Further it is convenient to use the deformation parameter α_a. This parameter has been introduced earlier in the papers [18],[19],[20]–[23] as a

deformation parameter (in terms of paper [24]) on going from the canonical quantum mechanics to the quantum mechanics at Planck's scales (Early Universe) that is considered to be the quantum mechanics with the minimal length (QMML):

$$\alpha_a = \ell^2/a^2, \tag{8}$$

where a is the measuring scale. It is easily seen that the parameter α_a from Equation (8) is discrete as it is nothing else but

$$\alpha_a = \ell^2/a^2 = \frac{\ell^2}{N_a^2 \ell^2} = \frac{1}{N_a^2}. \tag{9}$$

At the same time, from Equation (9) it is evident that α_a is irregularly discrete.

It should be noted that, physical quantities complying with **Definition 1** won't be enough for the research of physical systems.

Indeed, such a variable as

$$\alpha_{N_a \ell}(N_a \ell) = p(N_a)\frac{\ell}{\hbar} = \ell/N_a, \tag{10}$$

(where $\alpha_{N_a \ell} = \alpha_a$ is taken from formula (9) at $a = N_a \ell$, and $p(N_a) = \frac{\hbar}{N_a \ell}$ is the corresponding **primarily measurable** momentum), is fully expressed in terms *only* **Primarily Measurable Quantities** of **Definition 1** and that's why it may appear at any stage of calculations, but apparently doesn't comply with **Definition 1**. That's why it's necessary to introduce the following definition generalizing **Definition 1**:

Definition 2. Generalized Measurability
We shall call any physical quantity as **generalized-measurable** or for simplicity **measurable** if any of its values may be obtained in terms of **Primarily Measurable Quantities** of **Definition 1**.

In what follows, for simplicity, we will use the term **Measurability** instead of **Generalized Measurability**. It is evident that any **primarily measurable quantity (PMQ)** is **measurable**. Generally speaking, the contrary is not correct, as indicated by formula (10).

It should be noted that Heisenberg's Uncertainty Principle (HUP) [14] is fair at low energies $E \ll E_P$. However it was shown that at the Planck scale a high-energy term must appear:

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' l_p^2 \frac{\triangle p}{\hbar} \tag{11}$$

where l_p is the Planck length $l_p^2 = G\hbar/c^3 \simeq 1,6\ 10^{-35}m$ and α' is a constant. In [25] this term is derived from the string theory, in [26] it follows from the simple estimates of Newtonian gravity and quantum mechanics, in [27] it comes from the black hole physics, other methods can also be used [29],[28],[34]. Relation (11) is quadratic in Δp

$$\alpha' l_p^2 \, (\Delta p)^2 - \hbar \, \Delta x \Delta p + \hbar^2 \leq 0 \tag{12}$$

and therefore leads to the minimal length

$$\Delta x_{min} = 2\sqrt{\alpha'} l_p \doteq \ell \tag{13}$$

Inequality (11) is called the Generalized Uncertainty Principle (GUP) in Quantum Theory.
Let us show that the **generalized-measurable** quantities are appeared from the **Generalized Uncertainty Principle (GUP)** [25]–[36] (formula (11)) that naturally leads to the minimal length ℓ (13).
Really solving inequality (11), in the case of equality we obtain the apparent formula

$$\Delta p_{\pm} = \frac{(\Delta x \pm \sqrt{(\Delta x)^2 - 4\alpha' l_p^2})\hbar}{2\alpha' l_p^2}. \tag{14}$$

Next, into this formula we substitute the right-hand part of formula (2) for $L = x$. Considering (13), we can derive the following:

$$\begin{aligned}\Delta p_{\pm} &= \frac{(N_{\Delta x} \pm \sqrt{(N_{\Delta x})^2 - 1})\hbar\ell}{\frac{1}{2}\ell^2} = \\ &= \frac{2(N_{\Delta x} \pm \sqrt{(N_{\Delta x})^2 - 1})\hbar}{\ell}.\end{aligned} \tag{15}$$

But it is evident that at low energies $E \ll E_p; N_{\Delta x} \gg 1$ the plus sign in the nominator (15) leads to the contradiction as it results in very high (much greater than the Planck's) values of Δp. Because of this, it is necessary to select the minus sign in the numerator (15). Then, multiplying the left and right sides of (15) by the same number $N_{\Delta x} + \sqrt{N_{\Delta x}^2 - 1}$, we get

$$\Delta p = \frac{2\hbar}{(N_{\Delta x} + \sqrt{N_{\Delta x}^2 - 1})\ell}. \tag{16}$$

Δp from formula (16) is the **generalized-measurable** quantity in the sense of **Definition 2.** However, it is clear that at low energies $E \ll E_p$, i.e. for $N_{\Delta x} \gg 1$, we have $\sqrt{N_{\Delta x}^2 - 1} \approx N_{\Delta x}$. Moreover, we have

$$\lim_{N_{\Delta x} \to \infty} \sqrt{N_{\Delta x}^2 - 1} = N_{\Delta x}. \tag{17}$$

Therefore, in this case (16) may be written as follows:

$$\Delta p \doteq \Delta p(N_{\Delta x}, HUP) = \frac{\hbar}{1/2(N_{\Delta x} + \sqrt{N_{\Delta x}^2 - 1})\ell} \approx \frac{\hbar}{N_{\Delta x}\ell} = \frac{\hbar}{\Delta x}; N_{\Delta x} \gg 1, \tag{18}$$

in complete conformity with HUP. Besides, $\Delta p \doteq \Delta p(N_{\Delta x}, HUP)$, to a high accuracy, is a **primarily measurable** quantity in the sense of **Definition 1**.

And vice versa it is obvious that at high energies $E \approx E_p$, i.e. for $N_{\Delta x} \approx 1$, there is no way to transform formula (16) and we can write

$$\Delta p \doteq \Delta p(N_{\Delta x}, GUP) = \frac{\hbar}{1/2(N_{\Delta x} + \sqrt{N_{\Delta x}^2 - 1})\ell}; N_{\Delta x} \approx 1. \tag{19}$$

At the same time, $\Delta p \doteq \Delta p(N_{\Delta x}, GUP)$ is a **Generalized Measurable** quantity in the sense of **Definition 2**.

Thus, we have

$$GUP \to HUP \tag{20}$$

for

$$(N_{\Delta x} \approx 1) \to (N_{\Delta x} \gg 1). \tag{21}$$

Also, we have

$$\Delta p(N_{\Delta x}, GUP) \rightarrow \Delta p(N_{\Delta x}, HUP), \tag{22}$$

where $\Delta p(N_{\Delta x}, GUP)$ is taken from formula (19), whereas $\Delta p(N_{\Delta x}, HUP)$ from formula (18).

Comment 2.*
From the above formulae it follows that, within GUP, the **primarily measurable** *variations (quantities) are derived to a high accuracy from the* **generalized-measurable** *variations (quantities) only in the low-energy limit* $E \ll E_P$

Next, within the scope of GUP, we can correct a value of the parameter α_a from formula (9) substituting a for Δx in the expression $1/2(N_{\Delta x} + \sqrt{N_{\Delta x}^2 - 1})\ell$.
Then at low energies $E \ll E_p$ we have the **primarily measurable** quantity $\alpha_a(HUP)$

$$\alpha_a \doteq \alpha_a(HUP) = \frac{1}{[1/2(N_a + \sqrt{N_a^2 - 1})]^2} \approx \frac{1}{N_a^2}; N_a \gg 1, \tag{23}$$

that corresponds, to a high accuracy, to the value from formula (9).
Accordingly, at high energies we have $E \approx E_p$

$$\alpha_a \doteq \alpha_a(GUP) = \frac{1}{[1/2(N_a + \sqrt{N_a^2 - 1})]^2}; N_a \approx 1. \tag{24}$$

When going from high energies $E \approx E_p$ to low energies $E \ll E_p$, we can write

$$\alpha_a(GUP) \stackrel{(N_a \approx 1) \rightarrow (N_a \gg 1)}{\longrightarrow} \alpha_a(HUP) \tag{25}$$

in complete conformity to *Comment 2*.*
Remark 3.1 What is the main difference between **Primarily Measurable Quantities (PMQ)** and **Generalized Measurable Quantities (GMQ)**? **PMQ** defines variables which may be obtained as a result of an immediate experiment. **GMQ** defines the variables which may be *calculated* based on **PMQ**, i.e. based on the data obtained in previous clause.

Remark 3.2. It is readily seen that a minimal value of $N_a = 1$ is *unattainable* because in formula (19) we can obtain a value of the length l that is below the minimum $l < \ell$ for the momenta and energies above the maximal ones, and that is impossible. Thus, we always have $N_a \geq 2$. This fact was indicated in [18],[19], however, based on the other approach.

The above mentioned formula result to the fact that **generalized measurable** momenta at all energies are the following:

$$p_{1/N} \doteq p(1/N, \ell), N \neq 0 \tag{26}$$

where $\ell = \kappa l_p$ and κ is the constant of order 1.
Therefore, $p_{1/N}$ depends only on three fundamental constants $c, \hbar, G$, constant κ and discrete parameters $1/N$.
However, at $N \gg 1$, i.e. at $E \ll E_p$ imaging $\tau : 1/N \Rightarrow p_{1/N}$ will be almost continuous, which provides high match accuracy of this discrete model coincidence with the initial continuous theory.
The main objective target by the author is to get the quantum theory and the gravitation within the concepts of **primarily measurable** quantities. As in this case the theories become discrete, there will be a need of further lattice representation.

3.2 Space and Momentum Lattices of Generalized Measurable Quantities, and $\alpha - lattice$

In this Subsection are refined and supplemented ed the results from [2],[10]. So, provided the minimal length ℓ exists, two lattices are naturally arising according to the formulas of the previous subsection.
I. At low energies (LE) $E \ll E_{max} \propto E_P$, lattice of the space variation—$Lat_S[LE]$ representing, for sets integers $|N_w| \gg 1$ to within the known multiplicative constants, in accordance with the above formulas for each of the three space variables $w \doteq x; y; z$.

$$Lat_S[LE] = (N_w \doteq \{N_x, N_y, N_z\}), |N_x| \gg 1, |N_y| \gg 1, |N_z| \gg 1. \tag{27}$$

At high energies (HE) $E \to E_{max} \propto E_P$ to within the known multiplicative constants too in accordance with the formulas previous subsection we have the lattice $Lat_S[HE]$ for each of the three space variables $w \doteq x; y; z$.

$$Lat_S[HE] \doteq (\pm 1/2[(N_w + \sqrt{N_w^2 - 1})]); 2 \leq (N_w \doteq \{N_x, N_y, N_z\}) \approx 1. \tag{28}$$

II. Next let us define the lattice momentum variation Lat_P as a set to obtain (p_x, p_y, p_z) for low energies $E \ll E_P$, where all the components of the above sets conform to the space coordinates (x, y, z) are given by corresponding formulae from the previous subsection.
From this it is inferred that, in analogy with point I of this subsection, within the known multiplicative constants, we have lattice $Lat_P[LE]$

$$Lat_P[LE] \doteq (\frac{1}{N_w}), \tag{29}$$

where N_w are integer numbers from Equation (27).
In accordance with formulas (19), (28), the high-energy (HE) momentum lattice $Lat_P[HE]$ takes the form

$$Lat_P[HE] \doteq (\pm \frac{1}{1/2[(N_w + \sqrt{N_w^2 - 1})]}), \tag{30}$$

where N_w are integer numbers from Equation (28).
It is important to note the following.

In the low-energy lattice $Lat_P[LE]$ all elements are varying very smoothly enabling the approximation of a continuous theory.
It is clear that lattices $Lat_S[LE]$ and $Lat_P[LE]$ are lattices **primarily measurable** quantities, while lattices $Lat_S[HE]$ and $Lat_P[HE]$ are lattices **generalized measurable** quantities.

We will expand the space lattice $Lat_S[LE]$ to space-time lattice $Lat_{S-T}[LE]$:

$$\begin{aligned} Lat_{S-T}[LE] &\doteq (N_w, N_t), N_w \doteq \{N_x, N_y, N_z\}, \\ &|N_x| \gg 1, |N_y| \gg 1, |N_z| \gg 1, |N_t| \gg 1 \end{aligned} \tag{31}$$

Now **primarily** lattice $Lat_{S-T}[LE]$ will be replaced with "$\alpha-lattice$", **measurable space-time quantities**, which will be denoted by $Lat^{\alpha}_{S-T}[LE]$:

$$Lat^{\alpha}_{S-T}[LE] \doteq (\alpha_{N_w\ell}N_w\ell, \alpha_{N_t\tau}N_t\tau) = (\frac{\ell^2}{\hbar}p(N_w), \frac{\ell^2}{\hbar}p(N_t)) = (\frac{\ell}{N_w}, \frac{\tau}{N_t}). \quad (32)$$

In the last formula by the variable $\alpha_{N_t\tau}$ we mean the parameter α corresponding to the length $(N_t\tau)c$:

$$\alpha_{N_t\tau} \doteq \alpha_{(N_t\tau)c}. \quad (33)$$

And $p(N_w)$ it is taken from formula (10), where N_t corresponds formula (32). As low energies $E \ll E_P$ are discussed, $\alpha_{N_w\ell}$ in this formula is consistent with the corresponding parameter from formula (23):

$$\alpha_{N_w\ell} = \alpha_{N_w\ell}(HUP) \quad (34)$$

As it was mentioned in the previous section, in the low-energy $E \ll E_{max} \propto E_P$ all elements of sublattice $Lat_{P-E}[LE]$ are varying very smoothly enabling the approximation of a continuous theory.
It is similar to the low-energy part of the $Lat^{\alpha}_{S-T}[LE]$ of lattice Lat^{α}_{S-T} will vary very smoothly:

$$Lat^{\alpha}_{S-T}[LE] = (\frac{\ell}{N_w}, \frac{\tau}{N_t}); |N_x| \gg 1, |N_y| \gg 1, |N_z| \gg 1, |N_t| \gg 1. \quad (35)$$

In Section 5 of [2] three following cases were selected:

(a) *"Quantum Consideration, Low Energies"*:

$$1 \ll |N_w| \leq \widetilde{\mathbf{N}}, 1 \ll |N_t| \leq \widehat{\mathbf{N}}$$

(b) *"Quantum Consideration, High Energies"*:

$$|N_w| \approx 1, |N_t| \approx 1;$$

(c) *"Classical Picture"*:

$$|N_w| \to \infty, |N_t| \to \infty.$$

Here $\widetilde{\mathbf{N}}, \widehat{\mathbf{N}}$ is a cutoff parameters, defined by the current task [2] and corrected in this paper.
Let us for three space coordinates $x_i; i = 1, 2, 3$ we introduce the following notation:

$$\begin{aligned} \Delta(x_i) \doteq \widetilde{\Delta}[\alpha_{N_{\Delta x_i}}] = \alpha_{N_{\Delta x_i}\ell}(N_{\Delta x_i}\ell) = \ell/N_{\Delta x_i}; \\ \frac{\Delta_{N_{\Delta x_i}}[F(x_i)]}{\Delta(x_i)} \equiv \frac{F(x_i + \Delta(x_i)) - F(x_i)}{\Delta(x_i)}, \end{aligned} \tag{36}$$

where $F(x_i)$ is **"measurable"** function, i.e function represented in terms of **measurable** quantities.
Then function $\Delta_{N_{\Delta x_i}}[F(x_i)]/\Delta(x_i)$ is **"measurable"** function too.
It's evident that

$$\lim_{|N_{\Delta x_i}| \to \infty} \frac{\Delta_{N_{\Delta x_i}}[F(x_i)]}{\Delta(x_i)} = \lim_{\Delta(x_i) \to 0} \frac{\Delta_{N_{\Delta x_i}}[F(x_i)]}{\Delta(x_i)} = \frac{\partial F}{\partial x_i}. \tag{37}$$

Thus, we can define a **measurable** analog of a vectorial gradient ∇

$$\nabla_{\mathbf{N_{\Delta x_i}}} \equiv \{\frac{\Delta_{N_{\Delta x_i}}}{\Delta(x_i)}\} \tag{38}$$

and a **measurable** analog of the Laplace operator

$$\boldsymbol{\Delta}_{(N_{\Delta x_i})} \equiv \nabla_{\mathbf{N_{\Delta x_i}}} \nabla_{\mathbf{N_{\Delta x_i}}} \equiv \sum_i \frac{\Delta^2_{N_{\Delta x_i}}}{\Delta(x_i)^2} \tag{39}$$

Respectively, for time $x_0 = t$ we have:

$$\begin{aligned} \Delta(t) \doteq \widetilde{\Delta}[\alpha_{N_{\Delta t}}] = \alpha_{N_{\Delta t}\tau}(N_{\Delta t}\tau) = \tau/N_{\Delta t}; \\ \frac{\Delta_{N_{\Delta t}}[F(t)]}{\Delta(t)} \equiv \frac{F(t + \Delta(t)) - F(t)}{\Delta(t)}, \end{aligned} \tag{40}$$

then

$$\lim_{|N_{\Delta t}|\to\infty}\frac{\Delta_{N_{\Delta t}}[F(t)]}{\Delta(t)}=\lim_{\Delta(t)\to 0}\frac{\Delta_{N_{\Delta t}}[F(t)]}{\Delta(t)}=\frac{dF}{dt}. \quad (41)$$

We shall designate for momenta $p_i; i=1,2,3$

$$\Delta p_i=\frac{\hbar}{N_{\Delta x_i}\ell};$$
$$\frac{\Delta_{p_i}F(p_i)}{\Delta p_i}\equiv\frac{F(p_i+\Delta p_i)-F(p_i)}{\Delta p_i}=\frac{F(p_i+\frac{\hbar}{N_{\Delta x_i}\ell})-F(p_i)}{\frac{\hbar}{N_{\Delta x_i}\ell}}. \quad (42)$$

From where similarly (37) we get

$$\lim_{|N_{\Delta x_i}|\to\infty}\frac{F(p_i+\Delta p_i)-F(p_i)}{\Delta p_i}=\lim_{|N_{\Delta x_i}|\to\infty}\frac{F(p_i+\frac{\hbar}{N_{\Delta x_i}\ell})-F(p_i)}{\frac{\hbar}{N_{\Delta x_i}\ell}}=$$
$$=\lim_{\Delta p_i\to 0}\frac{F(p_i+\Delta p_i)-F(p_i)}{\Delta p_i}=\frac{\partial F}{\partial p_i}. \quad (43)$$

Therefore, in low energies $E\ll E_P$, i.e. at $|N_{\Delta x_i}|\gg 1; |N_{\Delta t}|\gg 1, i=1,...,3$ in passages to the limit (37),(41),(43) it's possible to obtain from **"measurable"** functions partial derivatives like in case of continuous space-time. That is, the partial derivatives of from **"measurable"** functions can be considered as **"measurable"** functions with any given precision.
In this case the infinitesimal space-time variations (1) are appearing in the limit from **measurable** quantities too

$$(\alpha_{N_{\Delta t}\tau}N_{\Delta t}\tau=\frac{\tau}{N_{\Delta t}}=p_{N_{\Delta t}c}\frac{\ell^2}{c\hbar})\stackrel{N_{\Delta t}\to\infty}{\longrightarrow}dt,$$
$$(\alpha_{N_{\Delta x_i}\ell}N_{\Delta x_i}\ell=\frac{\ell}{N_{\Delta x_i}}=p_{N_{\Delta x_i}}\frac{\ell^2}{\hbar})\stackrel{N_{\Delta x_i}\to\infty}{\longrightarrow}dx_i, 1=1,...,3. \quad (44)$$

Remark 3.2.1
Thereinafter, as it is mentioned above, we suppose that energies E are low,

i.e. $E \ll E_p$.
Up to the present moment there was a default precondition that all numbers $N_{\Delta x_i}, N_{\Delta t}$ are integral, which means they produce **primarily measurable** spacetime quantities $N_{\Delta x_i}\ell$ and $N_{\Delta t}\tau$. Currently we realize that this limitation is irrelevant, taking into account the fact that unless specially noted otherwise, $N_{\Delta x_i}\ell, N_{\Delta t}\tau$ are **generalized measurable** (or simply **measurable**) quantities. At that, due to the fact that energies E are low $E \ll E_P$ the following condition is preserved:

$$|N_{\Delta x_i}| \gg 1; |N_{\Delta t}| \gg 1, i = 1, ..., 3. \tag{45}$$

Therefore, in the formula (44) momenta $p_{N_{\Delta x_i}}, p_{N_{\Delta t}c}$ from this moment are **generalized measurable** quantities. The evident example of such momenta can be accurate (not approximate) value from the equation (18)

$$p_{N_{\Delta x_i}} = \frac{\hbar}{1/2(N_{\Delta x_i} + \sqrt{N_{\Delta x_i}^2 - 1})\ell}; N_{\Delta x_i} \gg 1, \tag{46}$$

It is also obvious that if $N_{\Delta x_i}\ell$ and $N_{\Delta t}\tau$ are **measurable** quantities, then numeric coefficients $N_{\Delta x_i}$ and $N_{\Delta t}$ are also **measurable** quantities.
In this case any **measurable** triplet $N_q = \{N_{\Delta x_i}\}, |N_{\Delta x_i}| \gg 1, i = 1, ..., 3$ corresponds to small **measurable** momentum $\mathbf{p_{N_q}} \doteq \{p_{N_{\Delta x_i}}\}$, with components $p_{N_{\Delta x_i}}, |p_{N_{\Delta x_i}}| \ll P_{pl}$:

$$N_{\Delta x_i} \xrightarrow{\mathbf{p}} p_{N_{\Delta x_i}} = \frac{\hbar}{N_{\Delta x_i}\ell} \tag{47}$$

And vice versa any small **measurable** momentum $\mathbf{p_q}$ with non-zero components $\mathbf{p_q} = \{p_i\}; 0 \neq |p_i| \ll P_{pl}$ corresponds to **measurable** triplet $N_q = \{N_{\Delta x_i}\}, |N_{\Delta x_i}| \gg 1, i = 1, ..., 3$, satisfying the condition (45):

$$p_i \xrightarrow{\mathbf{x}} N_{\Delta x_i} = \frac{\hbar}{p_i\ell} \tag{48}$$

Then, for simplification, instead of $N_{\Delta x_\mu}$ we will use $N_{x_\mu}, \mu = 0, ..., 3$.

4 Quantum Mechanics in Term of Measurable Quantities

4.1 General Remarks on Wavefunction Representation

Now for any coordinate u from the set $q \doteq (x, y, z) \in \mathbb{R}^3$ and some **measurable** quantity $N_u \ell; |N_u| \gg 1$ one can correlate **measurable** quantity $\Delta_{N_u}(u) = \ell / N_u$, and for $N_q \doteq \{N_x, N_y, N_z\}$ – **measurable** product

$$\Delta_{N_q}(q) = |\Delta_{N_x}(x) \cdot \Delta_{N_y}(y) \cdot \Delta_{N_z}(z)| = \frac{\ell^3}{|N_x N_y N_z|} \tag{49}$$

Then it becomes clear that for **measurable** of the wave function $\Psi(q)$, ($\Psi(q)$ is determined within the framework of the concepts of **measurable** of the spatial coordinates q, i.e. all changes of q are **measurable**), we can determine the value

$$|\Psi(q)|^2 \Delta_{N_q}(q), \tag{50}$$

which is the probability that the measurement carried out with the system presents the coordinate value in **measurable** in volume element $\Delta_{N_q}(q)$ of configuration space.
At that, known condition for total probability in the continuous case [14]:

$$\int |\Psi(q)|^2 dq = 1 \tag{51}$$

with any predefined accuracy is replaced by the condition

$$\sum_q |\Psi(q)|^2 \Delta_{N_q}(q) = 1. \tag{52}$$

Actually, due to the equation (44) **measurable** the volume element $\Delta_{N_q}(q)$ of configuration space can be considered as close as it can to dq which means that **measurable** element $q + \Delta_{N_q}(q)$ can be considered close to **nonmeasurable** element $q + dq$.

It is obvious that set of **measurable** functions create space, in which integrals of the continuous theory, if any, are replaced into the correspondent sums for **measurable** values, and dq is replaced onto $\Delta_{N_q}(q)$. This space very close to the correspondent Hilbert space in the continuous theory in limit of large $|N_q|$.
In particular, normalization condition for **measurable** eigenfunction Ψ_n of the given **measurable** physical value f changes from continuous consideration to the **measurable** consideration, as follows:

$$(\int |\Psi_n|^2 dq = 1) \mapsto (\sum_q |\Psi_n|^2 \Delta_{N_q}(q) = 1) \tag{53}$$

We have similarly:

$$\int \Psi\Psi^* dq \mapsto \sum_q \Psi\Psi^* \Delta_{N_q}(q) \tag{54}$$

It is easily noticeable that for spaces **measurable** of the functions, all main properties of the canonical quantum mechanics can be redefinied with the replacement of the integrals by the corresponding sums and dq onto $\Delta_{N_q}(q)$ (as in the formula (53),(54)).

4.2 Schrodinger Equation and Other Equations of Quantum Mechanics in "Measurable" Format

4.2.1 Schrodinger Equation for Free Particle

Let us consider the Schrodinger Equation [14] in terns of **measurable quantities**. As it is shown in the formula (44) taking into account **Remark 3.2.1** in low energies $E \ll E_P$ (i.e. at $|N_{x_\mu}| \gg 1$), the infinitesimal space-time variations $dx_\mu, \mu = 0, ..., 3$ are occurred within the limits of $|N_{x_\mu}| \to \infty$ from **measurable** momenta $p_{N_{x_i}}, (p_{N_t c})$ multiplied by the constant $\frac{\ell^2}{\hbar}, (\frac{\ell^2}{c\hbar})$ which is nothing else than $\ell/N_{x_i}, \tau/N_t$.
Therefore in all cases we should comply to the following conditions: $|N_{x_i}| \gg 1, |N_t| \gg 1; i = 1, ..., 3$

Then **measurable** N_t-*analog* of the derivative **measurable** wavefunction $\Psi(t)$ in the continuous case will be nothing else than

$$\frac{\Delta_{N_t}[\Psi(t)]}{\Delta(t)} \doteq \frac{\Psi(t+\tau/N_t)-\Psi(t)}{\tau/N_t}, \tag{55}$$

and **measurable** N_t-*analog* of the Schrodinger Equation

$$\frac{d\Psi(t)}{dt} = \frac{1}{\imath\hbar}\widehat{H}\Psi(t) \tag{56}$$

will be the following:

$$\frac{\Delta_{N_t}[\Psi(t)]}{\Delta(t)} = \frac{\Psi(t+\tau/N_t)-\Psi(t)}{\tau/N_t} = \frac{1}{\imath\hbar}\widehat{H}_{meas}\Psi(t), \tag{57}$$

where $\widehat{H}_{meas}$ – some **measurable** analog of the Hamiltonian $\widehat{H}$ in the continuous case, which means $\widehat{H}_{meas}$ – operator, expressed in the terms of **measurable** values.
We consider the example of the Schrodinger Equation for a free particle [14]

$$\imath\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\mathbf{\Delta}\Psi(\mathbf{r},t), \tag{58}$$

where $\mathbf{\Delta} \equiv \nabla\nabla \equiv \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is a Laplace operator and m is a particle's mass.
The formula(39) initially considered for the case integral numbers $N_{x_i}, |N_{x_i}| \gg 1$. However, due to **Remark 3.2.1**, it remains right for any **measurable** numbers $N_{x_i}, |N_{x_i}| \gg 1$.
From this formula we can conclude that

$$\lim_{|N_{x_i}|\rightarrow\infty} \mathbf{\Delta}_{(N_{x_i})} = \mathbf{\Delta} \tag{59}$$

Then the condition $|N_t| \gg 1, |N_{x_i}| \gg 1$ allows to state that **measurable** Schrodinger Equation analog (58):

$$\imath\hbar\frac{\Delta_{N_t}}{\Delta(t)}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\mathbf{\Delta}_{(N_{x_i})}\Psi(\mathbf{r},t), \tag{60}$$

at rather large, but finite $|N_t|, |N_{x_i}|$ complies to the Schrodinger Equation in the continuous case with any preset accuracy.
Similary, from the formula for **measurable** momentum value at low energies $E \ll E_P$

$$p_{N_{x_i}} = \frac{\hbar}{N_{x_i}\ell} \tag{61}$$

as well as the equation (38) for **measurable** analog of a vectorial gradient $\nabla_{\mathbf{N_{\Delta x_i}}}$, and the equations (36),(37) leads to the fact that accordance in **measurable** case

$$\mathbf{p_{N_{x_i}}} \doteq \mathbf{p_{N_q}} \mapsto \frac{\hbar}{\imath}\nabla_{\mathbf{N_q}}, \tag{62}$$

can with any preset accuracy reproduce the accordance in the continuous case

$$\mathbf{p} \mapsto \frac{\hbar}{\imath}\nabla \tag{63}$$

As it is for **measurable** energy value

$$E_{N_q} = \frac{p_{N_q}^2}{2m} = \frac{p_{N_x}^2 + p_{N_y}^2 + p_{N_z}^2}{2m} \tag{64}$$

the accordance

$$E_{N_q} \mapsto \imath\hbar \frac{\Delta_{N_t}}{\Delta(t)} \tag{65}$$

reproduces the accordance

$$E \mapsto \imath\hbar \frac{\partial}{\partial t} \tag{66}$$

of the continuous theory.
So,in terms of **measurable** quantities we can get the discrete model as close as it can to the source continuous theory.

From this we can make a direct conclusion that **measurable** wavefunction $\Psi_{meas}(\mathbf{r}, t, \mathbf{N_q}, N_t)$, which has form

$$\Psi_{meas}(\mathbf{r}, t, \mathbf{N_q}, N_t) = Aexp\{\imath(\frac{\mathbf{p_{N_q} r}}{\hbar} - \frac{\mathbf{E_{N_q} r}}{\hbar})\}, \tag{67}$$

where $\mathbf{r}$ and t – **measurable** with an exact accuracy reproduces the correspondent wavefunction $\Psi(\mathbf{r}, t)$ in the continuous case [14].
The certain example is presented above in the text. However, it is absolutely obvious that on its basis we can make more common conclusions.
Measurable analog $\widehat{H}_{meas}$ of Hamiltonian $\widehat{H}$ from the equation (57) has the following form on the common case

$$\widehat{H}_{meas} = \widehat{H}_{meas}(N_q), \tag{68}$$

where N_q is **measurable** and

$$\lim_{|N_q| \to \infty} \widehat{H}_{meas} = \widehat{H}. \tag{69}$$

And as

$$\lim_{|N_t| \to \infty} \frac{\Delta_{N_t}[\Psi(t))]}{\Delta(t)} = \frac{d\Psi(t)}{dt}, \tag{70}$$

then in the common case in the passage to the limit at $|N_q| \to \infty, |N_t| \to \infty$ from **measurable** analog of the Schrodinger equation (57) we can get the Schrodinger equation (56) in the continuous picture.
At that we can suppose that all variables including time t, influencing the wavefunction ψ are **measurable** quantities, the similar supposition is correct for the Hamiltonian $\widehat{H}_{meas}$.
Now we can suppose without losing commonness that the values $|N_q| \gg 1$ are large enough and we can practically think that **measurable** the Hamiltonian analog $\widehat{H}_{meas}$ with an high accuracy is equal to the Hamiltonian in the continuous case

$$\widehat{H}_{meas} = \widehat{H} \tag{71}$$

Then at the fixed large module N_t and **measurable** ψ **measurable** analog of the Schrodinger equation (57) can be solved recurrently

$$\Psi(t+\tau/N_t)=(\frac{\tau}{\imath N_t\hbar}\widehat{H}+1)\Psi(t). \tag{72}$$

Taking as an some initial point t **measurable** value $\psi(t)$ (possibly $t=0$), placing it to the right side (72), and then repeating this procedure but for the calculated value from the left side $\Psi(t+\tau/N_t)$ we can get function $\Psi(t+\Delta t)$ for arbitrary $\Delta t=K\tau/N_t$, where K is any natural number. It is obviously that if N_t- integer number then **primarily measurable variations** in this case will correspondent to the integer $K; K=\mathcal{M}N_t$, where $\mathcal{M}$ – integer number. And as $(E\ll E_P)$, then $|\mathcal{M}|\gg 1$.
Further denoting

$$(\frac{\tau}{\imath N_t\hbar}\widehat{H}+1)\doteq\widehat{U}(\tau/N_t) \tag{73}$$

we receive that

$$\frac{1}{\imath\hbar}\widehat{H}=\frac{\widehat{U}(\tau/N_t)-1}{\tau/N_t} \tag{74}$$

Here we, as a matter of course, can suppose that $U(0)=1$ and according (57)

$$\frac{\Delta_{N_t}[\widehat{U}(t')]}{\Delta(t)}\doteq\frac{\widehat{U}(t'+\tau/N_t)-\widehat{U}(t')}{\tau/N_t}, \tag{75}$$

we receive, that

$$\frac{\Delta_{N_t}[\widehat{U}(t')]}{\Delta(t)}|_{t'=0}=\frac{1}{\imath\hbar}\widehat{H} \tag{76}$$

which is in an exact accordance to the known formula in the continuous case

$$\widehat{H}=\imath\hbar\frac{d\widehat{U}(t')}{dt'}|_{t'=0} \tag{77}$$

Operator $\widehat{U}(t')$, satisfying to the equations (73)–(76) can be denoted as $\widehat{U}_{N_t}$. It is trivial implication from the abovementioned formula that

$$\Psi(t+\tau/N_t) = \widehat{U}(\tau/N_t)\Psi(t) \tag{78}$$

The presented calculations can be generalized for non-autonomous systems, when the hamiltonian $\widehat{H}, (\widehat{H}_{meas})$ depends on time t, i.e. $\widehat{H} = \widehat{H}(t)$ and the condition (71) is preserved. In this case we can suppose that all values (operators and wavefunction) are **measurable** quantities,therefore we can receive:

$$\Psi(t+\tau') = \widehat{U}(t+\tau',t)\Psi(t),$$
$$\frac{\Delta_{N_t}}{\Delta(t)}\Psi(t) = \frac{\Delta_{N_t}[\widehat{U}(t+\tau',t)]}{\Delta(\tau')}|_{(\Delta(\tau')=\tau/N_t)}\Psi(t) = \frac{1}{\imath\hbar}\widehat{H}(t)\Psi(t),$$
$$\widehat{H}(t) = \imath\hbar\frac{\Delta_{N_t}[\widehat{U}(t+\tau',t)]}{\Delta(\tau')}|_{\Delta(\tau')=\tau/N_t} \tag{79}$$

Obviously, in the present equation one can reproduce all main formulas of the continuous case replacing dt onto τ/N_t, particular:

$$\widehat{U}^{\dagger}(t+\tau/N_t,t) = (\widehat{1}+\frac{\tau}{N_t}\frac{\widehat{H}}{\imath\hbar}+o(\frac{\tau}{N_t}))^{\dagger} = \widehat{1}-\frac{\tau}{N_t}\frac{\widehat{H}^{\dagger}}{\imath\hbar}+o(\frac{\tau}{N_t}) =$$
$$= \widehat{U}^{-1}(t+\tau/N_t,t) = (\widehat{1}+\frac{\tau}{N_t}\frac{\widehat{H}}{\imath\hbar}+o(\frac{\tau}{N_t}))^{-1} = \widehat{1}-\frac{\tau}{N_t}\frac{\widehat{H}}{\imath\hbar}+o(\frac{\tau}{N_t}) \tag{80}$$

What is the meaning of changing dt onto τ/N_t and transition from continuous case to discrete case in the terms of **measurable quantities**? It is assumed that the following **Hypothesis** is valid :

at low energies $E \ll E_P$, i.e. at $|N_t| \gg 1$ for any wavefunction $\Psi(t)$ exists such natural number $\mathbf{N}(\psi), |\mathbf{N}(\psi)| \gg 1$ which is dependent from $\Psi(t)$ with unimprovable approximation of the Schrodinger equation (56) by the discrete equation (57). Of course, obviously, that $1 \ll |N_t| \leq |\mathbf{N}(\psi)|$.

4.2.2 The Linear Momentum Operator

It is known, the task for eigenvalues and eigenfunctions of momentum projection $\hat{p}_{x_i}$ in case of continuous space-time can be reduced to the differential equation [37]:

$$-\imath\hbar\frac{\partial\Psi(x_i)}{\partial x_i}=p_{x_i}\Psi(x_i). \quad (81)$$

One can find continuous single-valued and bounded solutions of this equation of all real values of p_{x_i} in the interval $-\infty < p_{x_i} < \infty$ with eigenfunctions

$$\Psi_p(x_i)=Aexp(\imath\frac{p}{\hbar}x_i). \quad (82)$$

Thus there is one eigenfunction (no degeneracy) for each eigenvalue $p_{x_i}=p$. As it was stated above, in the **measurable** case under consideration in the left side (82) for some **measurable** fixed $|N_{x_i}|\gg 1$ replacing occurs

$$\frac{\partial}{\partial x_i}\mapsto\frac{\Delta_{N_{x_i}}}{\Delta(x_i)} \quad (83)$$

and the eigenvalues $p_{N_{x_i}}$ of the operator $\hat{p}_{x_i}$ become discrete numbers N_{x_i}

$$p_{N_{x_i}}=\frac{\hbar}{N_{x_i}\ell},|N_{x_i}|\gg 1 \quad (84)$$

but due to the condition $|N_{x_i}|\gg 1$ we receive **discrete** spectrum of operator $\hat{p}_{x_i}$, which is **almost continuous**.
Taking into account that at $|N_{x_i}|$ large enough with any preset accuracy we have

$$\frac{\Delta_{N_{x_i}}}{\Delta(x_i)}=\frac{\partial}{\partial x_i}, \quad (85)$$

and taking into account the formula (84), we can get the analog of formula (82) in the considered case

$$\Psi_{p_{N_{x_i}}}(x_i)=Aexp(\imath\frac{x_i}{N_{x_i}\ell}) \quad (86)$$

This shows that for the fixed x_i the correspondent discrete set of eigenfunctions also changes almost continuously.

It should be noted that the condition $-\infty < p_{x_i} < \infty$ in this case is non-correct, because

$$((p_{x_i} = p_{N_{x_i}}) \to \pm\infty) \equiv (|N_{x_i}| \to 1), \tag{87}$$

which contradicts to the condition $|N_{x_i}| \gg 1$.
However, for the real task the abstract condition $|N_{x_i}| \gg 1.$ is always replaced by some certain condition

$$|N_{x_i}| \geq \mathbf{N}_* \gg 1. \tag{88}$$

Then the condition $-\infty < p_{x_i} < \infty$ in the continuous case is replaced in the studied case with the condition $p_{-\mathbf{N}_*} \leq p_{x_i} \leq p_{\mathbf{N}_*}$ with the separated point $p_{x_i} = 0$, which is evidently doesn't belong to the equation (84) at the finite N_{x_i}.
It is clear that the case $N_{x_i} = \pm\infty$ appropriate of the point $p_{x_i} = 0$ is the degenerate case, that is why if we would like to consider the finite N_{x_i} the condition (88) should be replaced with the condition

$$\mathbf{N}^* \geq |N_{x_i}| \geq \mathbf{N}_* \gg 1. \tag{89}$$

and then $p_{x_i} \in [p_{-\mathbf{N}_*}, p_{-\mathbf{N}^*}] \bigcup [p_{\mathbf{N}^*}, p_{\mathbf{N}_*}]$
Further, we denote as $\Delta_{\mathbf{N}_*,\mathbf{N}^*}(p_{x_i})$ intervals union

$$\Delta_{\mathbf{N}_*,\mathbf{N}^*}(p_{x_i}) \doteq [p_{-\mathbf{N}_*}, p_{-\mathbf{N}^*}] \bigcup [p_{\mathbf{N}^*}, p_{\mathbf{N}_*}], \tag{90}$$

and as $\Delta_{\mathbf{N}_*}(\mathbf{p})$

$$\Delta_{\mathbf{N}_*,\mathbf{N}^*}(\mathbf{p}) = \prod_i \Delta_{\mathbf{N}_*,\mathbf{N}^*}(p_{x_i}) \tag{91}$$

4.2.3 The z-component of the Angular Momentum $\hat{L}_z$

In the accepted quantum mechanics the task of eigenvalues and eigenfunctions of angular momentum operator $\hat{L}_z$

$$\hat{L}_z = -\imath\hbar(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}) \tag{92}$$

is reduced to the differential equation solution [37]

$$-\imath\hbar\frac{\partial\Psi(\phi)}{\partial\phi}=L_z\Psi(\phi), \tag{93}$$

where $0\leq\phi\leq 2\pi$.
In the considered case we can suppose that $\phi=\phi(x,y,z)$–**measurable** function from variables $x,y,z,$ having in case of continuity well defined partial derivative for each of them.
It is obvious that substitution into the formula (37) $F(x_i)=\phi(x,y,z)$ gives

$$\lim_{|N_{\Delta x_i}|\rightarrow\infty}\frac{\Delta_{N_{\Delta x_i}}[\phi(x,y,z)]}{\Delta(x_i)}=\lim_{\Delta(x_i)\rightarrow 0}\frac{\Delta_{N_{\Delta x_i}}[\phi(x,y,z))]}{\Delta(x_i)}=\frac{\partial\phi}{\partial x_i}. \tag{94}$$

On the abovementioned basis we can state that there is **measurable** function $\Delta\Psi/\Delta\phi$ and we have

$$\lim_{\Delta\phi\rightarrow 0}\frac{\Delta\Psi}{\Delta\phi}=\lim_{|N_{\Delta x_i}|\rightarrow\infty}\frac{\Delta\Psi}{\Delta\phi}=\frac{\partial\Psi}{\partial\phi}, \tag{95}$$

where $\Delta\phi(x_i)=\sum_i(\phi(x_i+\Delta x_i)-\phi(x_i))$ and **measurable** increments Δx_i are taken from the formula (36).
Taking into account that for enough large $|N_{x_i}|$ with an high accuracy
$\Delta_{N_{x_i}}/\Delta(x_i)=\partial/\partial x_i$ and $\Delta\Psi(\phi)/\Delta\phi=\partial\Psi(\phi)/\partial\phi$ we conclude that the equation (93) with an high accuracy can be used in **measurable** case for $\phi(x,y,z))$ **measurable** function from **measurable** $\{x,y,z\}$.
Then the solution (93) are presented as an exponent

$$\Psi(\phi)=Aexp(\imath\frac{L_z}{\hbar}\phi), \tag{96}$$

where $\phi=\phi(x,y,z)$–**measurable** function from **measurable** variables x,y,z.
At that, eigenfunctions for discrete spectrum $L_z=\hbar m; m=0,\pm 1,\pm 2,...$ of operator $\hat{L}_z$ as in the continuous case will be

$$\Psi_m(\phi)=(2\pi)^{-1/2}e^{\imath m\phi}, \tag{97}$$

where ϕ is a **measurable** quantity.
However, at normalization condition in the continuous case [37], in the present form the integral is replaced by the sum:

$$(\int_0^{2\pi} |\Psi_m|^2 d\phi = 1) \Rightarrow (\sum_{0 \leq \phi \leq 2\pi} |\Psi_m|^2 \Delta(\phi) = 1), \tag{98}$$

where $\Delta(\phi)$ is taken from the formula (95).

4.3 Position and Momentum Representations and Fourier Transform in Terms of Measurability

Now, using the formulas of the previous sections we can analyze in terms of **measurably** quantities issue of quantum representations and the Fourier transformation. Scalar (inner) product in position representation in the continuous case is determined by the equation [14],[38]:

$$(\varphi_1, \varphi_2) = \int_{R^3} \varphi_1^*(\mathbf{x}) \varphi_2(\mathbf{x}) d\mathbf{x} \tag{99}$$

Both operators of coordinates $\mathbf{x_j}$ and momentum $\mathbf{p_j}$,$(j = 1, 2, 3)$ in position representation are introduced by the equations [14]:

$$\begin{aligned} \mathbf{x_j}.\varphi(\mathbf{x}) &= x_j \varphi(\mathbf{x}), \\ \mathbf{p_j}.\varphi(\mathbf{x}) &= -\imath\hbar \frac{\partial}{\partial x_j} \varphi(\mathbf{x}) \end{aligned} \tag{100}$$

In the abovementioned denotations $\mathbf{x} = q$ is taken from the formula (49) therefore integral from the equation (100) is replaced by the sum

$$(\varphi_1, \varphi_2)_{meas} = \sum_{\mathbf{x} \in R^3} \varphi_1^* \varphi_2 \Delta_{N_\mathbf{x}}(\mathbf{x}), \tag{101}$$

where $\mathbf{x}$ – **measurable** coordinates.
It is clear that the passage to the limit takes place

$$\lim_{N_{x_i} \to \infty} (\varphi_1, \varphi_2)_{meas} = (\varphi_1, \varphi_2) \tag{102}$$

where $\{N_{x_i}\} = N_q$ from the equation (49) and at enough large $\{N_{x_i}\} = N_q$ with high precision

$$(\varphi_1, \varphi_2)_{meas} = (\varphi_1, \varphi_2) \tag{103}$$

In the considered case the first equation from (100) is preserved for all **measurable** values of the left and right side, while the second one is replaced by

$$\mathbf{p_{N_{x_j}}}.\varphi(\mathbf{x}) = -\imath\hbar\frac{\Delta_{N_{x_j}}}{\Delta(x_j)}\varphi(\mathbf{x}) \doteq$$
$$\doteq -\imath\hbar\frac{\varphi(x_{i\neq j}, x_j + \ell/N_{x_j}) - \varphi(\mathbf{x})}{\ell/N_{x_j}}, \tag{104}$$

where $\mathbf{p_{N_{x_j}}}$ – **measurable** momentum j-component presented as follows:

$$p_{N_{x_j}} = \frac{\hbar}{N_{x_j}\ell}. \tag{105}$$

And the function $\varphi(x_{i\neq j}, x_j + \ell/N_{x_j})$ differs from $\varphi(\mathbf{x})$ only with its "shift" to ℓ/N_{x_j} in j-component.
From the formulas above and, particularly, the formula (37), we can make a clear supposition that in this case of low energies $E \ll E_P$, i.e. at $|N_{x_j}| \gg 1$ with an high precision we have

$$\frac{\Delta_{N_{x_j}}}{\Delta(x_j)} = \frac{\partial}{\partial x_j}. \tag{106}$$

Then, due to the formula (104)–(106) in the case of low energies $E \ll E_P$ for **measurable** quantities with an high precision we get

$$[\mathbf{x}, \mathbf{p}].\varphi(\mathbf{x}) = \mathbf{xp}.\varphi(\mathbf{x}) - \mathbf{px}.\varphi(\mathbf{x}) = \imath\hbar\varphi(\mathbf{x}) \tag{107}$$

In momentum representation in the continuous picture:

$$\mathbf{x_j}.\varphi(\mathbf{p}) = \imath\hbar\frac{\partial}{\partial p_j}\varphi(\mathbf{p}),$$
$$\mathbf{p_j}.\varphi(\mathbf{p}) = p_j\varphi(\mathbf{p}) \tag{108}$$

In the **measurable** case the second equation (108) for **measurable** momenta remains unchanged. According to the formula (42) and (43) in the **measurable** case in the first equation from (108) a replacement takes place

$$\frac{\partial}{\partial p_j} \mapsto \frac{\Delta_{p_j}}{\Delta p_j}, \tag{109}$$

where

$$\begin{aligned} p_j \doteq p_{N_{x_j}} &= \frac{\hbar}{N_{x_j}\ell}; \\ \frac{\Delta_{p_j}\varphi(\mathbf{p}))}{\Delta p_j} \equiv \frac{\varphi(\mathbf{p}+p_j)-\varphi(\mathbf{p})}{p_j} &= \frac{\varphi(\mathbf{p}+\frac{\hbar}{N_{x_j}\ell})-\varphi(\mathbf{p})}{\frac{\hbar}{N_{x_j}\ell}}, \end{aligned} \tag{110}$$

and $\varphi(\mathbf{p}+p_j)$ differs from $\varphi(\mathbf{p})$ with the value p_j only in j-component. Then from the expression (43) due to the fact $|N_{x_j}| \gg 1$ with an high exactness we get

$$\frac{\Delta_{p_j}}{\Delta p_j} = \frac{\partial}{\partial p_j} \tag{111}$$

Now let us consider $[\mathbf{x},\mathbf{p}].\varphi(\mathbf{p})$ in momentum representation. Taking into account the formula (111) we receive

$$\begin{aligned} [\mathbf{x}_j,\mathbf{p}_j].\varphi(\mathbf{p}) \ &= \mathbf{x}_j\mathbf{p}_j.\varphi(\mathbf{p}) - \mathbf{p}_j\mathbf{x}_j.\varphi(\mathbf{p}) = \\ = \imath\hbar(\varphi(\mathbf{p}) + p_j &\frac{\varphi(\mathbf{p}+p_j)-\varphi(\mathbf{p})}{p_j} - \\ -p_j &\frac{\varphi(\mathbf{p}+p_j)-\varphi(\mathbf{p}}{p_j}) = \\ &= \imath\hbar.\varphi(\mathbf{p}). \end{aligned} \tag{112}$$

Thus, the expressions (106)–(112) show that

$$[\mathbf{x}_i,\mathbf{p}_j] = \imath\delta_{ij}\hbar \tag{113}$$

takes place in **measurable** case both in position representation and momentum representation.

In the continuous picture the Fourier transformation has the following form [38]:

$$\varphi(\mathbf{x}) = (\frac{1}{2\pi\hbar})^{3/2} \int_{R^3} e^{\frac{\imath}{\hbar}\mathbf{px}} \varphi(\mathbf{p}) d\mathbf{p} \tag{114}$$

And the operator $\mathbf{p_j}$ applied to the formula (114), gives [38]:

$$\mathbf{p_j}.\varphi(\mathbf{x}) = -\imath\hbar \frac{\partial}{\partial x_j}\varphi(\mathbf{x}) = -\imath\hbar \frac{\partial}{\partial x_j}(\frac{1}{2\pi\hbar})^{3/2} \int_{R^3} e^{\frac{\imath}{\hbar}\mathbf{px}} \varphi(\mathbf{p}) d\mathbf{p} =$$
$$= (\frac{1}{2\pi\hbar})^{3/2} \int_{R^3} e^{\frac{\imath}{\hbar}\mathbf{px}} p_j \varphi(\mathbf{p}) d\mathbf{p} \tag{115}$$

However, as it was indicated in the formulas (87),(88) in the considered **measurable** case of low energies the $|\mathbf{p}|$ values are bounded, therefore $\mathbf{p}$ doesn't fill in all space R^3, and belongs only to its part $\Delta_{\mathbf{N_*},\mathbf{N^*}}(\mathbf{p})$ (formula (91)).
That is why the integral in the equation (114) should be replaced by the sum:

$$\varphi_{meas}(\mathbf{x}) = (\frac{1}{2\pi\hbar})^{3/2} \sum_{\mathbf{p}\in\Delta_{\mathbf{N_*},\mathbf{N^*}}(\mathbf{p})} e^{\frac{\imath}{\hbar}\mathbf{px}} \varphi_{meas}(\mathbf{p}) \Delta_p(p_{N_\mathbf{x}}), \tag{116}$$

where $\mathbf{x}$,$\mathbf{p}$ and $\varphi_{meas}(\mathbf{p})$ are **measurable** quantities and

$$\Delta_p(p_{N_\mathbf{x}}) = \prod_j p_{N_{x_j}}, \tag{117}$$

where $p_{N_{x_j}}$ is taken from the equation (110).
And as $|N_{x_j}| \gg 1$, then in the limit $|N_{x_j}| \to \infty$ the sum in the right side of the equation (116) is replaced by the integral that's why, with an high precision, we receive

$$(\frac{1}{2\pi\hbar})^{3/2} \int_{\Delta_{\mathbf{N_*},\mathbf{N^*}}(\mathbf{p})} e^{\frac{\imath}{\hbar}\mathbf{px}} \varphi(\mathbf{p}) d\mathbf{p} = \tag{118}$$
$$= (\frac{1}{2\pi\hbar})^{3/2} \sum_{\mathbf{p}\in\Delta_{\mathbf{N_*},\mathbf{N^*}}(\mathbf{p})} e^{\frac{\imath}{\hbar}\mathbf{px}} \varphi_{meas}(\mathbf{p}) \Delta_p(p_{N_\mathbf{x}})$$

It should be noted that in this case the domain of the function changes only for the momenta. Due to the abovementioned equations it is tapered: $\{\mathbf{p} \in R^3\} \mapsto \{\mathbf{p} \in \Delta_{\mathbf{N}_*,\mathbf{N}^*}(\mathbf{p})\}$. For coordinates it remains $\{\mathbf{x} \in R^3\}$.
The function $\varphi(\mathbf{p})$ in the continuous case is the following form [38]:

$$\varphi(\mathbf{p}) = (\frac{1}{2\pi\hbar})^{3/2} \int_{R^3} e^{-\frac{\imath}{\hbar}\mathbf{px}} \varphi(\mathbf{x}) d\mathbf{x} \tag{119}$$

As the definition domain in the position representation remains the same $\{\mathbf{x} \in R^3\}$, then for **measurable** case $\varphi_{meas}(\mathbf{p})$ has the following form

$$\varphi_{meas}(\mathbf{p}) = (\frac{1}{2\pi\hbar})^{3/2} \sum_{R^3} e^{-\frac{\imath}{\hbar}\mathbf{px}} \varphi_{meas}(\mathbf{x}) \Delta_{N_\mathbf{x}}(\mathbf{x}), \tag{120}$$

where $\mathbf{x} = q$ from the formula (49), i.e.

$$\Delta_{N_\mathbf{x}}(\mathbf{x}) = \prod_j \Delta_{N_{x_j}}(x_j) = \frac{\ell^3}{N_x N_y N_z} \tag{121}$$

In this case due to the condition $|N_{x_j}| \gg 1$ we produce the following:

$$(\frac{1}{2\pi\hbar})^{3/2} \int_{R^3} e^{-\frac{\imath}{\hbar}\mathbf{px}} \varphi(\mathbf{x}) d\mathbf{x} \approx (\frac{1}{2\pi\hbar})^{3/2} \sum_{R^3} e^{-\frac{\imath}{\hbar}\mathbf{px}} \varphi_{meas}(\mathbf{x}) \Delta_{N_\mathbf{x}}(\mathbf{x}), \tag{122}$$

where all values in the right side of (122) are **measurable**.
Thus, the equations (116) and (120) are analogues of direct and inverse Fourier transformation in terms of **measurable** quantities or better to say of **measurable** of the direct and inverse Fourier transformation.
In the present formalism we can easily produce **measurable** analog of the equation (115) with replacement $\mathbf{p_j} \mapsto \mathbf{p_{N_{x_j}}}$,$\partial/\partial x_j \mapsto \Delta_{N_{x_j}}/\Delta(x_j)$,$\varphi(\mathbf{x}) \mapsto \varphi_{meas}(\mathbf{x})$ and $\int_{R^3} \mapsto \sum_{\Delta_{\mathbf{N}_*}(\mathbf{p})}$.
Similar for the corresponding replacement in **measurable** variant it is possible to receive the analogue of the accordance

$$\mathbf{x_j}.\varphi(\mathbf{p}) \mapsto \imath\hbar \frac{\partial}{\partial p_j} \varphi(\mathbf{p}) \tag{123}$$

in the continuous picture.

Here it is necessary to make some important explanations:

Commentary 4.3.

4.3.1. As we considered minimal length ℓ and time τ at Plank level $\ell \propto l_p, \tau \propto t_p$, The use of the **measurable** quantities $\ell/N_{x_i}; i = 1, ..., 3$ and τ/N_t at $|N_{x_i}| \gg 1, |N_t| \gg 1$ as a replacement of dx_i, dt in the continuous case is absolutely correct and justified. Actually, as in this case ℓ has the order $\approx 10^{-33}cm$, then ℓ/N_{x_i} will have the order of $\approx 10^{-33-\lg|N_{x_i}|}cm$, which is, without doubts, will exceed any practical computations precision. The similar statement is true for the value τ/N_t as well, where τ has the order of Plank time t_p, i.e. $\approx 10^{-44}sec$. For this reason, it is correct to use $p_{N_{x_i}}$ instead of dp_i and $\Delta_{N_{x_i}}/\Delta(x_i), \Delta_{N_t}/\Delta(t), \Delta_{p_i}/\Delta p_i$ instead of $\partial/\partial x_i, \partial/\partial t, \partial/\partial p_i$, accordingly, in the continuous case.

4.3.2. For the sake of generality in **Remark 3.2.1** we supposed that N_{x_i}, N_t are **generalized measurable** quantities. However, due to $|N_{x_i}| \gg 1, |N_t| \gg 1$ we can regard without loss of generality the numbers N_{x_i} and N_t as **primarily measurable** quantities. It is clear that

$$[N_{x_i}] \leq N_{x_i} \leq [N_{x_i}] + 1, \qquad (124)$$

where $[\aleph]$ defines the entier of number $\aleph$. Then $|N_{x_i}|^{-1}$ gets into the interval with the points $|[N_{x_i}]|^{-1}$ and $|[N_{x_i}] + 1|^{-1}$ (which is larger among these numbers and which is less depends on sign of the number N_{x_i}). In any case we have $|N_{x_i}^{-1} - [N_{x_i}]^{-1}| \leq |([N_{x_i}] + 1)^{-1} - [N_{x_i}]^{-1}| = |([N_{x_i}] + 1)[N_{x_i}]|^{-1}$. In any case, the difference between ℓ/N_{x_i} and $\ell/[N_{x_i}]$ (accordingly between $\Delta_{N_{x_i}}/\Delta(x_i)$ and $\Delta_{[N_{x_i}]}/\Delta(x_i)$ and so on) is almost insignificant. The similar computations are correct for τ/N_t and $\tau/[N_t]$ as well.

4.3.3a. It should be noted that despite the fact that in **measurable** case there is an analogue of direct and inverse Fourier transformation set by the equations (116) and (120) the difference between position and momentum representations is significant. Indeed, the first one has all three dimensional space R^3 as domain definition, while the second one has some part of finite sizes $\Delta_{\mathbf{N}_*,\mathbf{N}^*}(\mathbf{p})$, "cut out" in three dimensional space $\Delta_{\mathbf{N}_*,\mathbf{N}^*}(\mathbf{p}) \subset R^3$

4.3.3b. Significant difference between position representation and momentum representation in **measurable** case lays in their different nature in this formalism. Position representation in this case is formed, in general, the same as the correspondent representation in the continuous case. Momentum representation in **measurable** case , as it follows from the formulas **Remark 3.2.1** is formed in the basis of **measurable variations** in the position representation.
It should be noted that as ℓ with an accuracy up to multiplicative constant corresponds to l_p, and $p_{N_{\mathbf{x}}}$ with an accuracy up to multiplicative constant corresponds to $\ell/N_{\mathbf{x}}$ (formula (44)), then the summing measures in **measurable** case in the equations (116) and (120) in momentum and position spaces also match with an accuracy up to multiplicative constant

$$\Delta_{N_{\mathbf{x}}}(\mathbf{x}) = \frac{\ell^6}{\hbar^3}\Delta_p(p_{N_{\mathbf{x}}}) \tag{125}$$

4.3.4. It can be easily noticed that the abovementioned formalism of the Schrodinger picture's studying in terms of **measurability** can be applied for Heisenberg picture [14],[38]. Indeed, in the paradigm of the continuous space and time the motion equation for Heisenberg operators $\hat{L(t)}$ are as follows [14],[38]:

$$\frac{d\hat{L}(t)}{dt} = \frac{\partial \hat{L}(t)}{\partial t} + [\hat{H}, \hat{L}(t)], \tag{126}$$

where $\hat{H}$ – Hamiltonian and $[\hat{H}, \hat{L}(t)] = \frac{1}{\imath\hbar}(\hat{L}(t)\hat{H} - \hat{H}, \hat{L}(t))$–quantum Poisson bracket [38].
In **measurable** case quantum Poisson bracket preserves its form for **measurable** quantities inside it. $\partial\hat{L}(t)/\partial t$ is replaced to $\Delta_{N_t}[\hat{L}(t)]/\Delta(t)$, where the operator $\Delta_{N_t}[\hat{L}(t)]/\Delta(t)$ can be produced from the equation (75) with the replacement $\widehat{U}(t')$ onto $\hat{L}(t)$ at $|N_t| \gg 1$.
Then the analogue (126) in **measurable** case will be the equation:

$$\frac{\tilde{\Delta}_{N_t}[\hat{L}(t)]}{\Delta(t)} \doteq \frac{\Delta_{N_t}[\hat{L}(t)]}{\Delta(t)} + [\hat{H}, \hat{L}(t)], \tag{127}$$

It is clear that

$$\lim_{|N_t|\to\infty} \frac{\tilde{\Delta}_{N_t}[\hat{L}(t)]}{\Delta(t)} = \frac{d\hat{L}(t)}{dt}. \quad (128)$$

Thus, at enough large $|N_t|$ the equation (127) matches with the equation (126) with the high accuracy.

5 More Overall Measurability Definition

Now, basing on the abovementioned information, we can give the definition **measurability**, which is, as we concern, is more general that the initial one.
We, as it was performed before, begin with some minimal (universal) unit for length measurement ℓ, which corresponds to some maximal energy $E_\ell = \frac{\hbar c}{\ell}$ and universal time measurement unit $\tau = \ell/c$. Without the loss of generality we can consider ℓ and τ at Plank level, i.e. $\ell = \kappa l_p, \tau = \kappa t_p$, where numeric constant κ is order of 1. Consequently, $E_\ell \propto E_p$ with the suitable coefficient of proportionality.
We intentionally use in this case for ℓ and τ besides the phrase "minimal measurement unit" the phrase "universal measurement unit" as well, because in our case it presents full coverage of its sense.
Now we shall consider in the space of the momenta $\mathbf{P}$ the domain defined by the conditions

$$\mathbf{p} = \{p_{x_i}\}, i = 1, .., 3; P_{pl} \gg |p_{x_i}| \neq 0, \quad (129)$$

where P_{pl}–Plank momentum. Then we can easily calculate the numeric coefficients N_{x_i}

$$N_{x_i} = \frac{\hbar}{p_{x_i}\ell}, or \quad (130)$$
$$p_{x_i} \doteq p_{N_{x_i}} = \frac{\hbar}{N_{x_i}\ell}$$
$$|N_{x_i}| \gg 1,$$

where the last part of the equation (130) is determined by the formula (129).

Definition 1*
1*.1 Let's call the momenta $\mathbf{p}$, set by the formula (129) **primarily measurable**, if all numbers N_{x_i} from the equation (130) are integer numbers.
1*.2 Let's call any variation of Δx_i coordinates x_i and Δt of time t for energies $E \ll E_p$ as **primarily measurable**, if

$$\Delta x_i = N_{x_i}\ell, \Delta t = N_t \tau, \tag{131}$$

where N_{x_i} satisfies the condition **1*.1** and $|N_t| \gg 1$ – natural number.
1*.3 Let us define any physical quantity **primarily or elementarily measurable** at low energies $E \ll E_p$, when its value is consistent with points **1*.1** and **1*.2** of this Definition.

Further for the sake of convenience we denote the momenta domain, satisfying the conditions (129) (or (130)) as $\mathbf{P}_{LE}$..
In Commentary **4.3.2** it is shown that at low energies $E \ll E_P$ ($|N_{x_i}| \gg 1$) **primarily measurable** of momenta are enough to, with the high accuracy, produce all domain of momenta $\mathbf{P}_{LE}$.
This means that in the abovementioned domain the discrete set **primarily measurable** of momenta $p_{N_{x_i}}; i = 1, ..., 3$, (where N_{x_i}-natural number, and $|N_{x_i}| \gg 1$), changes almost continuously, practically covering the whole this domain.
That is why further $\mathbf{P}_{LE}$ means the domain consisting of **primarily measurable** momenta, satisfying the conditions of the formula (129) (or (130)).

Then the boundaries of the region $\mathbf{P}_{LE}$ are determined by the condition (89) for each coordinate

$$\mathbf{N}^* \geq |N_{x_i}| \geq \mathbf{N}_* \gg 1,$$

where large positive numbers $\mathbf{N}^*, \mathbf{N}_*$ are determined by the task solvable.
The choice of number $\mathbf{N}^*$ has particular importance. If $\mathbf{N}^* < \infty$, then it is clear that the studied momenta lay within $\mathbf{P}_{LE}$. If to make a precondition

that $\mathbf{N}^* = \infty$, then for $\mathbf{P}_{LE}$ we should add for each coordinate x_i "non-intrinsic" (or "singular") point $p_{x_i} = 0$ (we name these cases **degenerate**). In any case for each coordinate x_i the boundaries of $\mathbf{P}_{LE}$ are as follows:

$$p_{\mathbf{N}^*} \leq |p_{N_{x_i}}| \leq p_{\mathbf{N}_*} \tag{132}$$

Therefore, for distinctness we can note $\mathbf{P}_{LE}$ with certain boundaries set by the formula (132) per $\mathbf{P}_{LE}[\mathbf{N}^*, \mathbf{N}_*]$.
It is obvious that in such formalism **small** increments for any component $p_{N_{x_i}}$ of momentum $\mathbf{p} \in \mathbf{P}_{LE}$ are momentum values $p_{N'_{x_i}}$, for which $|N'_{x_i}| > |N_{x_i}|$. And then, incrementing $|N'_{x_i}|$ we can receive **as much as desired small** increments for momenta $\mathbf{p} \in \mathbf{P}_{LE}$.
Therefore in this case the definition of "measurable partial derivative" for momentum $p_{N_{x_i}}$ shall be correct, denoted in the equation (42) and (43) through $\frac{\Delta_{p_{N_{x_i}}}}{\Delta p_{N_{x_i}}}$. As it was shown in the equations (42) and (43) and due to the contents of the previous paragraph at the values of $|N_{x_i}|$ large enough, with any predetermined precision the equality $\frac{\Delta_{p_{N_{x_i}}}}{\Delta p_{N_{x_i}}} = \frac{\partial}{\partial p_i}$ takes place (for example formula (111)).
Obviously, that **primarily measurable** measurements Δx_i of coordinates x_i and Δt of time t from **1*.2** of **Definition 1*** can't be considered as small variations of space and time. Still, the equation (44) and its application in the further text of the article gives us a basis to state that space and time values

$$\frac{\tau}{N_t} = p_{N_t c}\frac{\ell^2}{c\hbar}$$
$$\frac{\ell}{N_{x_i}} = p_{N_{x_i}}\frac{\ell^2}{\hbar}, 1 = 1, ..., 3, \tag{133}$$

are small values and, as it is shown, in (44) they can be as small as desired at enough large values of $|N_{x_i}|, |N_t|$. Here $p_{N_{x_i}}, p_{N_t c}$ are corresponding **primarily measurable** momenta.
It is clear that space and time quantities $\frac{\tau}{N_t}, \frac{\ell}{N_{x_i}}$ won't be **primarily measurable** space-time quantities despite the fact that they, with up to constant accuracy are equal **primarily measurable** momenta.

Therefore, the following definition makes sense:

Definition 2*.(Generalized Measurability in Low Energies).
We shall call any physical quantity at low energies $E \ll E_p$ as **generalized-measurable** or for simplicity **measurable** if any of its values may be obtained in terms of **Primarily Measurable Quantities** of **Definition 1***.

Now, withdrawing the restriction $P_{pl} \gg |p_{x_i}|$ in the equation (129) nd, the same option, $|N_{x_i}| \gg 1$ in the formula (130), i.e. considering momenta space $\mathbf{p}$ at **all energies scales**

$$\begin{aligned} \mathbf{p} = \{p_{x_i}\}, i = 1,..,3; |p_{x_i}| \neq 0; \\ N_{x_i} = \frac{\hbar}{p_{x_i}\ell}, or \\ p_{x_i} \doteq p_{N_{x_i}} = \frac{\hbar}{N_{x_i}\ell}, \\ 1 \leq |N_{x_i}| < \infty, or \quad E \leq E_\ell \end{aligned} \tag{134}$$

we we introduce the following definition

Definition 3*(Primarily and Generalized Measurability at All Energies Scales).
3*.1. Let us call the momenta $\mathbf{p}$, set by the formula (134) **primarily measurable**, of all numbers N_{x_i} from this formula (134) are integer.
3*.2. Any variation Δx_i of coordinates x_i and Δt of time t at all energies scales $E \leq E_\ell$ can be called **primarily measurable**, if

$$\Delta x_i = N_{x_i}\ell, \Delta t = N_t \tau, \tag{135}$$

where N_{x_i} satisfy the condition **3*.1** and the integer number N_t are within the interval of $1 \leq |N_t| < \infty$.
3*.3. Let us define any physical quantity **primarily or elementarily measurable** at all energies scales $E \leq E_\ell$, when its value is consistent with points **3*.1** and **3*.2** of this Definition.
3*.4. Finally, we shall call any physical quantity at all energies scales $E \leq E_\ell$, as **generalized-measurable** or for simplicity **measurable** if any

of its values may be obtained in terms of **Primarily Measurable Quantities** of points **3*.1–3*.3** in **Definition 3***.

"Non-intrinsic" points, at the values $|N_{x_i}| = \infty$ and $|N_t| = \infty$ can be added to the equation (134) and **Definition 3*** accordingly, as at the low energies case.
As it was shown above **Primarily Measurable Momenta** practically cover all momenta region $\mathbf{P}_{LE}$ at low energies $E \ll E_p$ (or same $E \ll E_\ell$). However, this is no longer the case at **all energies scales** $E \leq E_\ell$.
Therefore the main target of the author is quantum theory construction at all energies scales $E \leq E_\ell$ in terms of **measurable** (or same **primarily measurable**) quantities of **Definition 3***.
In this theory the values of physical quantity $\mathcal{G}$ can be represented as the numeric function $\mathcal{F}$ as follows

$$\mathcal{G} = \mathcal{F}(N_{x_i}, N_t, \ell) = \mathcal{F}(N_{x_i}, N_t, G, \hbar, c, \kappa), \tag{136}$$

where N_{x_i}, N_t–integer numbers from the formula (134),(135) and $G, \hbar, c$ are fundamental constants. The last equality in (136) is determined by the fact that $\ell = \kappa l_p$ and $l_p = \sqrt{G\hbar/c^3}$.
If $N_{x_i} \neq 0, N_t \neq 0$ (**non-degenerated** case), then it is clear that (136) can be rewritten as follows:

$$\mathcal{G} = \mathcal{F}(N_{x_i}, N_t, \ell) = \tilde{\mathcal{F}}((N_{x_i})^{-1}, (N_t)^{-1}, \ell) \tag{137}$$

And then at low energies $E \ll E_p$, i.e. at $|N_{x_i}| \gg 1, |N_t| \gg 1$ the function $\tilde{\mathcal{F}}$ is the function from variables, changing practically continuously, despite the fact that these variables run over discreet set of the values. It can be naturally supposed that $\tilde{\mathcal{F}}$ changes fluently, (it means practically continuously). As a result we get the model with discrete nature which as it is shown above, with an high accuracy reproduces the known theory in the continuous space-time.
Obviously, at low energies $E \ll E_p$ the formula (137) can be presented as follows:

$$\begin{aligned} \mathcal{G} = \mathcal{F}(N_{x_i}, N_t, \ell) = \tilde{\mathcal{F}}((N_{x_i})^{-1}, (N_t)^{-1}, \ell) = \\ = \tilde{\mathcal{F}}_{\mathcal{P}}(p_{N_{x_i}}, p_{N_{tc}}, \ell), \end{aligned} \tag{138}$$

where $p_{N_{x_i}}, p_{N_{tc}}$ are **primarily measurable** momenta from formula (44). It should be noted that the approach to the concept **measurability**, set forth in present Section is much more overall, then in Sections 2,3 for two reasons:
a)it is not connected directly with Heisenberg Uncertainty Principle and its generalizations;

b)it can be successfully used both for the non-relativistic case [14] and for the relativistic case [39].

6 Final Comments and Further Perspectives

6.1. Thus, at all energies scales we get some model (which should be constructed) depending on the same discrete parameters, which is at low energies E far from Planck $E \ll E_p$ is very close to the initial theory, that is why it reproduces with an high accuracy, all main results of canonic quantum theory in continuous spacetime. At high (Planck) energies $E \approx E_p$ the abovementioned discrete model will present new results.
The author supposes that this model will be deprived principal drawbacks of canonical quantum theory – ultraviolet and infrared divergences [39]. It will be finite at all orders of the perturbation theory and due to this reason it won't need renormalization [39].

6.2. The formula (44) and (133) show that **measurable** analogues small and infinitesimal space-time quantities coincide (up to constants) with the **primarily measurable** momenta.
This allows for gravity [40] to state the same problem as it was stated for the quantum theory in the paragraph **6.1.**:

Construction of **measurable** model of gravity, depending on the same discrete parameters N_{x_i}, N_t, which is at low energies $E \ll E_p$ is practically continuous and "very close" to General Relativity, and at high energies $E \approx E_p, (E \approx E_\ell)$ it will present the correct quantum theory without ultraviolet divergences.

Hovewer, the phrase "very close" in the last item doesn't mean exact correspondence of the abovementioned model with General Relativity [40]. According to my assumption in the studied model there should be no the "nonphysical" solutions of the General Relativity (for example, the solutions involving the **Closed Time-like Curves** (CTC) [41]–[44]).

6.3. At the moment each of the abovementioned theories – Quantum Theory and Gravity, considered within continuous space-time are presented by various theories at low energies $E \ll E_p$ and at high energies $E \approx E_p$. Therefore let us summarize the points **6.1.** and **6.2.** as follows:

In **measurable** format each of theories (quantum theory and gravity) will be unified theory at all energies scales $E \leq E_\ell$. Word "unified" means that at all energies scales they should be determined by the same discrete set of parameters N_{x_i}, N_t and constants $G, \hbar, c, \kappa$.
The main problem in this case will be correct determination and computations of functions $\mathcal{F}$ and $\tilde{\mathcal{F}}$ from formula (136)–(138).
In Subsection 3.1 within the framework of Generalized Uncertainty Principle we have already determined function $\mathcal{F}$ for all **measurable** momenta $p_{i,\mathbf{meas}}; i = 1, .., 3$ at **all energies scales** $E \leq E_\ell$ by formula (18),(19):

$$p_{i,\mathbf{meas}} = \mathcal{F}(N_{x_i}, \ell) = \frac{\hbar}{1/2(N_{x_i} + \sqrt{N_{x_i}^2 - 1})\ell}. \qquad (139)$$

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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