Emergence of Evolving Dimensions in Field Theory and Cosmology

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Abstract

We briefly elaborate on the definitions of *continuously evolving dimensions* and examine their ties to our unconventional research in field theory and relativistic cosmology.

Key words: complex dynamics, continuous dimensions, fractal spacetime, cosmological anomalies, Beyond the Standard Model Physics.

I. Introduction

The tacit assumption that spacetime has a fixed, integer dimensionality is deeply ingrained in both field theory and standard cosmology. General Relativity (GR) and Quantum Field Theory (QFT) are formulated on standard differentiable manifolds of constant Euclidean dimensions d_E, while cosmological observables are interpreted under the premise that the spatial dimension remains equal to $d_E^{space} = 3$ at all accessible observation scales. Nevertheless, a growing body of theoretical and observational evidence suggests that this assumption may only be an effective description, valid over a restricted range of scales and epochs [1 – 38].

From the theoretical side, several independent approaches to Quantum Gravity (QG) and Nonequilibrium Field Theory predict a *scale-dependent concept of dimension*. These include Asymptotic Safety [29], Causal Dynamical Triangulation [28, 30-31], Noncommutative Geometry [32], Fractional Field Theory [11-13], and multifractal spacetime models [33]. In such frameworks, the relevant parameter controlling diffusion, propagation, and phase space volume is an effective *spectral dimension*, which is part of a full *hierarchy of dimensions* that run with the observational scale or curvature.

On the experimental side, tensions with the standard ΛCDM paradigm raise the possibility that the underlying gravitational and kinetic equations may themselves undergo a mild but systematic deformation at late cosmic times.

Importantly, these deviations do not require a breakdown of locality or Lorentz symmetry at laboratory scales; instead, they may reflect a change in the effective dimensionality governing the low-energy regime of long-wavelength phenomena [35-37].

In this work, we introduce the definitions of continuously evolving dimensions expected to play the leading role in the *complex dynamics* of interacting fields, somewhere above the Standard Model scale of particle physics (SM). Interacting fields propagating on the relic of a primordial multifractal spacetime naturally induce running effective dimensions through Renormalization Group flow and anomalous diffusion. The resulting dimensional flow *is not arbitrary*: it is constrained by consistency with conservation laws, causality, and the recovery of standard four-dimensional physics in appropriate limits.

Evolving dimensions carry *long range spatial* and *memory effects* across vast regions of spacetime and cosmic epochs. In particular, continuous dimensions determine the large-scale structure formation, weak lensing, evolution of matter perturbations, the flavor content of particle physics and the rate of cosmic expansion described by the Hubble parameter [36]. Unforeseen explanations of the Sigma 8 anomaly and Hubble tension emerge: late-time cosmology behaves as if space becomes *geometrically thinner* (Hausdorff dimension $d_H < 3$) and *less connected dynamically* (lower spectral dimension d_s), causing the Universe to expand faster while structures grow at a slower rate [36-37].

We caution upfront that this tutorial is limited in scope and extent. Interested readers are encouraged to further explore, refute or refine the body of ideas included here and in the list of references.

1. Observation scale

It is defined as either one of the following:

- In QFT, as the Renormalization Group energy scale μ. Some textbooks refer to μ as "sliding scale" or "subtraction point".

- In Cosmology, as redshift (z) or the scale of density perturbations in momentum space (k).

2. Dimensions

2.1 Euclidean Dimension d_E

The Euclidean dimension is the integer dimension of the *embedding space*:

$$d_E = \dim(\mathbb{R}^{d_E})$$

Familiar examples include line: $d_E = 1$, plane: $d_E = 2$, physical space: $d_E = 3$. Euclidean dimension is a *fixed number* which determines the tensor structure, Lorentz group and locality in both QFT and relativistic dynamics.

2.2 Topological Dimension d_T

It represents the minimal number of coordinates needed to specify a point locally. Examples include Cantor set: $d_T = 0$, curve (even fractal): $d_T = 1$, surface: $d_T = 2$. Topological dimension is an integer, invariant under homeomorphisms and insensitive to dynamics or metric.

2.3 Hausdorff Dimension d_H

For any set X, there exists a critical value d_H such that the Hausdorff measure of the set is

$$\mathcal{H}^s(X) = \begin{cases} \infty, & s < d_H \\ 0, & s > d_H \end{cases}$$

The operational definition of the Hausdorff dimension reads

$$d_H = \lim_{\epsilon \to 0} \frac{\log N(\epsilon)}{\log(1/\epsilon)}$$

where $N(\epsilon)$ is the minimal number of balls of radius ϵ covering the set X. Here are few traditional examples of Hausdorff dimensions:

- **Cantor set**: $d_H = \log 2/\log 3$
- **Sierpiński gasket**: $d_H = \log 3/\log 2$
- **Menger sponge**: $d_H = \log 20/\log 3$

The Hausdorff dimension is a non-integer number dependent on metric and describing the scaling of measures, where the latter are extensions of the familiar concepts of length, area and volume. It is not a dynamic degree of freedom in conventional QFT, but it is an essential parameter of Fractional Field Theory [12], where it turns into a *scale dependent* parameter (as in $d_H \square d_H(\mu)$).

In the context of Cosmology, the relation between the topological, Euclidean and the Hausforff dimensions is given by

$$\boxed{d_T(z) \leq d_H(z) \leq d_E(z)}$$

We close this paragraph with a definition of *multifractals*. A multifractal is a complex system or object that cannot be described by a single fractal dimension because its scaling properties vary from point to point. While a standard "mono-fractal" looks essentially the same at every scale and is defined by one single number, a multifractal is like an *interwoven collection of fractals*, each with its own unique dimension. As a result, a multifractal set is characterized by an ensemble of Hausdorff dimensions α whose distribution is called a *singularity spectrum* and denoted by $f(\alpha)$.

2.4 Spectral Dimension d_s

Spectral dimension is defined through a *diffusion (or heat kernel) process*:

$$P(\sigma) \sim \sigma^{-d_s/2}$$

where $P(\sigma)$ is the return probability at diffusion time σ. Equivalently,

$$d_s = -2\frac{d\ln P(\sigma)}{d\ln \sigma}$$

Spectral dimension controls propagation and dispersion in fractal backgrounds and may be interpreted as the dimension "seen" by a generic random walker in those backgrounds. It represents a dynamic observable that is, in general, different from the Hausdorff dimension ($d_s \neq d_H$) and it is also scale dependent.

Spectral dimension is absent in conventional QFT, but it is frequently used in QG programs (such as Causal Dynamic Triangulation or Asymptotic Safety). The key property of spectral dimension is that it directly modifies the form of propagators in fractal backgrounds.

2.5 Generalized (Rényi) Dimensions D_q

The generalized dimension (also known as *Rényi entropy*) is based on a probability measure $\mu_i(\epsilon)$ defined in boxes of size ϵ:

$$D_q = \frac{1}{q-1} \lim_{\epsilon \to 0} \frac{\log \sum_i \mu_i^q(\epsilon)}{\log \epsilon}$$

Special cases of interest are:

$q = 0 \Rightarrow$ Capacity (or box-counting) dimension,

$q = 1 \Rightarrow$ Information dimension,

$q = 2 \Rightarrow$ Correlation dimension

The generalized dimension D_q has the following attributes:

- Encodes multifractality,
- Different moments of D_q probe *dense* vs *sparse* regions,
- Full spectrum of D_q characterizes fluctuations in the measure.

D_q has no analogue in conventional QFT, but it is an essential tool in the characterization of *non-Gaussian* and *non-ergodic* systems. It also emerges in Self-Organized Criticality (SOC), fluid turbulence and multifractal spacetime.

It is instructive to note that, building on the connection between Rényi entropy and multifractals, [38] points out that the generalized dimension of geodesic trajectories in GR reproduces the four-dimensionality of classical spacetime ($d_E = 4$).

3. Concluding remarks

While in conventional field theory dimension is single integer playing the role of a fixed *background parameter*, in fractal and multifractal frameworks dimension is a scale-dependent and non-integer observable having multiple components,

$$\boxed{d \,\square\, \{d_T, d_H, d_s, d_E, D_q\}}$$

A synthesis of evolving dimensions along with their properties is captured in the table below.

Dimension	Encodes	Dynamical?	Exists in standard QFT?
d_E	Embedding space	No	✓
d_T	Topology	No	✓
d_H	Measure scaling	No	✗
d_s	Propagation / diffusion	✓	⚠ (only in QG)
D_q	Measure fluctuations	✓	✗

The current hypothesis is that all evolving dimensions emerge somewhere above the SM scale and exhibit long-range spatial and memory effects across vast regions of spacetime and cosmological epochs. A crucial observation is that the scale dependence of all these dimensions is neither *monotonic* nor *deterministic.* That is to say that dimensions are allowed to flow with the observation scale in some unpredictable way. Stated differently, the topological, Haussdorf, spectral and generalized dimensions *coexist* with an embedding spacetime whose Euclidean dimension flows at a lower rate than the rates attributed to the Hausdorff and spectral dimensions. This condition amounts to,

$$\boxed{\frac{d}{dz}(d_E) < \frac{d}{dz}(d_H)}$$

$$\boxed{\frac{d}{dz}(d_E) < \frac{d}{dz}(d_s)}$$

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