The Equivalence Principle, Measurability, Gravity at Low and High Energies, and Spacetime Foam

Alexander Shalyt-Margolin [1]

Research Institute for Nuclear Problems,Belarusian State University, 11 Bobruiskaya str., Minsk 220040, Belarus

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Abstract

The present paper is devoted to further studies of quantum theory and gravity in terms of the **measurability** notion on the basis of the previous author's works. In the beginning the applicability limit of Einstein's Equivalence Principle (EP) is considered. It is noted that in this case a natural upper limit is associated with the Planck scales (or same Planck energies) because, due to the modern knowledge about these scales, the well-known spacetime geometry should be replaced by the spacetime foam on account of great fluctuations of the metric. It is shown that a real applicability limit of EP may be considerably lower than the Plank scales. Without due regard to this fact, one can obtain senseless results from estimation of the relevant quantities within the scope of the conventional Quantum Field Theory. In the second part of the paper the earlier obtained results are applied to study the spacetime foam in the case of a **measurable** consideration. It is demonstrated that **measurability** allows for a new approach to investigation of quantum fluctuations of the metric, especially at high (Planck) energies, i.e. in the quantum-gravitational region, leading to new approaches to studies of the spacetime foam.

1 Introduction

This paper is a continuation of previous works written by the author [1]–[6] with the use of the **measurability** concept. In Section 2 the author

[1]E-mail: a.shalyt@mail.ru; alexm@hep.by

considers the applicability limit of Einstein's Equivalence Principle (EP). It is clear that in this case the Plank scales present a natural upper limit because it is known: the Plank energies $E \approx E_p$ are associated with great spacetime quantum fluctuations, and the initial spacetime geometry determined by the particular metric $g_{\mu\nu}$ is replaced by the spacetime foam, the properties of which are still inadequately known. It is shown that a real applicability limit of EP may be lying considerably lower than the Plank scales. Disregarding this, one can obtain senseless results during estimation of the relevant quantities within the scope of the conventional Quantum Field Theory (QFT), in particular, of the cosmological term λ in General Relativity (GR).
The author gives some arguments in support of the statement that all the processes studied in QFT should be considered separately in two different energy ranges

$$\begin{aligned} E &\ll E_p \\ &and \\ E &\approx E_p. \end{aligned} \tag{1}$$

Sections 3,4 present the results earlier obtained in [1]–[6] but now the author lifts some initial restrictions (limiting conditions) imposed in the above-mentioned papers. Specifically, it is not supposed from the start that a theory involves some minimal length l_{min}. Instead, the **primary** length ℓ (formula (10) is introduced in Section 3).In the beginning of Section 3 the adequate argumentation of such replacement is given. It is noted that the whole formalism developed in [1]–[6] on condition that $\ell = l_{min}$ is a minimal length is fully valid for the case when ℓ is the *primary* length.
Finally, in Section 5 the obtained results are applied to study the spacetime foam in a **measurable** consideration. It is demonstrated that **measurability** allows for studies of quantum fluctuations of metrics on the basis of a new approach. In the case of high energies $E \approx E_p$, i.e.in the quantum-gravitational region, this approach leads to new investigation methods for the spacetime foam.

2 Equivalence Principle and QFT Ultraviolet Behavior in Canonical Theory

The Einstein's Equivalence Principle (EP) is a basic principle not only in the General Relativity (GR) [7]–[9], but also in the fundamental physics as a whole. In the standard formulation it is as follows: ([9],p.68):
«*at every space-time point in an arbitrary gravitational field it is possible to choose* **a locally inertial coordinate system** *such that, within a sufficiently small region of the point in question, the laws of nature take the same form as in accelerated Cartesian coordinate systems in the absence of gravitation*».
Then in ([9],p.68) «...There is also a question, how small is **"sufficiently small"**. Roughly speaking, we mean that the region must be small enough so that gravitational field in sensible constant throughout it...».
However, the statement **"sufficiently small"** is associated with another problem. Indeed, let $\overline{x}$ be a certain point of the space-time manifold $\mathcal{M}$ (i.e. $\overline{x} \in \mathcal{M}$) with the geometry given by the metric $g_{\mu\nu}(\overline{x})$. Next, in accordance with EP, there is some **sufficiently small** region $\mathcal{V}$ of the point $\overline{x}$ so that, within $\mathcal{V}$, we have

$$g_{\mu\nu}(\overline{x}) \equiv \eta_{\mu\nu}(\overline{x}), \tag{2}$$

where $\eta_{\mu\nu}(\overline{x})$ is Minkowskian metric.
In essence, **sufficiently small** $\mathcal{V}_r$ means that the region $\mathcal{V}^{'}$, for which $\overline{x} \in \mathcal{V}^{'}_{r'} \subset \mathcal{V}_r$ with $r' < r$ (here r, r' are characteristic spatial sizes of $\mathcal{V}_r$ and $\mathcal{V}^{\prime}_r$ correspondingly), satisfies (2) as well. In this way we can construct the sequence

$$\begin{aligned} ... \subset \mathcal{V}^{''}_{r''} \subset \mathcal{V}^{'}_{r'} \subset \mathcal{V}_r, \\ ... < r^{''} < r' < r \end{aligned} \tag{3}$$

The problem arises, is there any lower limit for the sequence in formula (3)? The answer is positive. Currently, there is no doubt that at very high energies (on the order of Planck's energies $E \approx E_p$), i.e. on Planck's scales, $l \approx l_p$ quantum fluctuations of any metric $g_{\mu\nu}(\overline{x})$ are so high that in this case the geometry determined by $g_{\mu\nu}(\overline{x})$ is replaced by the "geometry" following from **space-time foam** that is defined by great quantum fluctuations of $g_{\mu\nu}(\overline{x})$,i.e. by the characteristic spatial sizes of the quantum-gravitational region (for example, [10]–[15]). The above-mentioned geometry is drastically differing from the locally smooth geometry of continuous space-time and EP in it is no longer valid [16]–[24].

From this it follows that the region $\mathcal{V}_{\overline{r},\overline{t}}$ with the characteristic spatial size $\overline{r} \approx l_p$ (and hence with the temporal size $\overline{t} \approx t_p$) is the lower (approximate) limit for the sequence in (3).

It is difficult to find the exact lower limit for the sequence in formula (3)–it seems to be dependent on the processes under study. Specifically, when the involved particles are considered to be point, their dimensions may be neglected in a definition of the EP applicability limit. When the characteristic spatial dimension of a particle is $\mathbf{r}$, the lower limit of the sequence from formula (3) seems to be given by the region $\mathcal{V}_{\mathbf{r}'}$ containing the above-mentioned particle with the characteristic dimensions $\mathbf{r}' > \mathbf{r}$, i.e. the space EP applicability limit should always be greater than dimensions of the particles considered in this region. By the present time, it is known that spatial dimensions of gauge bosons, quarks, and leptons within the limiting accuracy of the conducted measurements $< 10^{-18}m$. Because of this, the condition $\mathbf{r}' \geq 10^{-18}m$ must be fulfilled. In addition, the radius of interaction of particles $\mathbf{r}_{int}$ must be taken into account in quantum theory. And this fact also imposes a restriction on considering concrete processes in quantum theory. However, the interactions radii of all known processes lie in the energy scales $E \ll E_p$.

Therefore, it is assumed that the Equivalence Principle is valid for the locally smooth space-time and this suggests that all the energies E of the particles in the most general form meet the necessary condition

$$E \ll E_p. \tag{4}$$

Then, if not stipulated otherwise, we can assume that the condition (4) is valid.

The canonical quantum field theory (QFT) [25]– [27] is a local theory considered in continuous space-time with a plane geometry, i.e with the Minkowskian metric $\eta_{\mu\nu}(\overline{x})$. In addition, it is assumed that all objects in QFT are point-like. However, as noted above, this assumption will be true to a certain *limit: the assumptions that (a) even local space-time geometry is plane and (b) all objects in QFT are point-like have natural applicability boundaries directly specifying the EP applicability boundary.*

In reality, any interaction introduces some disturbances, introducing an additional local (little) curvature into the initially flat Minkowskian space $\mathcal{M}$. Then the metric $\eta_{\mu\nu}(\overline{x})$ is replaced by the metric $\eta_{\mu\nu}(\overline{x})+o_{\mu\nu}(\overline{x})$, where the increment $o_{\mu\nu}(\overline{x})$ is small. But, when it is assumed that EP is valid, the increment $o_{\mu\nu}(\overline{x})$ in the local theory has no important role and, in a fairly small neighborhood of the point $\overline{x}$, formula (2) is valid.

Within the scope of the canonical QFT, the process of passage to more higher energies without a change in the local curvature has no limits [25]–[27], just this fact is the reason for ultraviolet divergences in QFT. But as follows from the previous section, this is not the case. Actually, on passage to the Planck energies $E \approx E_p$ (Planck scales $l \approx l_p$), the space in the Planck neighborhood $\mathcal{V}_{\overline{r},\bar{t}}$ of the point $\overline{x}$ one cannot consider flat even locally and in this case (as noted above) EP is not valid.

Then we introduce the following assumption:

Assumption 2.1

In the canonical QFT in calculations of the quantities it is wrong to sum (or same consider within a single sum) the contributions corresponding to space-time manifolds with locally nonzero or zero curvatures since these contributions are associated with different processes: (1) with the existence of a gravitational field that, in principle, can hardly be excluded; (2) in the absence of a gravitational field.

From the start, we can isolate the case when EP is valid (at sufficiently low energies, specifically satisfying the condition (4)) from the cases when EP becomes invalid (for example, Planck energies $E \approx E_p$).

Let us consider a widely known example when **Assumption 2.1** is not fulfilled leading to the senseless results.

In his well-known lectures [28] at the Cornell University Steven Weinberg considered an example of calculating, within the scope of QFT, the expected value for the vacuum energy density $<\rho_{vac}>$ that is proportional to the cosmological term λ. To this end, zero-point energies of all normal modes of some field with the mass m are summed up to the wave number cutoff $\Lambda \gg m$ for the selected normalization $\hbar = c = 1$ (formula (3.5) in [28]):

$$< \rho_{vac} > \sim \int_0^{\Lambda} \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m^2} \simeq \frac{\Lambda^4}{16\pi^2}. \qquad (5)$$

Assuming, similar to [28], that GR is valid at all the energy scales up to the Planck's, we have the cutoff $\Lambda \simeq (8\pi G)^{-1/2}$ and hence (formula (3.6) in [28]) leads to the following result:

$$< \rho_{vac} > \propto 2 \cdot 10^{71} GeV^4, \qquad (6)$$

that by 10^{118} orders of magnitude differs from the well-known experimental value for the vacuum energy density

$$< \rho_{vac,exp} > \preceq 10^{-29} \mathrm{g}/cm^3 \propto 10^{-47} GeV^4. \qquad (7)$$

Here G is a gravitational constant.
It is clear that in this case **Assumption 2.1** fails as Planck's scales and those close to them at lower energies are included into consideration. By the author's opinion, this is impermissible because for Planck's scales the quantum rather than classical gravity is true and the space even in a small neighborhood of the point is hardly flat. But in formula (5) for the cutoff $\Lambda \simeq (8\pi G)^{-1/2}$ this fact is not included because all calculations in the canonical QFT [27] are valid for the locally flat space and hence (5) in this case leads to senseless results.
Of particular interest is the **inverse problem**: if the experimental value of the vacuum energy density $< \rho_{vac,exp} >$ is known from (7), substituting it into formula (5), we can estimate Λ_{exp} at the upper limit of integration by the above formula

$$< \rho_{vac,exp} > \sim \int_0^{\Lambda_{exp}} \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m^2} \simeq 10^{-47} GeV^4. \qquad (8)$$

Note that Λ_{exp} may be found in other way. Denoting by Λ_{UV} the quantity $\simeq (8\pi G)^{-1/2}$ from formula (5), corresponding to the cutoff at Planck's scale $\approx 1,6 \cdot 10^{-33} cm$ that is taken as the ultraviolet cutoff, denoting the required quantity $<\rho_{vac}>$ by $<\rho_{vac,UV}>$, by Λ_{IF} denoting the quantity from the same formula, that corresponds to the cutoff at the scale of a visible part of the Universe $\approx 10^{28}cm$, and the corresponding quantity $<\rho_{vac}>$ denoting as $<\rho_{vac,IF}>$ (infrared limit), in accordance with [29],[30], we obtain

$$<\rho_{vac,exp}>=\sqrt{<\rho_{vac,UV}><\rho_{vac,IF}>}. \quad (9)$$

Obviously, Λ_{exp} derived from formulae (8), (9) satisfies the condition (4) and in this case **Assumption 2.1** is fulfilled.

Remark 2.2

In this work we, in fact, consider two two limiting case:
a)low energies $E \ll E_p$ and
b)very high (essentially maximal) energies $E \approx E_p$.
Then it should be noted that, as all the experimentally involved energies E are low, they satisfy condition a). Specifically, for LHC maximal energies are $\approx 10TeV = 10^4GeV$, that is by 15 orders of magnitude lower than the Planck energy $\approx 10^{19}GeV$.
Moreover, the characteristic energy scales of all fundamental interactions also satisfy condition a). Indeed, in the case of strong interactions this scale is $\Lambda_{QCD} \sim 200MeV$; for electroweak interactions this scale is determined by the vacuum average of a Higgs boson and equals $\upsilon \approx 246GeV$; finally, the scale of the (Grand Unification Theory (GUT)) M_{GUT} lies in the range of $\sim 10^{14}GeV--10^{16}GeV$. It is obvious that all the above figures satisfy condition a).
Thus, only the expected characteristic energy scale of quantum gravity satisfies condition b).

From **Remark 2.2** it directly follows that even very high energies arising on unification of all the interaction types $M_{GUT} \approx 10^{14}GeV- \sim 10^{16}GeV$,(except of gravitational),satisfy the condition (4).
At the same time, it is clear that the requirement of the Lorentz-invariant QFT, due to the action of Lorentz boost (or same hyperbolic rotations) (formula (3) in [8]), results in however high momenta and energies. But it has been demonstrated that unlimited growth of the momenta and energies is impossible because in this case we fall within the energy region, where

the conventional quantum field theory [25]– [27] is invalid.
Note that at the present time there are experimental indications that Lorentz-invariance is violated in QFT on passage to higher energies (for example, [31]).Proceeding from the above, the requirement for Lorentz-invariance and EP is possible only within the scope of the condition (4). Besides, one should note important recent works associated with EP applicability boundaries and violation in nuclei and atoms at low energies (for example [32]). By the present time, numerous papers have been devoted to the applicability limits of EP to the processes of different nature in high energy physics within the scope of the condition (4) (for example [33]–[38]). Of course, the list of these papers is not in the least complete.

3 Preliminary Information about the Measurability Concept

In this Section we briefly consider some of the results from [1]–[6] which are essential for subsequent studies. Without detriment to further consideration, in the initial definitions we lift some unnecessary restrictions and make important specifications.
Presently, many researchers are of the opinion that at very high energies (Plank's or trans-Planck's) the ultraviolet cutoff exists that is determined by some maximal momentum.
Therefore, it is further assumed that there is a maximal bound for the measurement momenta $p = p_{max}$ represented as follows:

$$p_{max} \doteq p_{\ell} = \hbar/\ell, \qquad (10)$$

where ℓ is some small length and $\tau = \ell/c$ is the corresponding time. Let us call ℓ the *primary* length and τ the *primary* time.
Without loss of generality, we can consider ℓ and τ at Plank's level, i.e. $\ell \propto l_p, \tau = \kappa t_p$, where the numerical constant κ is on the order of 1. Consequently, we have $E_{\ell} \propto E_p$ with the corresponding proportionality factor, where $E_{\ell} \doteq p_{\ell} c$.

Explanation. In the theory under study it is not assumed from the start that there exists some minimal length l_{min} and that ℓ is such. In fact, the minimal length is defined with the use of Heisenberg's Uncertainty Principle (HUP) $\Delta x \cdot \Delta p \geq \frac{1}{2}\hbar$ or of its generalization to high (Planck) energies – Generalized Uncertainty Principle (GUP) [40]–[48], for example, of the form [40]

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' l_p^2 \frac{\Delta p}{\hbar}, \tag{11}$$

where α' is a constant on the order of 1. Evidently this formula (11) initially leads to the minimal length $\widetilde{\ell}$ on the order of the Planck length $\widetilde{\ell} \doteq 2\sqrt{\alpha'} l_p$. Besides, other forms of GUP [48] also lead to the minimal length. Thus, we should note that in all the works l_{min} is actually (but not explicitly) introduced on the basis of some measuring procedure (different forms of the Generalized Uncertainty Principle (GUP)). In any form GUP in turn is a high-energy generalization of HUP. But in the original proof of HUP a planar geometry of the initial space-time was actively used [49]. Extension of this principle to other pairs of conjugate variables is also valid only for quantum mechanics in the planar geometry space [50]. As HUP is a local principle, at low energies in the curved space-time, by virtue of Einstein's Equivalence Principle, we can consider that in a fairly small neighborhood of any point the geometry is planar an hence HUP is valid too. But all the results obtained point to the fact that l_{min} should be at a level of l_p,i.e. $l_{min} \propto l_p$, or even should be smaller. As noted in the previous section, at the Planck scales Einstein's Equivalence Principle is obviously inapplicable, and there is no way to use the measuring procedure ignoring the space geometry at these scales. Meantime, none of the GUP forms [48] makes an effort to include it and is hardly completely correct. Moreover, there are some serious arguments against GUP as demonstrated in Section IX of the review paper [48]. The foregoing considerations support argumentation against the introduction of l_{min} from the start.
Because of this, in the present work the validity of this principle is not implied from the start too. GUP is given merely as an example. As p_{max} (10) is taken at Planck's level, it is clear that HUP is inapplicable. Taking this into consideration, the existence of a certain minimal length $\widetilde{\ell}$ is not mandatory. So, we start from the *primary* length ℓ and the *primary* time τ. The whole formalism, developed in [1]–[6] on condition that ℓ is the minimal

length, is valid for the case when ℓ is the *primary* length but now we can lift the formal requirement for involvement of l_{min} in the theory from the start.

There is one more barrier for the use of l_{min} in the theory as indicated in [47] and other works (for example, [48]). In the above-mentioned papers, it has been noted that there is a nonzero minimal uncertainty in position, i.e. l_{min} implies that there is no physical state which is a position eigenstate since an eigenstate would, of course, have zero uncertainty in position. So, in this case in a quantum theory we have the momentum representation rather than the position representation, and the quantum theory becomes very depleted.

The question arises whether the introduction of p_{max} is naturally associated with the involvement of a minimal length. But this is the case only when at the energies E_{max} corresponding to p_{max} we have the substantiated measuring procedure. Unfortunately, this is not the case.

Note that in the canonical QFT in continuous space-time (i.e. without l_{min}) [25] –[27] measurements of the contributions in the loop amplitudes involve the standard cut-off procedure for some large (maximal) momentum $p_{cut} \doteq p_{max}$. Then it is demonstrated that the theory at low energies $p \ll p_{cut}$ is in fact independent of the selection of $p_{cut} \doteq p_{max}$. Of course, the theory still remains to be continuous [25] –[27]. In this case we make another step forward, relating the corresponding length $\ell = \hbar/p_{max}$ to p_{max} and constructing on its basis a low-energy theory very close to the initial continuous theory. Now we have the naturally derived parameter ℓ for the construction of a high-energy deformation of this theory at the energies $E \approx E_{max}$ within the scope of determining the physical theory deformation [39]. So, we start from the *primary* length ℓ and the *primary* time τ. The whole formalism, developed in [1]–[6] on condition that ℓ is the minimal length, is valid for the case when ℓ is the *primary* length but now we can lift the formal requirement for involvement of l_{min} in the theory from the start.

Evidently that for the correctness of the theory it is necessary that at low energies $E \ll E_p$ all results should not depend on the choice p_{max}.

3.1. The **primarily measurable** space-time quantities (variations) are understood as the quantities Δx_i and Δt taking the form

$$\Delta x_i = N_{\Delta x_i}\ell, \Delta t = N_{\Delta t}\tau, \tag{12}$$

where $N_{\Delta x_i}, N_{\Delta t}$ are integer numbers. Further in the text we use both $N_{\Delta x_i}, N_{\Delta t}$ and the equivalent N_{x_i}, N_t.

3.2. Similarly, the **primarily measurable** momenta are considered as a subset of the momenta characterized by the property

$$p_{x_i} \doteq p_{N_{x_i}} = \frac{\hbar}{N_{x_i}\ell}, \tag{13}$$

where N_{x_i} is a nonzero integer number and p_{x_i} is the momentum corresponding to the coordinate x_i.

3.3. Finally, let us define any physical quantity as **primarily or elementary measurable** when its value is consistent with point **3.1**,**3.2** and formulae (12), (13).
Then we consider formula (13) with the addition of the momenta $p_{x_0} \doteq p_{N_0} = \frac{\hbar}{N_{x_0}\ell}$, where N_{x_0} is an integer number corresponding to the time coordinate ($N_{\Delta t}$ in formula (12)).
For convenience, we denote **Primarily Measurable Quantities** satisfying **3.1**–**3.3** in the abbreviated form as **PMQ**. Also, for the **Primarily Measurable Momenta** we use the abbreviation **PMM.**

First, we consider the case of **Low Energies**, i.e. $E \ll E_\ell$ (same $E \ll E_p$. It is obvious that all the nonzero integer numbers N_{x_i}, N_t (or same $N_{x_\mu}; \mu = 0, ..., 3$) from formulae (12),(13) should satisfy the condition $|N_{x_\mu}| \gg 1$. It is clear that all the momenta p_i at **low energies** $E \ll E_p$ meet the condition $p_i = \hbar/(N_i\ell)$, where $|N_i| \gg 1$ but is not necessarily an integer. With regard for smallness of ℓ and for the condition $|N_i| \gg 1$, we can easily show that the difference $1/(N_i\ell) - 1/([N_i]\ell), (\hbar/(N_i\ell) - \hbar/([N_i]\ell))$ is negligible and in this way all momenta in the region of low energies $E \ll E_p$ may be taken as **PMM** with a high accuracy.
It is obviously that the case of **Low Energies** in this section is coincident

with the "low energies" condition from **Remark 2.2.**
It is assumed that a theory we are trying to resolve is a deformation of the initial continuous theory.

Remark 3.0
The deformation is understood as an extension of a particular theory by inclusion of one or several additional parameters in such a way that the initial theory appears in the limiting transition [39].

Then it should be noted that **PMQ** is inadequate for studies of the physical processes. In fact, among **PMQ**, we have no quantities capable to give the infinitesimal quantities $dx_\mu, \mu = 0, ..., 3$ in the limiting transition in a continuous theory.
Therefore, it is reasonable to use notion of **Generalized Measurability**
We define any physical quantity at all energy scales as **generalized measurable** *or, for simplicity,* **measurable** *if any of its values may be obtained in terms of* **PMQ** *specified by points* **3.1**–**3.3**.
The **generalized measurable** quantities will be denoted as **GMQ**.
Note that the space-time quantities

$$\frac{\tau}{N_t} = p_{N_tc}\frac{\ell^2}{c\hbar}$$
$$\frac{\ell}{N_i} = p_{N_i}\frac{\ell^2}{\hbar}, 1 = 1, ..., 3, \tag{14}$$

where p_{N_i}, p_{N_tc} are **Primarily Measurable** momenta, up to the fundamental constants, are coincident with p_{N_i}, p_{N_tc} and they may be involved at any stage of the calculations but, evidently, they are not **PMQ**, but they are **GMQ**.
So, in the proposed paradigm at low energies $E \ll E_p$ a set of the **PMM** is discrete, and in every measurement of $\mu = 0, ..., 3$ there is the discrete subset $\mathbf{P}_{\mathbf{x}_\mu} \subset$ **PMM**:

$$\mathbf{P}_{\mathbf{x}_\mu} \doteq \{..., p_{N_{x_\mu}-1}, p_{N_{x_\mu}}, p_{N_{x_\mu}+1}, ...\}. \tag{15}$$

In this case, as compared to the canonical quantum theory, in continuous space-time we have the following substitution:

$$\mathbf{\Delta p}_\mu \mapsto dp_\mu, \mathbf{\Delta p}_{\mathbf{N_{x_\mu}}} = \mathbf{p}_{\mathbf{N_{x_\mu}}} - \mathbf{p}_{\mathbf{N_{x_\mu}+1}} = \mathbf{p}_{\mathbf{N_{x_\mu}(N_{x_\mu}+1)}};$$
$$\frac{\mathbf{\Delta}}{\mathbf{\Delta p}_\mu} \mapsto \frac{\partial}{\partial \mathbf{p}_\mu}; \frac{\mathbf{\Delta F}(\mathbf{p}_{\mathbf{N_{x_\mu}}})}{\mathbf{\Delta p}_\mu} = \frac{\mathbf{F}(\mathbf{p}_{\mathbf{N_{x_\mu}}}) - \mathbf{F}(\mathbf{p}_{\mathbf{N_{x_\mu}+1}})}{\mathbf{p}_{\mathbf{N_{x_\mu}}} - \mathbf{p}_{\mathbf{N_{x_\mu}+1}}} = \frac{\mathbf{F}(\mathbf{p}_{\mathbf{N_{x_\mu}}}) - \mathbf{F}(\mathbf{p}_{\mathbf{N_{x_\mu}+1}})}{\mathbf{p}_{\mathbf{N_{x_\mu}(N_{x_\mu}+1)}}}. \quad (16)$$

And

$$\frac{\ell}{\mathbf{N_{x_\mu}}} \mapsto dx_\mu;$$
$$\frac{\mathbf{\Delta}}{\mathbf{\Delta_{N_{x_\mu}}}} \mapsto \frac{\partial}{\partial x_\mu}, \frac{\mathbf{\Delta F}(\mathbf{x}_\mu)}{\mathbf{\Delta_{N_{x_\mu}}}} = \frac{\mathbf{F}(\mathbf{x}_\mu + \ell/\mathbf{N_{x_\mu}}) - \mathbf{F}(\mathbf{x}_\mu)}{\ell/\mathbf{N_{x_\mu}}}. \quad (17)$$

It is clear that for sufficiently high integer values of $|N_{x_\mu}|$, formulae (16),(17) reproduce a continuous paradigm in the momentum space to any preassigned accuracy. However, at low energies $E \ll E_\ell$ a set of **PMM** clearly is not a space. Considering this, the formulae at low energies offer **the Correspondence to Continuous Theory (CCT).**

It is important to make the following remarks in medias res:

Remark 3.1.
In this way any point $\{x_\mu\} \in \mathcal{M} \subset \mathbf{R^4}$ and any set of integer numbers high in absolute values $\{N_{x_\mu}\}$ are correlated with a system of the neighborhoods for this point $(x_\mu \pm \ell/N_{x_\mu})$. It is clear that, with an increase in $|N_{x_\mu}|$, the indicated system converges to the point $\{x_\mu\}$. In this case all the ingredients of the initial (continuous) theory the partial derivatives including are replaced by the corresponding finite differences.

Remark 3.2.
It is further assumed that at low energies $E \ll E_\ell$ (same $E \ll E_p$) all the *observable quantities* **are PMQ**.
Because of this, values of the length ℓ/N_i and of the time ℓ/N_t from formula (14) could not appear in expressions for *observable quantities*, being involved only in intermediate calculations, especially at the summation for

replacement of the infinitesimal quantities $dt, dx_i; i = 1, 2, 3$ on passage from a continuous theory to its measurable variant.

Further it is assumed that at **High Energies**, $E \approx E_p$, **PMQ** are *inadequate* for studies of the theory at these energies. The assumption follows quite naturally. For example, if GUP (11) is valid and if $\ell = \widetilde{\ell}$, then at high energies formula (11) creates the momenta $\Delta p(N_{\Delta x}, GUP)$ which are not **primarily measurable** [4] –[6]:

$$\Delta p \doteq \Delta p(N_{\Delta x}, GUP) = \frac{\hbar}{1/2(N_{\Delta x} + \sqrt{N_{\Delta x}^2 - 1})\ell}. \tag{18}$$

Naturally, formula (18) represents only a particular case of variations in the **generalized measurable** momenta at high energies $E \approx E_p$. Suppose, we know that in the general case at high energies $E \approx E_p$ minimal variations of momenta are given by a set of the **generalized measurable** quantities $p_{N_{x_\mu}}$, where we have the integer numbers $N_{x_\mu}, |N_{x_\mu}| \approx 1$. Then it is reasonable to assume that minimal variations of "coordinates" at high energies are given by the following formula:

$$l_H(p_{N_{x_\mu}}) \doteq \frac{\ell^2}{\hbar} p_{N_{x_\mu}}, \tag{19}$$

where $p_{N_{x_\mu}}$ are the above-mentioned **generalized measurable** momenta at high energies.

The main target of the author is to form a quantum theory and gravity only in terms of **generalized measurable** quantities (or of **PMQ**).
In conclusion of this Section we summarize the principal results.

Remark 3.3
When at low energies $E \ll E_p$ we lift restrictions on integrality of N_{x_μ}, from formulae (16),(17) it directly follows that in this case we have a continuous analog of the well-known theory with the only difference: all the used small quantities become dependent on the existent energies and we can correlate

them. In this way formula (17) may be written as

$$dx_{\mu} \leftrightarrow \frac{\ell}{\mathbf{N_{x_{\mu}}}} \rightarrow \frac{\ell}{[\mathbf{N_{x_{\mu}}}]},$$
$$\frac{\partial}{\partial x_{\mu}} \leftrightarrow \frac{\mathbf{\Delta}}{\mathbf{\Delta_{N_{x_{\mu}}}}} \rightarrow \frac{\mathbf{\Delta}}{\mathbf{\Delta_{[N_{x_{\mu}}]}}} \tag{20}$$

where $|N_{x_{\mu}}| \gg 1$ is a sufficiently large number that varies continuously. It is clear that in formula (20) the first arrow corresponds to the continuous theory with a specific selection of values of the infinitesimal quantities dx_{μ}. As noted above, the difference $\ell/N_{x_{\mu}} - \ell/[N_{x_{\mu}}]$ is negligible and hence the second arrow corresponds to passage from the initial continuous theory to a similar discrete theory. Of course, formula (16) may be rewritten in the like manner. In what follows, formula (20) plays a crucial part in derivation of the results and is greatly important for their understanding.
The main target of the author is to form a quantum theory and gravity only in terms of **PMQ**.

Measurable form arbitrary metric and Minkowskian metric

According to the previous works, the **measurable** variants of quantum theory and gravity at low energies $E \ll E_p$ should be formulated in terms of the **measurable** space-time quantities $\ell/N_{\Delta x_{\mu}}$ or **primary measurable** momenta $p_{N_{\Delta x_{\mu}}}$.
Let us consider the case of the random metric $g_{\mu\nu} = g_{\mu\nu}(x)$ [7],[8], where $x \in R^4$ is some point of the four-dimensional space R^4 defined in **measurable** terms. Now, any such point $x \doteq \{x_{\chi}\} \in R^4$ and any set of integer numbers $\{N_{x_{\chi}}\}$ dependent on the point $\{x_{\chi}\}$ with the property $|N_{x_{\chi}}| \gg 1$ may be correlated to the "bundle" with the base R^4 as follows:

$$B_{N_{x_{\chi}}} \doteq \{x_{\chi}, \frac{\ell}{N_{x_{\chi}}}\} \mapsto \{x_{\chi}\}. \tag{21}$$

It is clear that $\lim\limits_{|N_{x_{\chi}}| \rightarrow \infty} B_{N_{x_{\chi}}} = R^4$.
As distinct from the normal one, the "bundle" $B_{N_{x_{\chi}}}$ is distinguished only by the fact that the mapping in formula (21) is not continuous (smooth) but

discrete in fibers, being continuous in the limit $|N_{x_\chi}| \to \infty$.
Then as a *canonically measurable prototype* of the infinitesimal space-time interval square [7],[8]

$$ds^2(x) = g_{\mu\nu}(x)dx^\mu dx^\nu \tag{22}$$

we take the expression

$$\Delta s^2_{N_{x_\chi}}(x) \doteq g_{\mu\nu}(x, N_{x_\chi})\frac{\ell^2}{N_{x_\mu}N_{x_\nu}}. \tag{23}$$

Here $g_{\mu\nu}(x, N_{x_\chi})$ – metric $g_{\mu\nu}(x)$ from formula (22) with the property that minimal **measurable** variation of metric $g_{\mu\nu}(x)$ in point x has form

$$\Delta g_{\mu\nu}(x, N_{x_\chi})_\chi = g_{\mu\nu}(x + \ell/N_{x_\chi}, N_{x_\chi}) - g_{\mu\nu}(x, N_{x_\chi}). \tag{24}$$

Let us denote by $\Delta_\chi g_{\mu\nu}(x, N_{x_\chi})$ quantity

$$\Delta_\chi g_{\mu\nu}(x, N_{x_\chi}) = \frac{\Delta g_{\mu\nu}(x, N_{x_\chi})_\chi}{\ell/N_{x_\chi}}. \tag{25}$$

It is obvious that in the case under study the quantity $\Delta g_{\mu\nu}(x, N_{x_\chi})_\chi$ is a **measurable** analog for the infinitesimal increment $dg_{\mu\nu}(x)$ of the χ-th component $(dg_{\mu\nu}(x))_\chi$ in a continuous theory, whereas the quantity $\Delta_\chi g_{\mu\nu}(x, N_{x_\chi})$ is a **measurable** analog of the partial derivative $\partial_\chi g_{\mu\nu}(x)$.
In this manner we obtain the (21)-formula induced bundle over the metric manifold $g_{\mu\nu}(x)$:

$$B_{g,N_{x_\chi}} \doteq g_{\mu\nu}(x, N_{x_\chi}) \mapsto g_{\mu\nu}(x). \tag{26}$$

Referring to formula (14), we can see that (23) may be written in terms of the **primary measurable** momenta $(p_{N_i}, p_{N_t}) \doteq p_{N_\mu}$ as follows:

$$\Delta s^2_{N_{x_\mu}}(x) = \frac{\ell^4}{\hbar^2} g_{\mu\nu}(x, N_{x_\chi}) p_{N_{x_\mu}} p_{N_{x_\nu}}. \tag{27}$$

Considering that $\ell \propto l_P$ (i.e., $\ell = \kappa l_P$), where $\kappa = const$ is on the order of 1, to within the constant $\ell^4/\hbar^2$, we have

$$\Delta s^2_{N_{x_\mu}}(x) = g_{\mu\nu}(x, N_{x_\chi}) p_{N_{x_\mu}} p_{N_{x_\nu}}. \tag{28}$$

As follows from the previous formulae, the **measurable** variant of General Relativity should be defined in the bundle $B_{g,N_{x_\chi}}$.
Let us consider any coordinate transformation $x^\mu = x^\mu(\bar{x}^\nu)$ of the space–time coordinates in continuous space-time. Then we have

$$dx^\mu = \frac{\partial x^\mu}{\partial \bar{x}^\nu}\, d\bar{x}^\nu. \tag{29}$$

As mentioned at the beginning of this section, in terms of **measurable** quantities we have the substitution

$$dx^\mu \mapsto \frac{\ell}{N_{\Delta x_\mu}}; d\bar{x}^\nu \mapsto \frac{\ell}{\bar{N}_{\Delta \bar{x}_\nu}}, \tag{30}$$

where $N_{\Delta x_\mu}, \bar{N}_{\Delta \bar{x}_\nu}$ – integers $(|N_{\Delta x_\mu}| \gg 1, |\bar{N}_{\Delta \bar{x}_\nu}| \gg 1)$ sufficiently high in absolute value, and hence in the **measurable** case (29) is replaced by

$$\frac{\ell}{N_{\Delta x_\mu}} = \Delta_{\mu\nu}(x^\mu, \bar{x}^\nu, 1/N_{\Delta x_\mu}, 1/\bar{N}_{\Delta \bar{x}_\nu})\, \frac{\ell}{\bar{N}_{\Delta \bar{x}_\nu}}. \tag{31}$$

Equivalently, in terms of the **primary measurable** momenta we have

$$p_{N_{\Delta x_\mu}} = \Delta_{\mu\nu}(x^\mu, \bar{x}^\nu, 1/N_{\Delta x_\mu}, 1/\bar{N}_{\Delta \bar{x}_\nu})\, p_{\bar{N}_{\Delta \bar{x}_\nu}}, \tag{32}$$

where $\Delta_{\mu\nu}(x^\mu, \bar{x}^\nu, 1/N_{\Delta x_\mu}, 1/\bar{N}_{\Delta \bar{x}_\nu}) \doteq \Delta_{\mu\nu}(x^\mu, \bar{x}^\nu, p_{N_{\Delta x_\mu}}, p_{\bar{N}_{\Delta \bar{x}_\nu}})$ – corresponding matrix represented in terms of **measurable** quantities.
It is clear that, in accordance with formula (14), in passage to the limit we get

$$\lim_{|N_{\Delta x_\mu}| \to \infty} \frac{\ell}{N_{\Delta x_\mu}} = dx^\mu =$$
$$= \lim_{|\bar{N}_{\Delta \bar{x}_\nu}| \to \infty} \Delta_{\mu\nu}(x^\mu, \bar{x}^\nu, 1/N_{\Delta x_\mu}, 1/\bar{N}_{\Delta \bar{x}_\nu})\, \frac{\ell}{\bar{N}_{\Delta \bar{x}_\nu}} = \frac{\partial \bar{x}^\mu}{\partial x^\nu}\, dx^\nu. \tag{33}$$

Equivalently, passage to the limit (33) may be written in terms of the **primary measurable** momenta $p_{N_{\Delta x_\mu}}, p_{\bar{N}_{\Delta \bar{x}_\nu}}$ multiplied by the constant $\ell^2/\hbar$.
How we understand formulae (30)–(33)?
The initial (continuous) coordinate transformations $x^\mu = x^\mu(\bar{x}^\nu)$ gives the

matrix $\frac{\partial x^\mu}{\partial \bar{x}^\nu}$. Then, for the integers sufficiently high in absolute value $\bar{N}_{\Delta \bar{x}_\nu}, |\bar{N}_{\Delta \bar{x}_\nu}| \gg 1$, we can derive

$$\frac{\ell}{N_{\Delta x_\mu}} = \frac{\partial x^\mu}{\partial \bar{x}^\nu} \frac{\ell}{\bar{N}_{\Delta \bar{x}_\nu}}, \tag{34}$$

where $|N_{\Delta x_\mu}| \gg 1$ but the numbers for $N_{\Delta x_\mu}$ are not necessarily integer.
Then using the formula (20) from **Remark 3.3** and substitution of $[N_{\Delta x_\mu}]$ for $N_{\Delta x_\mu}$ in the left-hand side of (34) leads to replacement of the initial matrix $\frac{\partial x^\mu}{\partial \bar{x}^\nu}$ by the matrix $\Delta_{\mu\nu}(x^\mu, \bar{x}^\nu, 1/N_{\Delta x_\mu}, 1/\bar{N}_{\Delta \bar{x}_\nu})$ represented in terms of **measurable** quantities and, finally, to the formula (31). Clearly, for sufficiently high $|N_{\Delta x_\mu}|, |\bar{N}_{\Delta \bar{x}_\nu}|$, the matrix $\Delta_{\mu\nu}(x^\mu, \bar{x}^\nu, 1/N_{\Delta x_\mu}, 1/\bar{N}_{\Delta \bar{x}_\nu})$ may be selected no matter how close to $\frac{\partial x^\mu}{\partial \bar{x}^\nu}$.
Similarly, in the **measurable** format we can get the formula

$$d\bar{x}^\mu = \frac{\partial \bar{x}^\mu}{\partial x^\nu} dx^\nu. \tag{35}$$

and correspondingly the matrix $\widetilde{\Delta_{\mu\nu}}(x^\mu, \bar{x}^\nu, 1/N_{\Delta x_\mu}, 1/\bar{N}_{\Delta \bar{x}_\nu})$ instead of the matrix $\Delta_{\mu\nu}(x^\mu, \bar{x}^\nu, 1/N_{\Delta x_\mu}, 1/\bar{N}_{\Delta \bar{x}_\nu})$.
Thus, any coordinate transformations may be represented, to however high accuracy, by the **measurable** transformation (i.e., written in terms of **measurable** quantities), where the principal components are the **measurable** quantities $\ell/N_{\Delta x_\mu}$ or the **primary measurable** momenta $p_{N_{\Delta x_\mu}}$
Analogously, a *canonically measurable prototype* of the **relativistic** infinitesimal space-time interval square

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu. \tag{36}$$

is given by

$$\Delta s^2_{N_{x_\chi}}(x) \doteq \eta_{\mu\nu}(x, N_{x_\chi}) \frac{\ell^2}{N_{x_\mu} N_{x_\nu}}, \tag{37}$$

where $\eta_{\mu\nu}$ is the Minkowskian metric

$$||\eta_{\mu\nu}|| = ||\eta^{\mu\nu}|| = Diag\,(1, -1, -1, -1)\,. \tag{38}$$

Here the integers N_{x_χ} naturally satisfy the condition $|N_{x_\chi}| \gg 1$, components of the **measurable** Minkowskian metric $||\eta_{\mu\nu}(x, N_{x_\chi})||$ are "close" to $||\eta_{\mu\nu}||$,i.e. we have

$$\lim_{(|N_{x_\chi}|)\to\infty} \eta_{\mu\nu}(x, N_{x_\chi}) = \eta_{\mu\nu}. \tag{39}$$

Without loss of generality, we can assume that $\eta_{\mu\nu}(x, N_{x_\chi}) = 0, \mu \neq \nu$.
Thus $||\eta_{\mu\nu}(x, N_{x_\chi})||$ is the diagonal matrix too and $||\eta^{\mu\nu}(x, N_{x_\chi})||$ is its inverse matrix, i.e.

$$||\eta^{\mu\nu}(x, N_{x_\chi})|| \cdot ||\eta_{\mu\nu}(x, N_{x_\chi})|| = 1 \tag{40}$$

Further we assume that the integers N_{x_χ} are sufficiently large in absolute value and, due to formula (39), the metric $||\eta_{\mu\nu}(x, N_{x_\chi})||$, to a high accuracy, is equal to $||\eta_{\mu\nu}||$; then formula (37) is as follows:

$$\Delta s^2_{N_{x_\chi}}(x) \doteq \eta_{\mu\nu} \frac{\ell^2}{N_{x_\mu} N_{x_\nu}}, \tag{41}$$

4 General Relativity in Terms of Measurable Quantities and Its High-Energy Deformations

At low energies $E \ll E_p$ for connectivity coefficients in gravity, i.e. *Christoffel symbols*, and for the fixed set $\{N\} \doteq (N_{x_\chi})$ in his paper [6] the author has derived their expressions in the **measurable** form (formula (50) in [6]):

$$\Gamma^{\alpha}_{\mu\nu}(x, \{N\}) = \frac{1}{2}\, g^{\alpha\beta}(x, \{N\})\, (\Delta_\nu g_{\beta\mu}(x, \{N\}) + \Delta_\mu g_{\nu\beta}(x, \{N\}) - \\ -\Delta_\beta g_{\mu\nu}(x, \{N\})). \tag{42}$$

Here, to make it short, the author denotes the operator $\mathbf{\Delta/\Delta_{N_{x_\chi}}}$ from formula (17) as Δ_χ, and N_{x_χ}–corresponding element from the set $\{N\}$.
In [6] it is shown that, with the use of (42) in the **measurable** form, one can obtain all the base quantities of General Relativity (GR), in particular the *Riemann tensor* $R^{\mu}{}_{\nu\alpha\beta}(x, \{N\})$ and, finally, the **measurable** form of

Einstein Equations, for short denoted as $(\mathcal{EEM})$ (abbreviation for *Einstein Equations Measurable*) (formula (57) in [6]):

$$\mathcal{EEM}\{N\} \doteq R_{\mu\nu}(x,\{N\}) - \frac{1}{2}\,R(x,\{N\})\,g_{\mu\nu}(x,\{N\}) + \lambda(x,\{N\})\,g_{\mu\nu}(x,\{N\}) = \\ = 8\,\pi\,G\,T_{\mu\nu}(x,\{N\}). \quad (43)$$

Considering the properties of $\{N\}$, for the **measurable** form of GR the *Bianchi identity* may be written, to a high accuracy, as follows:

$$\widetilde{D}_{\rho,\{N\}}R^{\chi}_{\lambda\mu\nu}(x,\{N\}) + \widetilde{D}_{\mu,\{N\}}R^{\chi}_{\lambda\mu\rho}(x,\{N\}) + \widetilde{D}_{\nu,\{N\}}R^{\chi}_{\nu\alpha\beta}(x,\{N\}) = 0, \quad (44)$$

where $\widetilde{D}_{\alpha,\{N\}} = \frac{\mathbf{\Delta}}{\mathbf{\Delta_{N_{x_\alpha}}}} + \Gamma^{\mu}_{\nu\alpha}(x,\{N\})$ and $N_{x_\alpha} \in \{N\}$.
Actually, it is clear that $(\mathcal{EEM})$ given by formula (43) represents **deformation** of the canonical Einstein equations $(\mathcal{EE})$ [7] in the sense of the Definition given in [39] with the deformation parameter $\{N\}$ (or $1/\{N\}$), and we evidently have [6])

$$\lim_{|\{N\}|\rightarrow\infty} \mathcal{EEM}\{N\} = \mathcal{EE}$$
$$\text{or} \quad \text{same} \quad \lim_{1/|\{N\}|\rightarrow 0} \mathcal{EEM}\{N\} = \mathcal{EE}. \quad (45)$$

It should be noted that the understanding of "high energies" in gravity and in other theories (in particular in gauge theories) is different. According to the current knowledge, in gravity these energies are at a level of the Planck energies $E \approx E_p$ (or same $E \approx E_\ell$) which are associated with origination of the quantum-gravitational effects. In [6], using the definitions given in **Remark 2.0**, the author has constructed a high-energy (Planck) deformation of GR of the form

$$\mathcal{EEM}[N_q] \doteq R_{\mu\nu}(x,\{N_q\}) - \frac{1}{2}\,R(x,\{N_q\})\,g_{\mu\nu}(x,\{N_q\}) - \\ +\lambda(x,\{N_q\})\,g_{\mu\nu}(x,\{N_q\}) = \\ = 8\,\pi\,G\,T_{\mu\nu}(x,\{N_q\}). \quad (46)$$

Here $\{N_q\} \doteq \{N_{x_\chi}\}, \chi = 0,...,3$ is a set of the integer numbers N_{x_χ} the absolute values of which are close to 1.

The small quantity $\ell/N_{x_\chi}=\frac{\ell^2}{\hbar}p_{N_{x_\chi}}$,where $p_{N_{x_\chi}}$ is a **primarily measurable** momentum and $|N_{x_\chi}|\gg 1$, at low energies $E\ll E_\ell$ in the case under study has its analog at high energies $E\approx E_\ell$–the quantity $l_H(p_{N_{x_\mu}})$ that is given by formula (19) in the present paper (or formula (113) in [6]).
As absolute values of the integers N_{x_μ} are small, the quantities $l_H(p_{N_{x_\mu}})$ are varying discretely (for example similar to the denominator in the right-hand side of formula (18)) and hence the high-energy deformation of GR specified by $\mathcal{EEM}[N_q]$ (formula (46)) is in fact a *discrete* theory.
It is clear that in this case the limit

$$p_{N_{x_\chi}},(|N_{x_\chi}|\approx 1)\stackrel{|N_{x_\chi}|\approx 1\rightarrow |N_{x_\chi}|\gg 1}{\Rightarrow} p_{N_{x_\chi}},(|N_{x_\chi}|\gg 1), \qquad (47)$$

where momenta in the right-hand side of formula (47), i.e. $p_{N_{x_\chi}},(|N_{x_\chi}|\gg 1)$, are the **primarily measurable** momenta at low energies $E\ll E_p$ and $p_{N_{x_\chi}},(|N_{x_\chi}|\approx 1)$ – corresponding **generalized measurable** momentum from formula (19), should be valid. Obviously, the momentum from formula (18) for $N_{\Delta x}\doteq N_{x_\chi}$ satisfies this condition.
Then formula (23) for the *canonically measurable prototype* of the infinitesimal space-time interval at low energies $E\ll E_p$ is replaced by its quantum analog or the *canonically measurable quantum prototype* for $E\approx E_p$ taking the form

$$\Delta s^2_{\{N\}}(x,\mathbf{q})\doteq g_{\mu\nu}(x,\{N\},\mathbf{q})l_H(p_{N_{x_\mu}})l_H(p_{N_{x_\nu}})=\frac{\ell^4}{\hbar^2}g_{\mu\nu}(x,\{N\},\mathbf{q})p_{N_{x_\mu}}p_{N_{x_\nu}}. \qquad (48)$$

Here there is no doubt that the numbers N_{x_μ},N_{x_ν} belong to the set $\{N\}$, all the components of this set are integers with small absolute values, $p_{N_{x_\chi}}$ are the **generalized measurable** momenta at high energies corresponding to formula (47) and $g_{\mu\nu}(x,\{N\},\mathbf{q})$ meets the condition

$$g_{\mu\nu}(x,\{N\},\mathbf{q}),(|\{N\}|\approx 1)\stackrel{|\{N\}|\approx 1\rightarrow|\{N\}|\gg 1}{\Rightarrow} g_{\mu\nu}(x,\{N\}),(|\{N\}|\gg 1), \qquad (49)$$

where $g_{\mu\nu}(x,\{N\})=g_{\mu\nu}(x,N_{x_\chi})$ is a metric in the **measurable** form at low energies (formula (23)).
Thus, at high energies $E\approx E_p$ we have

$$l_H(p_{N_{x_\chi}})\doteq\frac{\ell^2}{\hbar}p_{N_{x_\chi}};|N_{x_\chi}|\approx 1. \qquad (50)$$

Then by the substitution $\ell/N_{x_\chi} \mapsto l_H(p_{N_{x_\chi}})$ in formulae (24),(25) we can have quantum analogs of *minimal measurable variations* of the metric and of the partial derivative

$$\Delta_{\mathbf{q}} g_{\mu\nu}(x, N_{x_\chi}, \mathbf{q})_\chi \doteq g_{\mu\nu}(x + l_H(p_{N_{x_\chi}}), N_{x_\chi}, \mathbf{q}) - g_{\mu\nu}(x, N_{x_\chi}, \mathbf{q}),$$
$$\Delta_{\chi,\mathbf{q}} g_{\mu\nu}(x, N_{x_\chi}, \mathbf{q}) \doteq \frac{\Delta_{\mathbf{q}} g_{\mu\nu}(x, N_{x_\chi}, \mathbf{q})_\chi}{l_H(p_{N_{x_\chi}})}. \qquad (51)$$

Using the substitution in formula (17)

$$\frac{\ell}{\mathbf{N_{x_\mu}}} \mapsto l_H(p_{N_{x_\mu}}); \frac{\mathbf{\Delta}}{\mathbf{\Delta_{N_{x_\mu}}}} \mapsto \frac{\mathbf{\Delta_q}}{\mathbf{\Delta_{N_{x_\mu},q}}},$$
$$\frac{\mathbf{\Delta_q F(x_\mu)}}{\mathbf{\Delta_{N_{x_\mu},q}}} = \frac{F(x_\mu + l_H(p_{N_{x_\mu}})) - F(x_\mu)}{l_H(p_{N_{x_\mu}})} \qquad (52)$$

and applying this substitution to all corresponding formulae in the **measurable** format of GR at low energies, we can derive at planck energies $E \approx E_p$ all the components high-energy deformation of Einstein Equations in the **measurable** form $\mathcal{EEM}[N_q]$ (46) (or formula (117) in [6])
As a result, we have

$$\lim_{E \ll E_p} \mathcal{EEM}[N_q] = \mathcal{EEM} \quad or \lim_{|\{N_q\}| \gg 1} \mathcal{EEM}[N_q] = \mathcal{EEM}. \qquad (53)$$

For $\mathcal{EEM}[N_q]$, the metrics $g_{\mu\nu}(x, N_{x_\chi}, \mathbf{q})$ (formula (48)) represent the solution.

5 Spacetime Foam and Measurability

In accordance with the modern understanding of the problem, at high energies $E \approx E_p$ the space geometry, due to high Space-Time Quantum Fluctuations (*stqf*), represents the «space-time foam»(*stf*) [10]–[24]. The notion of «space-time foam» was introduced by J. A. Wheeler about 60 years ago for the description and investigation of physics at Planck's scales (Early Universe). Actually, because of high **quantum fluctuations** of the metric $g_{\mu\nu}$, the space has a quantity of geometries. Despite the fact that in the last time numerous works have been devoted to physics at Planck's scales within the scope of this notion, by this time still their no clear understanding of *stf* as it is.

And it should be noted that the proposed approach can be considered as a development of the idea of *stf*,i.e. space-time geometry *stf* [10]–[24] but for the case of **discrete consideration**. Really, at low energies $E \ll E_p$ the canonical metric components in a continuous consideration $g_{\mu\nu}(x)$ may be taken as components of the metric in the **measurable** form $g_{\mu\nu}(x, N_{x_\chi})$ (formula (23) for $N_{x_\chi} = \infty$, i.e. we have $g_{\mu\nu}(x) = g_{\mu\nu}(x, \infty)$). But, as at low energies $|N_{x_\chi}| \gg 1$, the theory may be considered continuous to a high accuracy due to **Remark 2.5**. Then, expanding the quantity $g_{\mu\nu}(x, N_{x_\chi})$ into a series in terms of the small parameter $1/N_{x_\chi}$ close to the point $g_{\mu\nu}(x)$ and retaining only the zero- or first-order terms (due to obvious smallness of all the remaining terms), in fact, we arrive at the formula for fluctuation of the metric g in a region with the size L ([12],formula (43.29)):

$$\Delta g \sim \frac{l_p}{L}. \tag{54}$$

Indeed, as $l_p \propto \ell$, considering that the energies are low and with due regard for **Remark 2.2**, L represents **PMQ**. Then, setting $L = N_{x_\chi}\ell$ and substituting it into (54),we get the following:

$$\Delta g \sim \frac{l_p}{L} \sim \frac{\ell}{N_{x_\chi}\ell} = \frac{1}{N_{x_\chi}}. \tag{55}$$

So, at low energies the indicated *quantum fluctuations* are very small, actually being coincident with the basic parameters in the **measurable** approach (parameters of the corresponding deformation).

But, as demonstrated by formulae (46)–(52), at high energies $E \approx E_p$ this is not the case, and *quantum fluctuations*
$g_{\mu\nu}(x, \{N\}, \mathbf{q}), (|\{N\}| \approx 1)$ of the metric $g_{\mu\nu}(x, \{N\}), (|\{N\}| \gg 1)$ are great.

In this case in the **measurable** form the notion **"space-time foam"** is absolutely adequate because the only restriction imposed on
$g_{\mu\nu}(x, \{N\}, \mathbf{q}), (|\{N\}| \approx 1)$ is (49). It is clear that in this case there is a great deal of different $g_{\mu\nu}(x, \{N\}, \mathbf{q}), (|\{N\}| \approx 1)$. As the **measurable**

analogs of Einstein Equations at low energies $\mathcal{EEM}$ (43) and at high energies $\mathcal{EEM}[N_q]$ (46), according to the above formulae, are determined by the quantities $p_{N_{x_\chi}}$, where $|N_{x_\chi}| \gg 1, |N_{x_\chi}| \approx 1$, respectively, at low energies for the given metric
$g_{\mu\nu}(x, \{N\}, \mathbf{q}), (|\{N\}| \gg 1)$ its *quantum fluctuations* in the general case are determined by the functions $\mathcal{G}_\mu(N_{x_\mu}), \mu = 0, ..., 3$ which are dependent on integer values of N_{x_μ} so that

$$p_{N_{x_\mu}} \doteq \frac{\hbar}{\mathcal{G}_\mu(N_{x_\mu})\ell}, \tag{56}$$

and

$$\lim_{|N_{x_\mu}| \to \infty} \mathcal{G}_\mu(N_{x_\mu}) = N_{x_\mu}. \tag{57}$$

Still, some models based on *micro-black holes* are very interesting and fairly promising. In particular, the models studied in [15]–[21] and based on *micro-black holes*, i.e. black holes with a Schwarzschild radius of several Planck's units of length.
It should be noted that the case of *micro-black holes* with the Schwarzschild metric in terms of **measurable** quantities has been already studied by the author in his paper [4]. In this paper, within the scope of validity of the Generalized Uncertainty Principle (GUP) of Section 3, in terms of the **measurability** notion the gravitational equations at the event horizon surface of these holes have been derived and their basic thermodynamic characteristics (temperature, entropy) have been obtained.
It is obvious that these holes form a discrete finite set, provided their Schwarzschild radii r_{mbh} are considered **primarily measurable** quantities:

$$r_{mbh} = N_{r_{mbh}}\ell, N_{r_{mbh}} \approx 1, \tag{58}$$

where $N_{r_{mbh}}$ is an integer number.
As, in accordance with GUP of Section 3, we have

$$p(N_{x_i}, GUP) = \frac{\hbar}{1/2(N_{x_i} + \sqrt{N_{x_i}^2 - 1})\ell}, i = 1, ..., 3, \tag{59}$$

on passage from high energies $E \approx E_p$ to low energies $E \ll E_p$, formulae (56),(57) are valid and we can, to a high accuracy, obtain at low energies the **primarily measurable** momenta $p(N_{x_\mu}), |N_{x_\mu}| \gg 1$.
However in [4], due to the validity of GUP, initially it has been supposed that ℓ is a minimal length, that is a rather restrictive condition as noted in the very beginning of Section 3. But in the case under study we assume *only* that ℓ is a *primary* length, i.e. it satisfies the formula of (10).
Then formula (59) for the integer N_{x_i} takes the following form:

$$p_{N_{x_i}} = \frac{\hbar}{1/2(N_{x_i} + \sqrt{N_{x_i}^2 - 1})\ell}, i = 1, ..., 3, \tag{60}$$

and formulae (56),(57) are valid too. This is in line with remark from Section 3: "the whole formalism, developed in [1]–[6] on condition that ℓ is the minimal length, is valid for the case when ℓ is the *primary* length".
In the terms and notations from [14], for the fluctuations $\widetilde{\delta l}$ of the distance l, the estimate is as follows:

$$(\widetilde{\delta l})_\gamma \gtrsim l_p^\gamma l^{1-\gamma} = l_p(\frac{l}{l_p})^{1-\gamma} = l(\frac{l_p}{l})^\gamma = l\lambda_l^\gamma, \tag{61}$$

or that same

$$|(\widetilde{\delta l})_\gamma|_{min} = \beta l_p^\gamma l^{1-\gamma} = \beta l_p(\frac{l}{l_p})^{1-\gamma} = \beta l\lambda_l^\gamma, \tag{62}$$

where $0 < \gamma \leq 1$, coefficient β is of order 1 and $\lambda_l \equiv l_p/l$.
From (61),(62), we can derive the quantum fluctuations for all the primary characteristics, specifically for the time $(\widetilde{\delta t})_{\gamma,l}$, energy $(\widetilde{\delta E})_{\gamma,l}$, and metrics $(\widetilde{\delta g_{\mu\nu}})_{\gamma,l}$. In particular, for $(\widetilde{\delta g_{\mu\nu}})_{\gamma,l}$ we can use formula (10) in [14]

$$(\widetilde{\delta g_{\mu\nu}})_{\gamma,l} \gtrsim \lambda_l^\gamma. \tag{63}$$

As it is assumed that $\ell \propto l_p$, i.e. $\ell = \kappa l_p$, where $\kappa \approx 1$, in formulae (61)–(63), without loss of generality, the following substitution is justified:

$$l_p \mapsto \ell. \tag{64}$$

Then at low energies $E \ll E_p$ in the **measurable** form it is natural to assume that the distance l is the **primarily measurable** quantity, i.e. $l = N_l \ell$, where $N_l \gg 1$ is an integer number, in this case $\lambda_l \equiv \ell/l = N_l^{-1}$, and all the quantities in formulae (61)–(63) are expressed in terms of N_l. Specifically, formula (63) takes the form

$$(\widetilde{\delta g_{\mu\nu}})_{\gamma,l} \gtrsim N_l^{-\gamma}. \tag{65}$$

Obviously, because in this case we have $N_l \gg 1$, $(\widetilde{\delta g_{\mu\nu}})_{\gamma,l}$ is weakly dependent on N_l.

On the contrary, at high energies $E \approx E_p$, according to formulae (56),(57), the distance l is not a **primarily measurable** quantity

$$l = \mathcal{G}(N_l)\ell, \tag{66}$$

where $\mathcal{G}_i = \mathcal{G}_j = \mathcal{G}, i \neq j; i, j = 1, ..., 3,$ N_l is a small integer number, and the fluctuation $(\widetilde{\delta g_{\mu\nu}})_{\gamma,l}$ in this case is evidently strongly dependent on N_l (or l).

Note that, within the constant factor $\hbar/\ell$, the parameter λ_l is coincident with the momentum $p_l = \hbar/(\mathcal{G}(N_l)\ell)$ from formula (56) that, in accordance with formula (57), is a **primarily measurable** momentum at low energies $E \ll E_p$ and a **generalized measurable** momentum at high energies $E \approx E_p$.

In [6] at low energies $E \ll E_p$ for the **measurable** form of gravity $\mathcal{EEM}\{N\}$ (43) the author has derived the Least Action Principle and the Lagrangian formalism .

The action for GR in the **measurable** format can be derived from the action for the canonical GR in continuous space-time

$$S_{EH} = -\frac{1}{16\pi G} \int d^4x \sqrt{|g|}\ (R + \lambda) \tag{67}$$

And **"measurable"** action has the following form (formula (79) from [6])

$$\begin{aligned} S_{EH}(\{N\}) = -\frac{1}{16\pi G} \sum \Delta_{(\{N\})}\Omega \sqrt{|g(\{N\})|}\, \cdot \\ \cdot\ (R(x,\{N\}) + \Lambda(x,\{N\}))\,, |\{N\}| \gg 1, \end{aligned} \tag{68}$$

where $\Delta_{(\{N\})}\Omega$ is the volume element in a **measurable** variant of GR (formulae (44)-(46) in [6]).
It is clear that at high energies $E \approx E_p$, due to real discreteness of the theory, the Least Action Principle in the general case is no longer valid for this theory. We can note only the Planck deformation $S_{EH}(N_{x_\chi}, q)$ of the **"measurable"** action $S_{EH}(N_{x_\chi})$ (68):

$$S_{EH}(N_{x_\chi}, \mathbf{q}) \doteq -\frac{1}{16\pi G} \sum \Delta_{(N_{x_\chi}),\mathbf{q}}\Omega \sqrt{|g(N_{x_\chi}, \mathbf{q})|} \cdot \\ \cdot \left(R(x, N_{x_\chi}, \mathbf{q}) + \lambda(x, N_{x_\chi}, \mathbf{q})\right), |N_{x_\chi}| \approx 1, \quad (69)$$

with substitution of all components in formula (68) in accordance with the formulae in Section 4.
Of course, in this case the condition

$$S_{EH}(N_{x_\chi}, \mathbf{q}), (|N_{x_\chi}| \approx 1) \stackrel{|N_{x_\chi}|\approx 1 \rightarrow |N_{x_\chi}| \gg 1}{\Rightarrow} S_{EH}(N_{x_\chi}), (|N_{x_\chi}| \gg 1) \quad (70)$$

must be fulfilled. It should be noted that the above-mentioned results may be applied for the derivation of a **measurable** variant of gravitational thermodynamics for horizon spaces and Schwarzschild's black holes [4]
Besides, we also have

$$\lim_{|\{N\}| \rightarrow \infty} S_{EH}(\{N\}) = S_{EH} \quad (71)$$

Then at low energies $E \ll E_p$, due to formulae (45) and (71), at sufficiently large $|\{N\}|$ all the results for the spacetime foam valid in the continuous pattern remain valid in a **measurable** consideration, to a high accuracy, with the adequate replacement of the quantities used in the continuous case by the **measurable**-form quantities.
In particular, all the results from [23] devoted to evolution of quantum low-energy fields (i.e. the fields at the energies $E \ll E_p$) in a foam-like spacetime may be expressed in the **measurable** form within a high accuracy by substitution of ℓ for l_p with the use of the above-mentioned formula $\ell = \kappa l_p$.
In the case under study, using formulae (46),(56),(60),... , one can extend the results from [23] to evolution of quantum high-energy fields (i.e. the

fields at the energies $E \approx E_p$).
It is important: taking into account that at low energies $E \ll E_p$ in the **measurable** form $|\{N\}| < \infty$ gives hope that in a **measurable** picture, with the specific restrictions imposed on the set $\{N\}$, we should have no pathological solutions of GR (specifically, Closed Timelike Curves (CTC) [51]–[54]), which are not excluded in a continuous consideration (Section 3 **Loss of quantum coherence** in [23]).
Passage in the **measurable** form to high energies $E \approx E_p$ leads to a completely discrete picture and hence in this case the notion of CTC in its initial formulation becomes senseless. Possibly, due to the limiting transition in formula (53), this furnishes the clue to understanding of the conditions resulting in the absence of CTC in gravity and, generally, in the absence of the *loss of quantum coherence* in the **measurable** form of gravity.

6 Conclusion

Let us summarize the results obtained in this work.

6.1 The Einstein's Equivalence Principle (EP) has a natural applicability limit, the upper bound of which lies at the Planck scales $E \approx E_p$, though, or the specific processes in high-energy physics, this bound is always considerably lower than the Planck's, lying within the energy region $E \ll E_p$.

6.2 The **measurability** notion enables one to represent gravity at all the energy scales (i.e. at low energies $E \ll E_p$ and at high energies $E \approx E_p$) in terms of one and the same parameters dependent on the integer variables N_{x_μ}. It should be noted that in the earlier works by the author (for example, [55],[56])similar parameters were in the form of the quantity $\alpha_l \doteq \ell^2/l^2$,yet beyond the **measurability** concept.

6.3 The **measurability** concept generates a new approach to studies of quantum fluctuations of metrics and, finally, of the spacetime foam at high energies $E \approx E_p$. Therewith, quantum fluctuations of metrics are determined by the **generalized measurable** momenta which, on passage to low energies $E \ll E_p$, become **primarily measurable momenta**.

6.4 As noted in the beginning of Section 3, for the correctness of this approach, it is necessary that at low energies $E \ll E_p$ all the results should be independent of the choice of p_{max}. The problem arises: and what about high energies $E \approx E_p$?
It is clear that, in accordance with the first part of this point, for any choice of p_{max}, we should have the same quantities in the right-hand sides of formulae (53) and (70)within a high accuracy. Provided at the Planck energies $E \approx E_p$ the results are also weakly dependent on choice of p_{max}, there should be discrete symmetries relating **measurable** pictures at high energies for different values of p_{max}.

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