Electron gas under the non-homogeneous magnetic field of cylindrical symmetry

P.N. Romanets*

(Dated: November 15, 2011)

We study the states of 3D-electron gas in non-homogeneous magnetic field. It is supposed that the step of magnetic field, at which field changes its sign, lies on the cylindrical surface. The eigen value problem is solved for are different parameters of the system. The equilibrium statistics in the limit of large quantum lifetime are considered as well. It is predicted that the system characterized by quasi-zero-dimensional spectra with large density of states; snakes orbits lead to the effective charge transfer; an appearance of the magnetic field step strongly modify the coordinate dependence of carriers concentration.

PACS numbers:

I. INTRODUCTION

The fabrication technologies for making non-homogeneous magnetic field in 2DEG are well developed. Micromagnets, superconducting micro samples providing Meissner effect, or magnetic impurities are used to obtain non-homogeneous magnetic field with given strength and geometry [1, 2].

Different phenomena related to 2D-electron transport under non-homogeneous perpendicular magnetic field have been studied intensively within recent years both experimentally [2–6] and theoretically [2, 7, 10**?** –12]. If the characteristic geometry scales of non-homogeneous magnetic field l_{ch} satisfy the condition $l_{ch} > l_e$ (l_e is the electron mean free path), it leads to the drift diffusion phenomena. If the contrary condition $l_{ch} < l_e$ is satisfied, but the temperature T or characteristic relaxation energy $\sim \hbar^2(l_e^2\widetilde{m})^{-1} \sim \tau_{ql}^{-1}\hbar$ of the system is large enough [2], than classical ballistic transport takes place (τ_{ql} is the quantum life time, $\widetilde{m}$ is the effective mass of electron). The quantum phenomena takes place if the average of absolute value of cyclotron frequency satisfy $\overline{|\omega_c|} > T\hbar^{-1},\ \tau_{ql}^{-1}$.

In this paper we propose to examine properties of 3D-electron gas in non-homogeneous magnetic field in quantum and semiclassical ballistic regimes $\omega_c \gtrsim T\hbar^{-1} \gg \tau_{ql}^{-1}$.

The ways to obtain 3D-electron gas under strongly non-homogeneous magnetic field are not widely discussed to the best of our knowledge. The appearing of the high density currents which are arranged in are special way may cause strongly non-homogeneous magnetic field of requisite geometry.

The superconducting inclusions in bulk semiconductors have been examined for several decades. Moreover, present-day nanotechnology permit to obtain hight-quality superconducting nanowires of given geometric parameters, using different superconducting materials and substrates [13, 14]. But most of the studies that had been carried out, aimed to study superconducting phase transition, and tunneling effects (proximity) of the system under different parameters and conditions. There are number of conditions which could be responsible for the phase transition in the real system with superconducting sample (sample geometry, temperature, magnetic field, irradiation etc.)[15–20]. One of the most significant parameters for superconducting nanowires is their diameter. The decreasing of the diameter of nanowire to the order of superconducting coherence length or lower may dramatically change or completely vanish the superconducting properties, more over quantum confined effects are also possible [21–25].

Specially arranged currents integrated into a bulk semiconductor may be the source of strong non-homogeneous magnetic field with given geometry. The examples of flat and cylindrical geometry of the magnetic field step are shown in Fig. 1(a) and (b) correspondingly. One can use classical limit of the model to make it clear. In classical limit far from rupture of magnetic field charged particle is in homogeneous magnetic field and has spiral-like trajectory with parallel to the magnetic field axis. On the other hand, if the particle closer to the currents than its cyclotron radius, the trajectory, which is schematically described in Fig 1(d), become possible (so-called snake orbit) [8]. Obviously, the flat geometry case (a) for the most part is similar to the studied in [8, 9] case of perpendicular to the 2D-electron gas non-homogeneous magnetic field. Therefor the second case [Fig. 1 (b-d)] is more interesting

The currents $I_1,\ I_2,\ ..., I_n$ are arranged on the cylindrical surface and of the same directions and values. The current $I_0 = -(\sum_{k=1}^{N} I_k)/2$ (opposite direction) is fixed to the axis of the cylindrical surface.

We make the next simplifications. The semiconductor electrons has parabolic isotropic dispersion law. Any reasons to proximity effect appearing (contact peculiarities etc.) are absent. The Fermi energy of the electron gas is much lower then threshold of superconductor. The superconducting inclusion with I_0 (see Fig.1(d)) may be treated as impermeable barrier. The distance between the currents on cylindrical surface $I_1,\ I_2$,...,I_N and the diameters of the corresponding inclusions is lower or in order of particle wave length ($l_{ch} \lesssim \lambda_e$). We also suppose that the system may be treated as infinite one and has

*Electronic address: pn˙romanets@yahoo.com

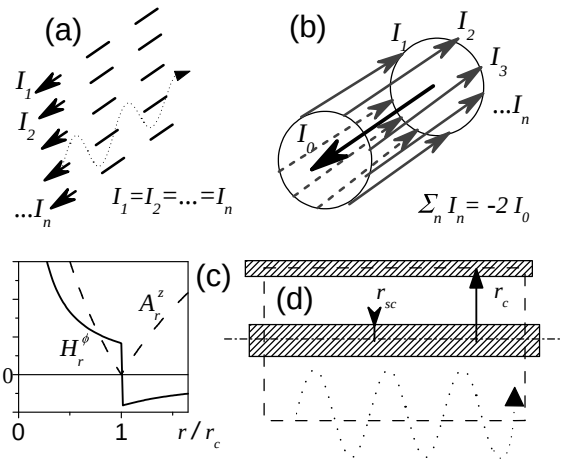

FIG. 1: Schematic description of the systems with magnetic field step; a – plane arranged parallel currents, and possible trajectory of the charged (negatively) particle; b – currents are arranged on the cylindrical surface and one fixed to the its axis; c – coordinate dependence of the magnetic field and vector potential (arbitrary units); d – the section of the described in (b) geometry and possible trajectory of the motion of the particle.

continues translational symmetry along the the cylinder axis. The spin effects are not considered as well.

Also we are not considering any mechanisms of relaxation which cause electron spectra transformation and broadening of the energy levels (limit of large quantum lifetime).

II. SCHRODINGER EQUATION

Now we turn to the quantum mechanical description of the problem. The Schrodinger equation for one-electron wave function in approach of effective mass has the next form [**?**]:

$$\frac{(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A_R})^2}{2\widetilde{m}}\Psi^E_{\mathbf{R}} = E_{m,k^z}\Psi^E_{\mathbf{R}}, \qquad (1)$$

here $\widetilde{m}$ is the effective mass of electron, $e = -|e|$ is the electron charge, c is the light velocity, $\mathbf{A_R}$ is the vector-potential. The wave function $\Psi^E_{\mathbf{R}}$ that describes the eigenstate with with eigenvalues of the quantum numbers k^z, m, E_{m,k^z}. One has choose the vector-potential to not break continuous translational symmetry along cylindrical axis, for example $\mathbf{A_R} = (0,\ 0,\ A^z_r)$ with nonzero z-component $A^z_r = (p_c c/e)|\ln(r/r_c)|$, where $p_c \equiv \hbar k_c = 2eI_0/c^2$ [here and below we use cylindrical coordinates $\mathbf{r} = (r,\ \phi,\ z)$, all vector components has corresponding upper indexes]. Notice, that we are not considering field inside superconductors, and so gauge invariance has trivial form. The momentum operators $\hat{p}^z - eA^z/c$ and $\hat{p}^r$ are not commuting with hamiltonian, but $\hat{p}^z$ and $r\hat{p}^\phi = -i\hbar\partial/\partial\phi$ are commuting. The latter implies that we can search the solutions in the next form $\Psi^{m,k^z}_{\mathbf{R}} = \exp(-ik^z z - im\phi)\Phi^{m,k^z/k_c}(rk_c)$. Before substituting this form into eq. (1) we turn to the next dimensionless parameters, wave number $\kappa^z \equiv k^z/k_c$, radial coordinate $\rho \equiv rk_c$ [$\rho_c \equiv r_c k_c$, $\rho_{sc} \equiv r_{sc}k_c$ see Fig. 1(d)]. Substituting definite above form, one obtains next homogeneous differential equations for Φ^{m,κ^z}_ρ

$$\left[\frac{1}{\rho}\frac{d}{d\rho}\rho\frac{d}{d\rho} - \frac{m^2}{\rho^2} + \kappa^2 + 2\kappa^z\left|\ln\left(\frac{\rho}{\rho_c}\right)\right| - \ln\left(\frac{\rho}{\rho_c}\right)^2\right]\Phi^{\kappa,m,\kappa^z}_\rho = 0, \qquad (2)$$

where $\kappa^2 = 2\widetilde{m}E_{m,k^z}/(\hbar k_c)^2 - \kappa^{z\,2}$. The boundary condition (i) – $\Phi^{\kappa,m,\kappa^z}_{\rho=\rho_c} = 0$ is the sequence of impermeable barrier approach (see above), other one is in standard requirements. The standard requirement for such solutions are next (ii) – the existence of the integral $\int_0^\infty |\Phi^{\kappa,m,\kappa^z}_\rho|^2\rho d\rho$, in particular expects for $\rho|\Phi^{\kappa,m,\kappa^z}_\rho|^2_{\rho\to\infty} \to 0$; (iii) – normalization condition $\int_0^\infty |\Phi^{\kappa,m,\kappa^z}_\rho|^2\rho d\rho = 1$, always may be satisfied if (ii) is satisfied, but exclude one constant from the general solution of eq.(2). (iv) – the continuity of the first derivative $[d\Phi^{m,\kappa^z}(\rho+o)/d\rho - d\Phi^{m,\kappa^z}(\rho-o)/d\rho]_{o\to 0} = 0$ (or existence of the second derivative) for $\rho \in (\rho_{sc},\ \infty)$; The condition of the orthogonality of different eigenstates is also satisfied.

It is convenient to make substitution $x = \ln(r/r_c)$ that results to the usual form of one-dimensional Schrodinger equation

$$\left(\frac{d^2}{dx^2} + K^{\kappa,m,\kappa^z\,2}_x\right)\breve{\Phi}^{\kappa,m,\kappa^z}_x = 0, \qquad (3)$$

were $K^{\kappa,m,\kappa^z\,2}_x = \exp(2x)\rho_c^2\left(\kappa^2 + 2\kappa^z|x| - x^2\right) - m^2$ (introducing one-dimensional potential energy $U(x)$, one obtains $K^{\kappa,m,\kappa^z\,2}_x = \frac{2\widetilde{m}}{\hbar^2}[E_{m,k^z} - U(x)]$) Below we analyze this equation using quasiclassical approach. First, we suppose condition $x_2 - x_1 \gg 1$, were $x_{1,2}$ are turning points: $K^{\kappa,m,\kappa^z}_{x=x_{1,2}} = 0$. The solutions inside the region $x_1 \ll x \ll x_2$ tend to the functions [27]

$$\breve{\Phi}^{\kappa,m,\kappa^z}_x \simeq \frac{\sum_\pm C_\pm \exp\{\pm i\int_{x_0}^x K^{\kappa,m,\kappa^z}_{x'}dx'\}}{\sqrt{K^{\kappa,m,\kappa^z}_x}}, \qquad (4)$$

where C_+ and C_- are constants. The condition (i) transforms to $\breve{\Phi}^{\kappa,m,\kappa^z}_{x_{sc}} = 0$ [$x_{sc} = \ln(r_{sc}/r_c)$]. The solutions for $x_2 \gg x$ approaches the functions [conditions (ii)]:

$$\breve{\Phi}^{\kappa,m,\kappa^z}_x \simeq \frac{B\exp\{-\int_{x_0}^x \mathcal{K}^{\kappa,m,\kappa^z}_{x'}dx'\}}{\sqrt{\mathcal{K}^{\kappa,m,\kappa^z}_x}}, \qquad (5)$$

where $\mathcal{K}^{\kappa,m,\kappa^z\,2}_x = m^2 - \exp(2x)\rho_c^2\left(\kappa^2 + 2\kappa^z|x| - x^2\right)$. The condition (ii) for approach (4)-(5) means that if x ($x < x_2$) is increasing and passing by point x_2 in complex space ($x \to x' = x_2 + \delta x e^{\pm i\psi}$, $\delta x = |x - x_2| > o$) then (4)

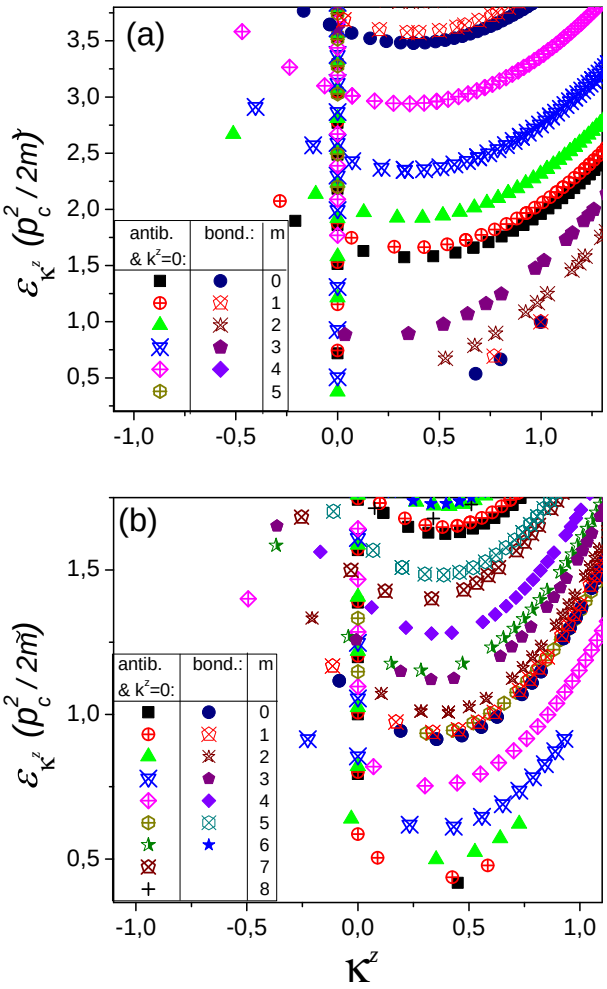

FIG. 2: Spectra of the system versus κ^z for different I_0: (a) I_0 correspond to $\rho_{sc}=4$, $\rho_c=10$ and (b) I_0 correspond to $\rho_{sc}=2$, $\rho_c=5$.

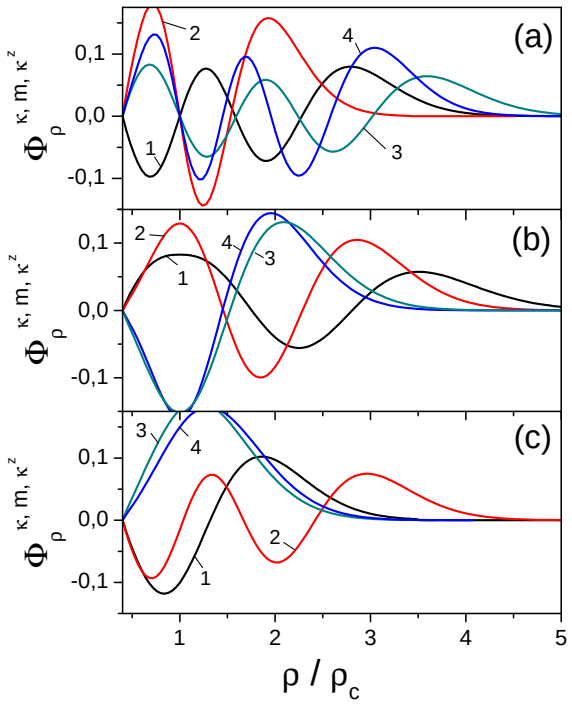

FIG. 3: The radial wave functions versus ρ for $\rho_c=10$, $\rho_{sc}=4$ and different numbers n_1, n_2 or n [see eqs. (6), (7) or (8)] and m: (a) antibonding states ($n1$, $n2$, m) = (0, 2, 0), (0, 1, 1), (0, 3, 0), (0, 3, 4); (b) bonding states ($n1$, $n2$, m) = (0, 2, 0), (0, 2, 4), (0, 1, 3), (0, 1, 4); states with $\kappa^z=0$ (n, m) = (1, 0), (3, 0), (0, 2), (0, 3);

turns into (5) [27]. There are three kinds of quasiclassical solutions which satisfy (i)-(iv) and correspond to a different choice of $C_\pm$, κ, κ^z in (4).

The first one ($\kappa^z\neq 0$, $\breve{\Phi}_{x_c}^{\kappa,m,\kappa^z}$ – antibonding states):

$$\breve{\Phi}_x^{\kappa,m,\kappa^z}\simeq A\frac{\sin\left(\int_0^x K_{x'}^{\kappa,m,\kappa^z}dx'\right)}{\sqrt{K_x^{\kappa,m,\kappa^z}}},\qquad(6)$$

with $\int_0^{x_2}K_{x'}^{\kappa,m,\kappa^z}dx'=\pi(n_2+\frac{3}{4})$ and $\int_{x_{sc}}^0 K_{x'}^{\kappa,m,\kappa^z}dx'=\pi(n_1+1)$ if $x_{sc}>x_1$, or $\int_{x_1}^0 K_{x'}^{\kappa,m,\kappa^z}dx'=\pi(n_1+\frac{3}{4})$ if $x_{sc}<x_1$.

The second one ($\kappa^z\neq 0$, $\breve{\Phi}_{x_c}^{\kappa,m,\kappa^z}$ – bonding states):

$$\breve{\Phi}_x^{\kappa,m,\kappa^z}\simeq B\frac{\cos\left(\int_0^x K_{x'}^{\kappa,m,\kappa^z}dx'\right)}{\sqrt{K_x^{\kappa,m,\kappa^z}}},\qquad(7)$$

with $\int_0^{x_2}K_{x'}^{\kappa,m,\kappa^z}dx'=\pi(n_2+\frac{1}{4})$ and $\int_{x_{sc}}^0 K_{x'}^{\kappa,m,\kappa^z}dx'=\pi(n_1+1)$ if $x_{sc}>x_1$, or $\int_{x_1}^0 K_{x'}^{\kappa,m,\kappa^z}dx'=\pi(n_1+\frac{1}{4})$ if $x_{sc}<x_1$. Here, in eq. (7) we suppose $|x|>+o$ (see Appendix B and [**?**]).

The third one ($\kappa^z=0$):

$$\breve{\Phi}_x^{\kappa,m,\kappa^z=0}\simeq C\frac{\sin\left(\int_{x_{sc}}^x K_{x'}^{\kappa,m,\kappa^z=0}dx'\right)}{\sqrt{K_x^{\kappa,m,\kappa^z=0}}},\qquad(8)$$

with $\int_{x_1}^{x_2}K_{x'}^{\kappa,m,\kappa^z=0}dx'=\pi(n+\frac{3}{4})$ if $x_{sc}<x_1$ and $\int_{x_{sc}}^{x_2}K_{x'}^{\kappa,m,\kappa^z=0}dx'=\pi(n+\frac{1}{2})$ if $x_{sc}<x_1$ (n, n_1, n_2 are integer numbers).

Using (5)-(7) we construct the solutions of the problem (2), (i)–(iv) in the first approximation. Next approximations performed numerically. In Fig. 2(a),(b) one can see the spectra of the system versus quantum number k^z. The numerical solutions which correspond to the first approximations (5)-(7) are shown in Fig. 3(a)-(c) correspondingly.

All states with $m\neq 0$ are twice degenerated $E_{|m|,k^z}=E_{-|m|,k^z}$. The lager r_c the lower difference $E_{|m|+1,k^z}-E_{|m|,k^z}>0$. The bonding states and the states with $\kappa^z=0$ may be strongly modified or suppressed because $\Phi_{\rho_c}^{\kappa,m,\kappa^z}\neq 0$ and they interact with the currents on cylindrical surface $r=r_c$. On the other hand the states with $\kappa^z=0$ plays an essential role in intersubband photoexci-

tation processes if the Fermi energy of electron gas and photoexcitation energy E_F, $\hbar\omega < p_c^2/2\widetilde{m}$.

III. EQUILIBRIUM DENSITY MATRIX

Now we can turn to the consideration of the statistics of the system under equilibrium conditions. We neglect any effect of the scattering mechanisms on eigenstates and eigenvalues of the system, supposing that feeble interaction between electrons and thermostat, as well as electron-electron interaction impose equilibrium state of the system that may be characterized by diagonal density matrix:

$$\hat{\varrho} = \frac{|E, k^z, m\rangle\langle E', k'^z, m'|}{\exp[(E_{m,k^z} - \mu)/T] + 1}\delta_{E,E'}\delta_{k^z,k'^z}\delta_{m,m'}, \qquad (9)$$

here $\delta_{a,b}$ is the Kronecker symbol, μ and T is the chemical potential and the temperature of the system. The the coordinate-dependent concentration is calculated with the usual formula

$$\mathrm{N}(r) = 2\hat{S}p\hat{\varrho}, \qquad (10)$$

where $\hat{S}p\hat{A}$ denotes the sum of diagonal matrix element, multiplier 2 takes into account spin. In Fig. 4 one can see the concentration versus dimensionless radial coordinate ρ calculated using eq. (10) for different I_0 (variation of I_0 means proportional variation of ρ_c and ρ_{sc}) μ and T. Comparing Fig. 4 (c) and (d) show the variation of the concentration with magnetic field step appearing. In the case $I_0 \neq 0$, $\sum_{k=1}^{N} I_k = 0$ and $\rho_c \gtrsim 10$ if μ, $T \lesssim p_c^2/2\widetilde{m}$ the majority of electrons localized inside the cylinder $r < r_c$. Classically they are not effected by the magnetic field modification after switching on the currents in the cylindrical surface $\sum_{k=1}^{N} I_k \neq 0$, but their coordinate distribution are modified essentially. The sketchy description of the effect one can see in Fig. 5.

Using the current operator $\hat{j}_z = e(\hat{p}_z - p_c|\ln(r/r_c)|)/\widetilde{m}$, we obtain

$$j_r^z = 2\hat{S}p(\hat{j}^z\hat{\varrho}). \qquad (11)$$

One can see the current density versus μ for different T that is calculated using eq.(11) in Fig.6. The current is increasing with the increasing of μ, T, and r_c as well. The the reason of the increasing are the same versus μ, T, and r_c – the higher one of this parameters the more states participate in charge transfer. The increasing of r_c leads to increasing of the density of states. And the increasing of T and μ leads to enabling of higher states.

IV. CONCLUDING REMARKS

To summarize, we have considered electronic gas in discontinues magnetic field under cylindrical geometry

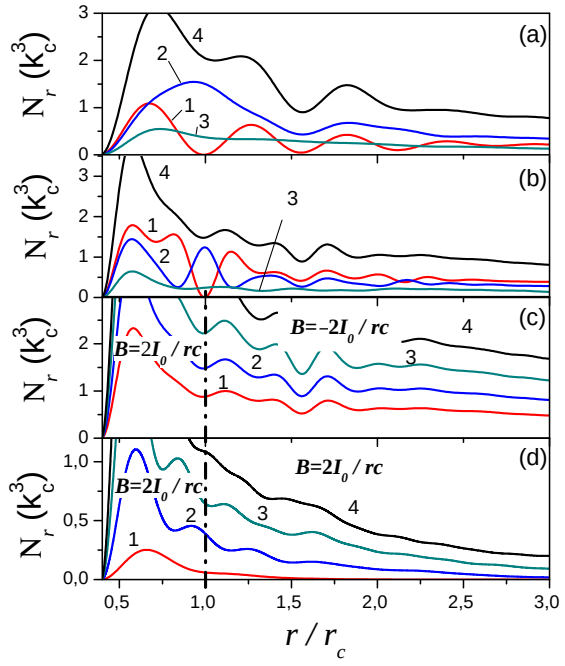

FIG. 4: Concentration versus ρ: (a) $T = 0$, $\mu = 0.8p_c^2/\widetilde{m}$ for $\rho_{sc} = 4$ and $\rho_c = 10$, the lines 1-3 take into account antibonding, bonding and the states with $\kappa^z = 0$, the line 4 includes all of them; (b) the same as in panel (a) for $\rho_{sc} = 2$ and $\rho_c = 5$; (c) $T = 0$ and different chemical potential value $\mu = 0.8\times$, $1.6\times$, $2.4\times$, $3.2\times$ $p_c^2/2\widetilde{m}$ marked 1-4 correspondingly; (d) the same as in panel (c) for the case $I_0 \neq 0$, $\sum_{k=1}^{N} I_k = 0$ (currents on cylinder are switched off) [compere (c) and (d)].

condition. The quasi-zero-dimensional spectrum was obtained for semiconductor electronic gas under considered condition. Nevertheless average value of the current along the preferential direction is predicted to be non-zero.

Now we discuss the numerical parameters that where used. For example, one can suppose that system based on bulk GaAs and lead nanowires. Under the helium temperatures characteristic thermal momentum of GaAs electronic gas $p_T = \sqrt{2\widetilde{m}T} \simeq 2.7 \times 10^{-22}\ g \cdot cm/s$. Supposing $r_{sc} \simeq 0.22\ \mu m$ we obtain the Ginzburg - Landau parameter for such lead nanowire under $T \sim 4.2$K estimated to be $0.31 < 1/\sqrt{2}$, so we deal with first type superconductor (see also [25]). The critical field of lead under helium temperature $H_c \simeq 530\ Gs$. Using the Silsbees rule one can obtain $p_c \lesssim 1.4p_T$ and $r_{sc}p_c/\hbar \simeq 4$.

Finally, we estimate the concentration and current values. Using estimated here above parameters one can make conclusion that $k_c^3 \simeq 5 \times 10^{16}$ cm^{-3}.

We performed consideration suppose very hight current densities I_0, $I_1, ...I_N$ (i.e. low ρ_{sc} and ρ_c) to emphasize quantum peculiarities. The realization of such systems is questionable, but the majority of discussed

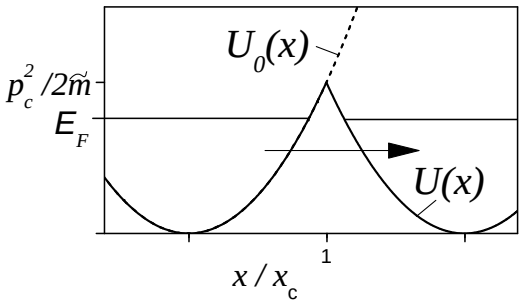

FIG. 5: The sketchy description of the magnetic field step effect on the electrons coordinate distribution. The effective one-dimensional potential energy $U_0(x) = \hbar^2(k^{z2} + 2k^z k_c x - k_c^2 x^2)/2\widetilde{m}$ correspond to the case $\sum_{k=1}^{N} I_k = 0$, $k^z < 0$ [see eq. (3)]. After the currents are switched on $\sum_{k=1}^{N} I_k \neq 0$ the effective potential energy is changed to the $U(x) = \hbar^2(k^{z2} + 2k^z k_c|x| - k_c^2 x^2)/2\widetilde{m}$ (k^z is the same). The electrons tunneling to the right hand side (outside the cylinder).

peculiarities remain unique even for the case of low current density [i.e. $1 \ll \rho_{sc} \ll \rho_c$, see appendix A and the discussion of Fig. 4(c),(d), Fig.5].

Appendix A: Flat geometry as the limit case

Here we consider eq. (1) in the limit case $r_c \to \infty$. We locate the Cartesian coordinates in the point $\phi = 0$, $r = r_c$, z = 0 and make asymptotical transition from the cylindrical to the orthogonal coordinates $\mathrm{x} = r - r_c$, $\mathrm{y} = r_c\phi$, z = z (ϕ posses low values $\sim r_c^{-1}$, for finite y). Making expansion of the vector-potential up to second order on x/r_c and substituting wave function $\Psi(\mathbf{r}) = \Phi^{\kappa,\kappa^z}(\mathrm{x})\exp(ik^y\mathrm{y} + ik^z\mathrm{z})$ one could obtain

$$\left[\frac{d^2}{d\mathrm{x}^2} + k^2 + 2k^z k_c\frac{|\mathrm{x}|\mathrm{x}}{2r_c^2} + \frac{\omega_c^2\widetilde{m}^2}{\hbar^2}(|\mathrm{x}| - \mathrm{x}_0)^2\right]\Phi^{k^y,k^z}(\mathrm{x}) = 0 \qquad \text{(A1)}$$

here $k^2 = 2\widetilde{m}E/\hbar^2 - k^{y2}$, $\mathrm{x}_0 = r_c k^z/k_c = p_z/(\widetilde{m}\omega_c)$, and $\omega_c = p_c/(r_c\widetilde{m})$. This equation still do not correspond to flat geometry case Fig.1 (a). One has to use the next additional restrictions: (i) $k^z \ll k_c = \omega_c r_c\widetilde{m}/\hbar$, (ii) $r_c\exp(-k^z/k_c - k/k_c) > r_{sc}$ to obtain flat geometry case. The restriction (i) we need to neglect the term $2k^z k_c|\mathrm{x}|\mathrm{x}/(2r_c^2)$. This term means that system remembers cylindrical symmetry even for large r_c if quantum number k^z is large enough. The restriction (ii) we need to omit boundary condition for $r = r_{sc}$. Using (i) and (ii) one obtains

$$\left[\frac{d^2}{d\mathrm{x}^2} + k^2 + \frac{\omega_c^2\widetilde{m}^2}{\hbar^2}(|\mathrm{x}| - \mathrm{x}_0)^2\right]\Phi^{k^y,k^z}(\mathrm{x}) = 0, \qquad \text{(A2)}$$

with boundary conditions $\Phi^{k^y,k^z}(\mathrm{x})|_{|\mathrm{x}|\to\infty} = 0$. This problem also can be solved in the framework of quasiclassical approach. One has to use quasiclassical function in the next form (antibonding)

$$\Phi^{k^y,k^z}(\mathrm{x}) = A\frac{\sin\left[\int_0^{\mathrm{x}} d\mathrm{x}'\sqrt{k^2 - \left(\frac{\omega_c\widetilde{m}}{\hbar}\right)^2(|\mathrm{x}'| - \mathrm{x}_0)^2}\right]}{[k^2 - \left(\frac{\omega_c\widetilde{m}}{\hbar}\right)^2(|\mathrm{x}| - \mathrm{x}_0)^2]^{1/4}} \qquad \text{(A3)}$$

or (bonding) for $|\mathrm{x}| > +o$ (see Appendix B and [**?**]):

$$\Phi^{k^y,k^z}(\mathrm{x}) = B\frac{\cos\left[\int_0^{\mathrm{x}} d\mathrm{x}'\sqrt{k^2 - \left(\frac{\omega_c\widetilde{m}}{\hbar}\right)^2(|\mathrm{x}'| - \mathrm{x}_0)^2}\right]}{[k^2 - \left(\frac{\omega_c\widetilde{m}}{\hbar}\right)^2(|\mathrm{x}| - \mathrm{x}_0)^2]^{1/4}}. \qquad \text{(A4)}$$

The standard conditions are satisfied if $\int_{\mathrm{x}_a}^{0} d\mathrm{x}'\sqrt{k^2 - (\omega_c\widetilde{m}/\hbar)^2(|\mathrm{x}'| - \mathrm{x}_0)^2} = \int_0^{\mathrm{x}_b} d\mathrm{x}'\sqrt{k^2 - (\omega_c\widetilde{m}/\hbar)^2(|\mathrm{x}'| - \mathrm{x}_0)^2} = \pi(n + 1/2 \pm 1/4)$ {here $n = 0, 1, 2..$; "+" for antibonding and "−" for bonding states; $\mathrm{x}_{a,b} = \pm[\mathrm{x}_0 + \hbar k/(\omega_c\widetilde{m})]$}

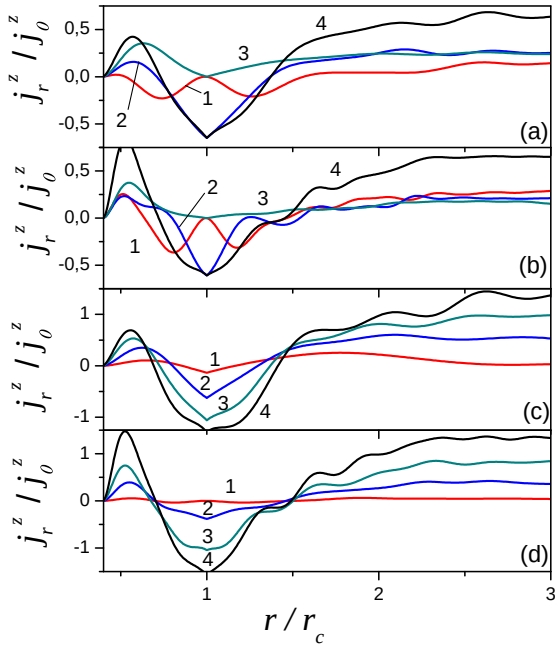

FIG. 6: Current density versus ρ in units of $j_0^z e = |e|k_c^2 p_c/\widetilde{m}$: (a) $T = 0$, $\mu = 0.8p_c^2/\widetilde{m}$ for $\rho_{sc} = 4$ and $\rho_c = 10$, the lines 1-3 take into account antibonding, bonding and $\kappa^z = 0$ states, the line 4 includes all of them; (b) the same as in panel (a) for $\rho_{sc} = 2$ and $\rho_c = 5$; (c) $T = 0$ and different chemical potential value $\mu = 0.8\times$, $1.6\times$, $2.4\times$, $3.2\times$ $p_c^2/2\widetilde{m}$ marked 1-4 correspondingly ($\rho_{sc} = 4$ and $\rho_c = 10$); (d) the same as in panel (c) for $\rho_{sc} = 2$ and $\rho_c = 5$.

and $p^y < \sqrt{2\widetilde{m}E}$. For $p^y > \sqrt{2\widetilde{m}E}$ in terms of quasiclassical approach we have four turning points: $\mathrm{x}_{a_{1,2}} = -[\mathrm{x}_0 \pm \hbar k/(\omega_c\widetilde{m})]$, $\mathrm{x}_{b_{1,2}} = (\mathrm{x}_0 \pm \hbar k/(\omega_c\widetilde{m}))$ and the Born-Sommerfeld quantization rule $\int_{\mathrm{x}_{a1}}^{\mathrm{x}_{a2}} d\mathrm{x}'\sqrt{k^2-(\omega_c\widetilde{m}/\hbar)^2(|\mathrm{x}'|-\mathrm{x}_0)^2} = \int_{\mathrm{x}_{b1}}^{\mathrm{x}_{b2}} d\mathrm{x}'\sqrt{k^2-(\omega_c\widetilde{m}/\hbar)^2(|\mathrm{x}'|-\mathrm{x}_0)^2} = \pi(n+1/2)$. This leads to the spectral relation ($p^y < \sqrt{2\widetilde{m}E}$):

$$\frac{(\hbar k)^2}{2\widetilde{m}}\left[\arcsin\left(\frac{\mathrm{x}_0\omega_c\widetilde{m}}{\hbar k}\right)+\frac{\pi}{2}+\right.$$
$$\left.\frac{\mathrm{x}_0\omega_c\widetilde{m}}{\hbar k}\sqrt{1-\left(\frac{\mathrm{x}_0\omega_c\widetilde{m}}{\hbar k}\right)^2}\right] = \hbar\omega_c\left(n+\frac{1}{2}\pm\frac{1}{4}\right)\pi. \quad \text{(A5)}$$

and $E = \hbar\omega_c(n+1/2)$ for $p^y > \sqrt{2\widetilde{m}E}$. The spectra that is definite by eq. (B5) well studied for 2D case, when $k^{\mathrm{y}} \equiv 0$. It gives usual Landau levels, when $\mathrm{x}_0 \to \hbar k/(\omega_c\widetilde{m})$; levels splitting takes place with decreasing of x_0; and $k \to \infty$, when $\mathrm{x}_0 \to -\hbar k/(\omega_c\widetilde{m})$. Notice, that performed transition is valid only for $k^{\mathrm{z}} \ll k_c$.

Appendix B: Solutions about magnetic field step

Lets consider eq. (3) for $|x| \ll 1$. Performing linearization $K_x^{\kappa,m,\kappa^{\mathrm{z}}2} = \exp(2x)\rho_c^2\left(\kappa^2+2\kappa^{\mathrm{z}}|x|-x^2\right) - m^2 \approx c_1x + c_2|x| + c_3$ [$c_1 = (\rho_c\kappa)^2/\sqrt{(\rho_c\kappa)^2-m^2}$ $c_2 = \rho_c^2\kappa^{\mathrm{z}}/\sqrt{(\rho_c\kappa)^2-m^2}$, $c_3 = \sqrt{(\rho_c\kappa)^2-m^2}$], we obtain

$$\left(\frac{d^2}{dx^2}+c_1x+c_2|x|+c_3\right)\breve{\Phi}_x = 0. \quad \text{(B1)}$$

The solution of this equation may be introduced through the Airy functions od first Ai and second kind Bi as

$$\breve{\Phi}_x = a_\pm Ai[-(c_1\pm c_2)^{1/3}x - c_3/(c_1\pm c_2|)^{2/3}] + b_\pm Bi[-(c_1\pm c_2)^{1/3}x - c_3/(c_1\pm c_2|)^{2/3}], \quad \text{(B2)}$$

here "+" for $x > 0$ and "−" for $x < 0$; the constant coefficients

$$a_\pm = Bi[-c_3/(c_1\pm c_2)^{2/3}],$$
$$b_\pm = -Ai[-c_3/(c_1\pm c_2)^{2/3}]; \quad \text{(B3)}$$

for asymmetric solutions, or

$$a_\pm = (c_1\pm c_2)^{1/3}Bi'[-c_3/(c_1\pm c_2)^{2/3}],$$
$$b_\pm = -(c_1\pm c_2)^{1/3}Ai'[-c_3/(c_1\pm c_2)^{2/3}]; \quad \text{(B4)}$$

for symmetric solutions. The coefficients do not satisfy normalization condition which is not essential for the discussed here characteristics. One can notice that first derivative of symmetric solution equivalent to the symmetric solution in the point $x = 0$ if c_1, c_2 and c_3 are the same in both cases.

Similar description for the 2D or flat geometry case one can find in [**?**]. To obtain it one has use the substitution $x = \mathrm{x}$, $c_1 = 0$, $c_2 = -2\mathrm{x}_0(\widetilde{m}\omega_c/\hbar)^2$, $c_3 = k^2 + \mathrm{x}_0^2(\widetilde{m}\omega_c/\hbar)^2$.

[1] Peeters F. M. and De Boeck J. 1999, *Handbook of Nanostructured Materials and Technology*, vol. 3, ed. N. S. Nalwa (New York: Academic) p. 345.
[2] A. Nogaret, J. Phys.: Condens. Matter 22, 253201 (2010)
[3] A. Nogaret, S. J. Bending, and M. Henini, Phys. Rev. Lett. 84, 2231 (2000).
[4] D. N. Lawton, A. Nogaret, and S. J. Bending, D. K. Maude and J. C. Portal, M. Henini, Phys. Rev. B 64, 033312 (2001)
[5] A.K. Geim, I.V. Grigorieva, S.V. Dubonos, J.G.S. Lok, J.C. Maan, A.E. Filippov, F.M. Peters, Nature 390 (1997) 259
[6] S. V. Dubonos, A. K. Geim, K. S. Novoselov, J. G. S. Lok, J. C. Maan, and M. Henini, Physica E 6, 746 (2000).
[7] J. Reijniers and F.M. Peeters, Phys. Rev. B 63, 165317 (2001).
[8] J. Reijniers and F.M. Peeters, J. Phys.: Condens. Matter 12, 9771-9786 (2000).
[9] Müller J. E. Phys Rev. Lett. 68 385 (1992).
[10] J. Reijniers, F.M. Peeters, and A. Matulis, Phys. Rev. B 59, 2817-2823 (1999).
[11] J. Reijniers, F.M. Peeters, and A. Matulis, Phys. Rev. B 64, 245314 (2001).
[12] F. M. Peeters, A. Matulis, and I. S. Ibrahim, Physica B 227, 131, (1996)
[13] M. Savolainen, V. Touboltsev, P. Koppinen, K.-P. Riikonen and K. Yu. Arutyunov, Appl. Phys. A 79 1769, (2004)
[14] M.Zgirski, K-P. Riikonen, V. Tuboltsev, P. Jalkanen, T. T. Hongisto and K. Yu. Arutyunov, Nanotechnology 19, 055301 (2008).
[15] T. Matsuda, S. Hasegawa, M. Igarashi, T. Kobayashi, M. Naito,H. Kajiyama, J. Endo, N. Odakabe, A. Tonomura, R. Aoki. Phys. Rev. Lett., 62, 2519 (1989).
[16] A. Lenard, T. Dietl, M. Sawicki, W. Dobrowolski, K. Dybko, T. Skoskiwicz, S. Miotkowska, A. Witek, W. Plesiewicz, A. Mycielski. J. Low. Temp. Phys., 80, 15 (1990).
[17] S. Takaoka, T. Sugita, K.Murase. Jap. J. Appl. Phys., 26 (Suppl. 26.3), 1345 (1987).
[18] O.N. Shevtsova, Superconductor Science and Technology, 2005, v. 18, p. 726729.
[19] J.M. Baranowski, Z. Liliental-Weber, W.-F. Yau, E.R. Weber,Phys. Rev. Lett.,1991, v. 66, N 23, p. 30793082
[20] S. Takaoka, T. Sugita, K. Murase. JJAP, Supplement. 1987, v. 26, p. 13451346.
[21] A. A. Shanenko, M. D. Croitru, M. Zgirski, F. M. Peeters

and K. Arutyunov, Phys. Rev. B 74, 052502 (2006).
[22] K. Yu. Arutyunov, D. S. Golubev, and A.D. Zaikin, Physics Reports 464, 1 (2008).
[23] M. Zgirski, K.-P. Riikonen, V. Touboltsev, and K. Arutyunov, Nano Letters 5, 1029 (2005).
[24] M. Zgirski, K.-P. Riikonen, V. Touboltsev and K.Yu. Arutyunov, Phys. Rev. B. 77, 054508 (2008).
[25] S. Michotte, L. Piraux, S. Dubois, F. Pailloux, G. Stenuit, and J. Govaerts, Physica C 377, (2002) 267-276
[26] Haug, R. J.,Semiconductor Science and Technology, v. 8, Issue 2, pp. 131-153 (1993).
[27] Landau, L. D., and E. M. Lifshitz, 1981, *Quantum Mechanics: Non-Relativistic Theory* (Pergamon Press(London)).