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Foundations of a theory of quantum gravity

by

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Chapter 1

Introduction

This adventure started out as a paper, but soon it grew considerably in size and there was no choice left anymore but to present it as a full blown book written in a style which is intermediate between that of an original research paper and that of a book. More precisely, I opted for a style which is somewhat between the historical and axiomatic approach and this manuscript can therefore be read from different perspectives depending upon the knowledge and skills of the reader. Since quantum gravity is more than a technical problem, the mandatory sections constitute the introduction as well as the technical and axiomatic framework of sections seven till eleven. However, the reader who is also interested in the philosophical aspects as well as a general overview of the problem is advised to study sections two and three as well. The critical reader who is not willing to take any statement for granted should include also sections four till six, since these are somewhat of a transitional nature closing the gap between the conservative initial point of view and the new theory developed later on. Lecturing about this work made me aware that there is also a more direct way to arrive in Rome and for that very reason, this introduction is also split into two parts. The first one takes the conservative point of view as it is done by the very large majority of researchers which necessitates a careful and precise way of phrasing the content; the second approach however is more bold and direct but goes, in my humble opinion, much more economic to the heart of the matter. I believe that the variety of presenting the same material in this introduction will allow the reader to choose which way he prefers to follow.

Let me also say from the outset what this book achieves and what it leaves as open issues, where the last phrase is to be understood in the sense that these issues are technically open but that the successful realization of them is motivated to some extend. As is always the case in science, the judgment of whether an argument is compelling or not depends upon the history and experience of the beholder and I certainly do not claim to be the oracle of Delphi in this regard. However, I deem these conjectures to be utterly reasonable and received no serious signs of doubt from those people I actually explained the content to in detail and who understood the material. If there were no major open *technical* issues anymore, at least the mathematical side of the theory would be fully specified and detailed leaving merely the duty to match experiment, something which remains at this point to be done. For example verifying the post Newtonian expansion as well as the emergence of QED are mandatory tasks. Nevertheless, let us start by the achievements: a new class of gravitational theories is presented which naturally incorporate a novel relativistic quantum theory whose formulation is entirely *local* on spacetime in a way which is identical to Einstein's original formulation of the theory of relativity. This means that we dismiss global Hamiltonian and path integral approaches to quantum mechanics and that causality, amongst other things, is an emergent property instead of a fundamental principle. Moreover, it is shown that on Minkowski, ordinary free quantum field theory is the "natural" limit of our theory in the absence of interactions. In section ten, we started the study of how the free theory behaves in a nontrivial gravitational background with a global spatial rotational symmetry and we impose natural boundary conditions at infinity for the quantum theory. Some of the main realizations, however, are that (a) we have a full nonperturbative formulation of a candidate theory of quantum gravity (b) we present a solution to the question of "Where the collapse takes place?" (c) our laws have a local four dimensional formulation which allows for a consistent treatment of singularities (d) we present a natural class of local physical observables (e) we give a natural interpretation to the Weinberg-Witten theorem and circumvent as well Haag's theorem as the Coleman-Mandula no-go argument (f) the local vacuum states are dynamically determined (g) we have shown that Newton's law and free Quantum Field Theory emerge in the suitable limits assuming natural boundary conditions. To the best knowledge of the author, string theory managed to solve (g) as well as some form of the post-Newtonian limit; however (a) till (c) are certainly open issues in that approach. An emergent virtue is that "new" and more general mathematical concepts and techniques enter the formulation and sections seven and eleven are entirely devoted to the introduction of these tools. Obviously, a lot of work will have to be done before these new mathematical gadgets are understood at an appropriate level but that is nothing extraneous to other approaches. In all honesty, I believe it is quite remarkable that someone can offer a complete new quantum theory, based upon totally different principles, which appears to have the right limits ninety years after birth of that same theoretical framework.

Certainly, these promises must arouse some skepticism and also I did not believe much of it in the beginning. However, as the work evolved, the inner coherence became stronger which reassured me that it was not all utter nonsense. At this point, it might be opportune to make some philosophical remarks as to why it is not very surprising that a singleton comes up with some novel ideas regarding this old problem which opposes modern culture. As an act of wisdom and cowardry resulting from the fear of a potential downfall on the sales ranks of this work, I shall refrain from doing so hoping that the intelligent reader understands what I am talking about. Let us now take the historical and conservative approach and say how the argumentation and gist behind this work is subdivided contentwise. Sections two and three have a rather special place in this work and reflect more my own way of thinking than anything else. Nevertheless, a reader who would finish the entire book might have the feeling that somehow these two sections already contained some of the main seeds of the later construction, albeit in a somewhat hidden form which is, at least, my humble intention. Why are sections of a "revisional" and "philosophical" nature important? Well, they reflect how one thinks about contemporary science; what its main lessons are, where reside the most important shortcomings and what logical gaps might imply a very different worldview which in turn generates new mathematics, hence new physics and the cycle starts again. In spite of the even more radical character of the end product, I decided not to change any word here because I want to convey that many cycles can lead to very different conclusions, but one has to go one "rotation" at a time. Especially the role of consciousness in physics, historically stressed by Von Neumann, and more recently revived by Penrose and others gets a more central place in the theory and as an amateur philosopher, I spur some resemblance to monism. In the third chapter, the focus is changed from relativity and quantum mechanics to quantum gravity; this chapter will contain technical arguments as well as metaphysical ones. I realize that this is a rather unconventional course of action for a physics book but sometimes it is good to be liberated from too restrictive formal rules.

Considering this philosophical and physical input, it requires a novel idea to save manifest background independence in the sense that we demand a well defined representation of the group of coordinate transformations as well as a covariant (hence dynamical) procedure for fixing the *local* vacuum state and particle interpretation. Loop quantum gravity certainly tries to construct this representation as well as vacuum state however unsuccessfully so far and the issue of a particle interpretation is nonexistent apart from some naive attempts trying to identify particles with knot like configurations in the spin network basis states. String theory follows a more conventional approach, however, to my knowledge the issue of the vacuum state has not recieved any answer. A radical new construction is presented in chapter eight which allows for a treatment of all these issues which appears to be consistent so far. However, these ideas are highly nontrivial if you look through conservative glasses and in chapters four and five, we present a representation in terms of background dependent physics. The germs of this theory, that is the kinematical setting and classical dynamics, are presented in chapter four. Here, I study a novel type of background dependent dynamics which resembles the Polyakov action but with the important difference that the worldsheet metric is not a dynamical variable. Therefore, we not have to consider the Virasoro constraints and a kinematical volume constraint is put in by hand. The motivation for committing this ugly crime comes from the technical idea that inverting a metric becomes an analytic operation if one does not have to divide trough its determinant (in either volume). The problems of causality and "localizability" are discussed and an old idea of how to retrieve matter from such framework is revived (just consider the Einstein-Cartan equations to be an identity). It turns out that Quantum Field Theory generates local degrees of freedom which are not present classically because the curvature tensor may be nonvanishing depending upon the type of Wick ordering one considers (something which one may call a quantum anomaly). However, this theory cannot be rescued but trying to so lead me to work done in chapter six which by itself formed an important corner stone for the ideas presented later on. A philosophical principle, which constitutes the very core of the reasoning behind that later work, is that there is no point in axiomatizing based upon representation prejudices. Indeed, all inequivalent representations should be investigated and therefore one should only *try* to formalize physical principles. There are plenty of examples in the literature of the first kind of activity: (a) the old Wightmann axioms (and more recently Wald's) of Quantum Field Theory (b) the work of Piron on some possible extensions of Quantum Mechanics (c) General Relativity as the Einstein equations (d) Dirac's Fermion theory (e) Weinberg's analysis of the implications of first principles of Quantum Field Theory [57] even if this work is by far superior to anything else in literature (and was actually the key motivator for my ideas). Indeed, the philosophical ideas explained in sections two and three do not change later on, only the mathematical representation does. In other words, this work is written in the old spirit of natural philosophy complemented with novel mathematical techniques exceeding the current use in mainstream physics. Valuable inspiration for these ideas originated from literature on quantum group theory, Von Neumann algebras, measure theory, Krein spaces, operator theory and many other branches of mathematics.

Chapter five starts with a general discussion about interpretational subtleties in quantum physics regarding observables which do not commute with the Hamiltonian and give rise to fairly complicated interpretations of pretty simple dynamical systems. Consequently, we apply this idea to the simple theory proposed in chapter three and, as said previously, define observed matter though calculation of relevant tensors in Einstein-Cartan theory. Those observables are highly nonlinear and noncommuting with the Hamiltonian and it could be hoped that the probability of decay for their low energy eigenstates on the time scale of observation is sufficiently low for no inconsistencies to arise. However, computation of the metric tensor and (anti)commutation relations thereof leads to unwanted infinties which I try to dissolve through a modification of the quantization procedure and particle statistics. This leads to a split in the content of the chapter where on one on side the question of statistics is readdressed and on the other the "quantization" of our preliminary theory is continued. I have decided to move the reinvestigation of the spin-statistics theorem, which is justified because Minkowski causality is not a valid assumption anymore, to a separate appendix in order to improve the general readership of this chapter. The outcome of this investigation is rather surprising since a consistent quantization of our theory (that is one without normal ordering infinities at fourth order) does not only require spin $\frac{1}{2}$ Clifford particles, but we must also allow for negative energies. The latter cannot be replaced by negative norm, positive energy Bosons as such particles would not cancel out the infinities in the Hamiltonian as well as the commutation relations of the metric. Given the importance of the Clifford numbers in this procedure, it is logical to study Clifford valued actions and quantize them; a study which is initiated in chapter six. Here, a trade off between negative energies and negative probabilities occurs and the resulting particles have genuinely different transformation properties under the Poincaré group than is allowed for by the analysis of Wigner [57]. Given that we have to work on indefinite Hilbert spaces, the spin statistics connection vanishes and we shall have better things to say about that later on. All this requires a first extension of Quantum Field Theory, that is one must study representation theory on indefinite Hilbert spaces and construct a consistent local and causal interpretation. At the same time, one might investigate the possibility of negative energies and study if this theory is really as screwed as most people believe. Although the quantization scheme in chapter six is the first example in the literature where negative probabilities are mandatory, since without them negative energy spin $\frac{1}{2}$ particles would have to be Bosons, indefinite Hilbert spaces have shown up in history on several other occasions such as Gupta-Bleuler quantization of gauge theories. Moreover, negative probabilities allow one to sidestep the famous Weinberg-Witten theorem, which states that there exists no theory with a Lorentz covariant energy momentum tensor containing massless spin two particles. There are plenty of other means for achieving this goal such as allowing for fat gravitons, or one might dismiss gravitons and recuperate the Newtonian gravitational force from virtual particle interactions[1]. Anyhow, all above results strongly indicate that indefinite Hilbert spaces do not only allow for a broader class of phenomena, but appear also to be necessary for quantum gravity. There is still another way of looking at the Weinberg-Witten theorem which does not seem to have been appreciated too much which is simply accepting its conclusion: that is, gravitons do not gravitate directly (they do nevertheless indirectly through interaction with matter)! This must appear nutty for someone who thinks in the conventional way about how gravitons arise (through quantization of a classical field theory), but as will become clear in chapter eight, it is completely consistent and physical within the new framework. Therefore, in my mind, we are left with essentially two possibilities : (a) gravitons on Nevanlinna spaces which do gravitate and (b) non-gravitating gravitons (on Nevanlinna spaces or not). In sections seven and eight, we will come to the conclusion that option (b) on Clifford-Nevanlinna modules is the right way to go[2]. In a nutshell, chapter four is a fairly ordinary analysis of a simple theory which realizes the ideas of chapter three in a straightforward way, while sections five and six are of a transitional nature; the "real" theory starts to be developed from chapter seven onwards.

So, chapter seven paves the way for a future study of representation theory of the Poincaré group on infinite dimensional Clifford-Nevanlinna modules which

[1] I acknowledge useful private correspondence with Alejandro Jenkins about the Weinberg-Witten theorem although he would not morally agree with all conclusions I draw here [68].

[2] To add to the reader's confusion, these non-gravitating gravitons can nevertheless scatter in a non-trivial way.

is an even wider first generalization of Quantum Field Theory. For starters, I was quite unhappy with the definition of Nevanlinna spaces by Krein and Jadczyk and decided to rigorously construct my own concept; the latter is a lot more advanced and relates to concepts such as an observer dependent topology. The definition suggests an even wider generalization to non-associative structures we baptise to be kroups, as opposed to groupoids and semi-groups. The construction of a rigorous definition of a Nevanlinna space constitutes the main body of the chapter as it currently stands while the study of *finite dimensional* Clifford-Nevanlinna modules and a suitable spectral theorem thereon is its primary stages. We learn for now that an Hermitian operator allows for *many* (approximate) decompositions of several inequivalent types, each with their own probability interpretation, but as it stands no general theorem is formulated. These preliminary results suggest such an interpretational "revolution" that is legitimate to spend many pages spend to it. The interpretation needs to be further worked out and generalization towards the infinite dimensional context needs to be made prior to studying representation theory of the Poincaré group.

The dynamics presented in chapter eight incorporates the idea of a quantum bundle in which the unitary relators form a group locally, but only have a kroup structure globally. As mentioned there, I foresee the possibility for a slight generalization of this to kroups with a special kind of connectedness property but I feel it would be hard, if not impossible, to construct a dynamics while assuming only a general kroup structure to hold. Hard computations will have to show whether the "postulate" of a local group structure can be sustained, otherwise one would have to give up associativity even locally; this is one of the issues I still need to adress in sections nine and ten, but this book is not going to give a final answer to this question. The second idea consists in putting free Quantum Field Theory on the tangent bundle instead of on spacetime itself: the physical and mathematical ideas behind this are nontrivial and I go through a great deal to explain them properly. Moreover, the setting discussed here is just a special case of an even much wider class of possibilities and only future work can tell to which extend our limitations are justified. The third idea deals with a totally nonperturbative treatment of particle interactions; particles originate from ultralocal "hidden variables" living on tangent space and the relators between those hidden variables are subject of the real dynamical content. In this sense, our approach is radically quantum and many ideas are natural continuations of suggestions made, even as early, by Von Neumann, Wigner and Heisenberg. We dismiss the path integral as a step back in the natural evolution of quantum theory in the sense that it hinges too close on concepts involving a classical reality and it is moreover not as relativistic as one would like it to be. Indeed, as mentioned previously, our theory really has a *local* formulation and global considerations like hypersurfaces, action principles with ill defined integration over noncompact spacetimes definitively belong to the past. Not only do the laws have a local formulation on spacetime, also the probability interpretation and state of the universe have a mere local meaning. It would be too much to simply explain these things at this point, but let me say that (a) a boundary value point of view is more natural for the theory of gravitation than initial values are (b) the holographic principle is directly reflected in the quantum and geometry theory. Many of the philosophical implications (which were not foreseen in chapter three) would simply be too mind stretching to explain without any understanding of the mathematical formalism and the chapter finishes with a more in depth discussion where physics could go from thereon. For all these reasons, I believe it is not a good idea to start at chapter eight or just even chapter six for that matter. Chapters nine and ten, which are currently under construction, will deal with phenomenology as well as some representation theory of the Poincaré group on Hilbert spaces in which an infinite number of copies of the same particles are allowed for. The latter involve a length scale which has to be sufficiently large so that the corresponding violations of the Pauli principle do not lead to conflicts with observation. Chapter nine in particular will deal with corrections to the Hawking effect as calculated in our novel quantum theory. A full mathematical investigation of integrability of the equations of motion is, as said previously, not treated in this work for the understandable reason that it would take too much work to fill all the gaps. Chapter eleven is meant as a teaser and provides an even wider mathematical implementation of the physical principles we enunciated before; a novel and universal concept developed in that direction is the notion of a quantum manifold. This concludes the overview from the conservative vantage point of view.

As an alternative way of reaching similar conclusions and of deepening ones understanding of the physical principles which go into the theory, let me present an exercise which is seldomly made but can have an illuminating effect after one has gone through all the painful derivations. That is, I shall first present the known principles behind Quantum Field Theory and General Relativity and comment upon which ones are to remain there and which should be the approximate result of a computation in weak gravitational fields instead of a fundamental law of nature. The physical principles behind Quantum Field Theory are (a) locality (b) Poincaré covariance (c) causality, in the sense that spacelike separated observables commute, (d) positive energies (e) the statistics assumption (f) cluster decomposition principle and the technical assumption made is that all representations should be on separable Hilbert spaces. Of course, some of these principles can be exchanged such as the statistics assumption which follows from the existence of a well defined number operator, Poincaré covariance and a relative isotropy condition while ignoring parastatistics. Now, there is no doubt that all these physical restrictions should apply in case all interactions are shut off, but there are no good indications for the technical requirements. Indeed, positive probability is tightened to the straightforward Born rule, but the latter can be extended to representations on Nevanlinna space; likewise, it is rather unnatural that the representation space should be separable since it is impossible to describe the situation with an infinite number of particles which should be allowed, in principle, if one is describing the whole universe. However, this puts doubt on the principle of causality since the spin statistics theorem fails if *any* one of the above restrictions is dropped; replacing causality by spin statistics as a fundamental principle of nature appears a better thing to do since the implication of causality would be much more robust (that is, not depend upon any of these technical assumptions). Another argument which leads to this conclusion is the desire to have a truly local, four dimensional formulation of quantum interactions; in that case, the commutation relations cannot be implemented since they depend upon a global apriori notion of spatiality. For quantum gravity therefore, we demand that the *interactions* satisfy laws which have a local formulation, are covariant under *local* Lorentz transformations and are "locally unitary". The free theory on the other hand should obey locality, Poincaré covariance, spin statistics, positive energies and cluster decomposition; the reader notices that we dropped the technical requirements as well as the statistics assumption. To merge these views, the free theory should live on the tangent bundle and the representation of the Poincaré group should live on the tangent plane and not on spacetime. This means that the translation symmetry of the free theory is broken by means of the interactions which single out a preferred origin.

On the side of Relativity, the main principles are (a) locality (b) background independence (c) local Lorentz covariance (d) general covariance (e) the equivalence of gravitational and inertial mass. Except for the last principle, all the latter are mathematically well defined and there is no reason to abandon them in a theory of quantum gravity and one has the choice whether to make the gravitational theory locally Lorentz covariant or locally Poincaré covariant (it does not really matter). However (e) is something which should only hold in the linearization of the theory around a Minkowski background and current work reveals it does not hold if nonlinear corrections are taken into account. From all the above, it follows that if one probes the world at small distance scales, the theory should become free and therefore asymptotic freedom is build into the construction right from the start. These constitute the very foundations upon which the construction in chapter eight hinges and we have more to say about these things in the course of this book. Most attention however is spend to the principle of locality which *appears* to necessitate the framework of classical abelian manifolds. However, there is a small caveat here and in section eleven we show how the standard locality concept can be canonically lifted to non-abelian manifolds. This is an extremely strong result since it allows for the construction of a "universal" differential calculus where the ambiguity in the derivative operators originates from a quantum connection. We shall not further treat this construction in this book since I feel that the more conservative theory is already more than complex enough to start with.

Chapter 2

On quantum mechanics and relativity

My first reaction when learning about quantum mechanics was that this could not be and that eventually quantum theory would prove to be an excellent approach to an otherwise deterministic theory. This (local) realist stance remained with me for a long time even in spite of Bell's theorem which strictly speaking doesn't prove anything since it assumes a nondetermistic feature of nature, namely "free will". This has recently been pointed out again by 't Hooft [1] and resulted in a debate with Conway and Kochen [2] [4]. Indeed, the textbook case for quantum mechanics is rather weak, first of all do you need to assume a two fold level of reality, the classical observer and the quantum system under consideration, but moreover is the dynamics presented as a procedure applied to a classical system. This is certainly so in the Dirac quantization scheme where classically meaningless Poisson brackets get promoted to physical statements about the quantum world; this situation, however, is already considerably improved upon - but not completely erased - in "the" path integral formulation. In that sense quantum mechanics is not even a theory, rather an algorithm, and the only argument in favor of it is that it manages to produce accurate outcomes of experiments. This is of course a very strong indication that something about it must be right but as long as we do not understand quantum mechanics "an sich" the situation is theoretically rather unsatisfying. That is, until we figure out *why* nature would prefer some of its ideas, the theorist must remain skeptical and open to alternatives. Before I proceed, let me stress that I am an unashamed realist in the sense that I believe some stuff to exist, but the question is what does and how it connects to our observations. Indeed, suppose you want to make a theory for the universe, then your Platonic objects might be a fixed four manifold $\mathcal{M}$ and the *definition* of a Lorentzian metric, i.e. a symmetric covariant two tensor with signature $(- + ++)$ or you might want to be more ambitious and take as Platonic object the definition of a causal set. Now, classical mechanics corresponds to a single universe which we need to find out by specifying initial conditions and by proposing a certain dynamics. The view on this procedure is rather limited since it allows only for globally hyperbolic universes and wouldn't allow us to think of black holes while we clearly can do that within general relativity. There, the Einstein equations should be thought of as a constraint on the universe and the initial value point of view must be entirely dropped. This leads one to propose that classical mechanics could be thought of as a singular probability measure with support on one Lorentzian metric on the space of all Lorentzian metrics on $\mathcal{M}$. Specifying which measures μ of that type are allowed is equivalent to formulating a dynamics; in that respect a single measure unifies the idea of "initial values" with the dynamics and putting physical demands on μ would constrain as well the kinematics and dynamics at the same time. A first, albeit limited, generalization of this would consist in studying nonsingular probability measures. This can give rise to a genuine stochastic *dynamics* with fixed initial boundary conditions such as happens in the Sorkin Rideout-dynamics for causal sets [5] [6] and does not need to be limited to measures expressing lack of knowledge of the initial data. One recognizes that this is already a higher form of physics since it involves the entire space of representations (usually called histories) of the Platonic theory. "Quantum mechanics" is another generalization of this idea which contains the latter as a special case; actually as Sorkin noticed, it is the next alternative in an infinite series of theories expressing higher types of correlations between alternate histories [7] [8]. More precisely, assume the space of histories is equipped with a topology and its subsequent sigma algebra Σ, then a function $\mu : \Sigma \to \mathbb{R}_+$ is said to be a measure of order $n-1$ if for every n tuple of disjoint elements $A_i \in \Sigma$, μ satisfies

$$\mu\,(A_1 \cup A_2 \ldots A_n) - \sum \mu\,(\text{n-1 tuples}) + \sum \mu\,(\text{n-2 tuples}) \ldots + (-1)^{n-1} \sum \mu(A_i) = 0.$$

Sorkin's generalization of quantum mechanics deals with measures of order 2. One can show that this implies the existence of a real valued function $I(A,B)$ satisfying for A and B disjoint

$$I(A \cup B, C) = I(A,C) + I(B,C)$$

and

$$\mu(A) = I(A,A).$$

This ties actually with the decoherence functional approach developed by amongst others Dowker and Halliwell [3]. A decoherence functional is a complex valued function D on $\Sigma \times \Sigma$ satisfying $D(A,B) = \overline{D(B,A)}$, $D(A \cup B, C) = D(A,C) + D(B,C)$ and for any n and n-tuple A_i, the matrix $D(A_i, A_j)$ is positive definite; $I(A,B)$ can be thought of as the real part of $D(A,B)$. The way all these notions tie with the ordinary path integral is as follows :

$$D(A,B) = \int_{\gamma \in A, \chi \in B} D\gamma D\chi e^{i(S(\gamma) - S(\chi))} \delta(\gamma(T), \chi(T))$$

where T is a so called truncation time and S is the ordinary action. A constrained history A is equivalent to the insertion of a (possibly distributional) operator in the Hamiltonian formalism. In this language, there is no room for operators and Hilbert spaces (just as in the path integral language) and one needs to figure out an (objective) interpretation based upon the measure alone. Likewise, the measurement problem in quantum mechanics needs to find a translation and resolution in this language. A promising framework for such interpretation has recently been proposed by Sorkin and Gudder [9] [10] [19]. The approach I will take later on is based upon a much more sophisticated operational formalism and is likewise genuinely quantum in the sense that it does not start from a classical action principle. But the unification of the "state" and "action" in a single measure of order 3 is certainly a nice idea which is also capable of encapsulating topology change in quantum gravity, as is our formulation of the quantum laws enunciated in chapter eight. In this framework, one recognizes that "quantum mechanics" is a higher order theory than classical mechanics is which in a certain sense respects more the Platonic world because it expresses pairwise relations between measurable sets of representations. However, the above discussion also puts into doubt the universality of quantum theory as a theory of nature and a three split experiment has been devised to verify if nature does not entail higher order correlations [11]. Let me mention here that all my comments concerning quantum gravity below also apply to these higher order theories.

The traditional physicist might now object that the Hilbert space framework with a well defined Hamiltonian or a more traditional path integral point of view ensures a *unitary* dynamics or at least a unitary scattering matrix. In the above interpretational framework, there is nothing which automatically ensures unitarity and one is left with the task of constructing theories in which the breakdown of unitarity is sufficiently small such that no reasonable contradiction with observation arises [12]. The acceptance of a lack of unitarity mainly stems from two different observations : (a) unitarity is not a logical requirement to have a consistent probability interpretation (b) Hawking radiation seems to suggest a violation of unitarity in quantum gravity albeit the opinions upon that are rather divided [14] [15] [16] [17] [18]. The dynamics I am about to propose in the fourth and fifth chapter is not unitary either due to a novel implementation of the commutation relations. In the usual path integral formulation, the measure μ is split into an infinite dimensional Lebesgue measure and the exponential of the action. The Lebesgue measure does however not exist and to make it precise, one has to start with a theory on a finite lattice and take the thermodynamic and continuum limit (in the right order) later while renormalizing at the same moment [13]. The same can be understood in -say- free Klein Gordon field theory starting from the Hamiltonian Fock space quantization. It might be an instructive exercise to explicitly construct formal "field" ψ and "field momentum" π eigenstates on the Fock space and calculate their inner products. Both operators are defined in a distributional sense (as a limit of bounded operators corresponding to a momentum cutoff) and the domain $\mathcal{D}$ of ψ is defined as the set of all vectors v in Fock space such that $\lim_{L \to \infty} \psi_L(v)$ is well defined (where the ψ_L are the cutoff operators). Hence, we may define