

Two Axioms of European Classical Music and Maximum Number of Polytonalities in Music

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Abstract

Article formulates two axioms of European classical music “*Every musical piece should be within 24 tonalities, specifically 12 major and 12 minor tonalities, with some temporary slight variations allowed*” and “*Every musical piece should be written so that all instruments play in the same tonality at any given moment.*” These axioms stayed unchallenged until the approximately end of 19th century.

First axiom was challenged by impressionists’ composers Claude Debussy, Maurice Ravel and other at the end of 19 century and later in the first half of 20 century by atonal, twelve-tone and other kinds of musical systems created by Arnold Schoenberg, Alban Berg, Anton Webern and other.

The second axiom was challenged by creation of polytonal music. One of the early examples of polytonality can be found in the ballet of Igor Stravinsky "The Rite of Spring" (1913), where different instruments play in different keys simultaneously. Another influential figure in the development of polytonality was the French composer Charles Ives.

The use of polytonal technique raised the theoretical question of how rich a contemporary composer's palette can be when the single-tonality axiom is set aside. In other words, what is the theoretical maximum number of tonalities that a composer can explore within a polytonality approach? Simple calculations show that total palette of polytonal music contains $24 \times 23 \times 22 \times 21 \times \dots \times 1 = 24!$ (twenty-four factorial) or 620,448,401,733,239,439,360,000 of theoretically available combinations of tonalities. While obviously most of these combinations will result in cacophony, some tonal combinations have the potential to yield fresh and pleasant impressions that have not been explored in the past. If we consider the complete list of 9 modes per note (Major, Natural Minor, Harmonic Minor, Melodic Minor, Dorian mode, Phrygian mode, Lydian mode, Mixolydian mode, Locrian mode), we obtain $12 \text{ notes} \times 9 \text{ modes} = 108 \text{ base tonalities/modes}$. Correspondingly in that case use of polytonalities produces $108!$ (108 factorial) combinations of tonalities/modes. This number appears as the theoretical upper limit of tonal palette in polytonality approach.

1. First axiom of European classical music

Until approximately the end of the 19th century, European music was based on the axiom that “*Every musical piece should be within 24 tonalities, specifically 12 major and 12 minor tonalities, with some temporary slight variations allowed.*”

These tonalities are represented by the 12 different notes: C, C#, D, D#, E, F, F#, G, G#, A, A#, and B. Each of these notes can serve as the starting point for a tonal center, and different tonalities are created by arranging these notes in various sequences and combinations. Most popular sequences known and used today are presented below (where W represents a whole step and H represents a half step).

1. Ionian mode (Major): This is the standard major scale, with a pattern of W-W-H-W-W-W-H.
2. Aeolian mode (Natural Minor): This is the standard natural minor scale, with a pattern of W-H-W-W-H-W-W.
3. Harmonic Minor: W-H-W-W-H-WH-H.

4. Melodic Minor: W-H-W-W-W-W-H
5. Dorian mode: Characteristic minor sound with a raised sixth degree: W-H-W-W-W-H-W.
6. Phrygian mode: This mode has a distinctive minor sound with a lowered second degree: H-W-W-W-H-W-W.
7. Lydian mode: It has a major sound with a raised fourth degree: W-W-W-H-W-W-H.
8. Mixolydian mode: This mode has a dominant or bluesy sound with a lowered seventh degree: W-W-H-W-W-H-W.
9. Locrian mode: It has a diminished sound with both a lowered second and lowered fifth degree: H-W-W-H-W-W-W.

In spite of rich enough palette comprising $12 \times 9 = 108$ different tonalities and modes in classical music, between 1600 and 1900, much of the music is written in major and minor tonalities. The major and natural minor modes, derived from the diatonic major and natural minor scales, form the basis of tonal harmony in classical music.

The major tonality (Ionian mode) with a bright and happy sound are commonly used in classical compositions. Many symphonies, sonatas, and other classical forms are written in major keys.

Natural Minor Tonality (Aeolian mode) with a somewhat darker and more melancholic sound, are also prevalent in classical music. Composers often utilized the natural minor mode for expressing a range of emotions.

While major and natural minor tonalities dominate classical music, composers also explored modal mixture, modulations to related keys, and chromaticism to add color and complexity to their compositions. Additionally, during the Classical and Romantic periods, composers occasionally experimented with other modes, such as the Dorian and Mixolydian modes, but these were less common than major and natural minor tonalities.

Overall, we can safely say that before end of 19 century the 99% of European music was written in 24 tonalities - 12 major and 12 minors. Of course, while major and minor tonalities were predominant, the use of modes, chromaticism, and other harmonic devices added further complexity to the tonal language. Also, not all tonalities were used equally. Depending on musical instruments construction, singer's diapasons and skills of performers certain tonalities were used more frequently and certain – less frequently.

These 12 majors and 12 minor tonalities form the so-called “quart-quint circle”, a.k.a. "circle of fifths". Moving clockwise around the circle, each key adds one more sharp to its key signature, indicating the addition of a leading-tone note.

Axiom that all music must be composed within these 24 tonalities (with some short deviations) was challenged by impressionist composers at the end of 19 century and later in the first half of 20 century by atonal, twelve-tone and other kinds of musical systems.

The impressionist movement, led by composers such as Claude Debussy and Maurice Ravel, questioned and expanded upon traditional harmonic structures, utilizing modes, scales, and

non-traditional chord progressions. Impressionist music often sought to evoke moods and atmospheres through novel harmonic language and unique orchestration.

Following the impressionist era, the early 20th century witnessed the emergence of more radical departures from tonality with movements like atonality and the use of twelve-tone techniques. Composers like Arnold Schoenberg, Alban Berg, and Anton Webern played pivotal roles in the development of atonal and dodecaphonic (twelve-tone) music. Atonal music abandons traditional tonal centers and often lacks a sense of traditional key, while twelve-tone music employs a tone row that uses all twelve pitch classes in a specific order, aiming to avoid a tonal center.

Composers during this time explored new ways of organizing pitch, rhythm, and form, often pushing the boundaries of what was considered acceptable in the realm of Western classical music. These developments marked a departure from the strict tonal paradigm that had dominated much of classical music for centuries.

2. The second axiom of European classical music

The second axiom of European classical music "*Every musical piece should be written so that all instruments play in the same tonality at any given moment*" was challenged by so called polytonality. Early examples of polytonality include Heinrich Ignaz Franz Biber's 'Battalia' (1673) [1] and Wolfgang Amadeus Mozart's 'Ein musikalischer Spass' (1787) [2], where polytonality is employed as a tool to introduce humor and satire into their musical pieces.

The wide use of polytonality in Western classical music emerged in the early 20th century as part of the broader exploration of new harmonic languages and breaking away from traditional tonal practices. Polytonality involves the simultaneous use of two or more tonal centers or keys. Composers began experimenting with polytonality as a way to create dissonance, ambiguity, and complexity in their compositions.

The simplest example of polytonal accord is augmented major triad. While inversions of a regular triad produce tertian-quartal and quartal-sextal chords in the same key, inversions of an augmented major triad produce augmented major triads in different keys. For example, inversions of the augmented C major triad (C, E, G#) produce the augmented E major triad (E, G#, B#) and the augmented G# major triad (G#, B#, D##), or enharmonically the same augmented Ab major triad (Ab, C, E). We can say that the augmented C major triad is simultaneously an augmented E major triad and an augmented Ab major triad. Obviously, this is just the simplest example.

One of the early examples of polytonality can be found in the works of Igor Stravinsky. Stravinsky's ballet "The Rite of Spring" (1913) is often cited for its use of polytonality, where different instruments play in different keys simultaneously, creating a sense of tension and primal energy.

Another influential figure in the development of polytonality was the French composer Charles Ives. His compositions explored polytonal elements, incorporating dissonant layers to convey complex musical ideas.

Polytonality became more widespread in the works of various composers during the early and mid-20th century, including Bela Bartok, Darius Milhaud, and Sergei Prokofiev. These composers experimented with different tonalities, often using polytonality as a means of expression and expanding the harmonic palette beyond traditional tonal boundaries. In summary, polytonality emerged as a notable harmonic innovation in the early 20th century, with composers exploring the simultaneous use of multiple tonal centers to create new and challenging musical landscapes.

3. *What is the theoretical maximum number of polytonalities?*

The use of polytonal technique raised the theoretical question of how rich a contemporary composer's palette can be when single-tonality axiom is put aside. In other words, what is the theoretical maximum number of tonalities that a composer can explore within a polytonality approach?

At the moment, we are excluding consideration of atonal or serial techniques. In the following example composers are assumed to be working with classical tonalities (major and minor), but they are not bound by the axiom that "all instruments must play in the same tonality at any given moment."

1. With a single musical instrument, there is flexibility to play in any of the 24 available tonalities, encompassing both major and minor keys.
2. When we introduce a second musical instrument, it can also play in any of the 24 tonalities. However, if it plays in the same tonality as the first instrument, it does not produce a polytonal sound. Consequently, only 23 tonalities are available for the second instrument in that specific case. Considering both instruments together, a composer can utilize a palette of $24 * 23 (=552)$ tonalities.
3. When we introduce a third musical instrument, it can also play in any of the 24 tonalities; however, if it plays in the same tonality as the first or second instrument, it does not produce a polytonal sound. As a result, only 22 tonalities are available for the third instrument in that specific case. Considering all three instruments together, the composer can use a palette of $24 * 23 * 22 (=12,936)$ tonalities.
4. For the fourth musical instrument, we have only 21 tonalities available, and the palette of these four instruments includes $24 * 23 * 22 * 21 = 285,144$ possible combinations of tonalities.
5. And so on.
6.
7. The last 24th musical instrument has no choice, only one tonality is available to it since other tonalities are already taken.

Total palette of these 24 instruments contains $24 * 23 * 22 * 21 * \dots * 1 = 24!$ (twenty-four factorial) or **620,448,401,733,239,439,360,000** of theoretically available combinations of tonalities.

So far, we have only considered the 24 base tonalities presented by the Ionian mode (Major) and Aeolian mode (Natural Minor) (12 notes * 2 modes = 24 tonalities). If we expand our consideration to the full list of 9 modes mentioned above, we obtain $12 \text{ notes} * 9 \text{ modes} = 108$

base tonalities/modes. Correspondingly use of polytonalities produces $108!$ (108 factorial) combinations of tonalities/modes. This number appears as the theoretical upper limit of palette in polytonality approach.

While obviously most of these combinations will result in cacophony (chaotic arrangement of sounds that lack a pleasing or harmonious quality), some tonal combinations have the potential to yield fresh and pleasant impressions that have not been explored in the past.

The theoretical lower limit of tonalities in the palette within the polytonality approach is obviously 2. The use of just 1 tonality at any given moment would bring the composer back to the second axiom of European classical music, where all instruments must play in the same tonality.

References

1. Heinrich Ignaz Franz Biber's 'Battalia' (1673)
<https://www.youtube.com/watch?v=5YBOMgi-qSs>
2. Wolfgang Amadeus Mozart's 'Ein musikalischer Spass' (1787)
<https://www.youtube.com/watch?v=wFPoRmsiFzc>