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Mathematical Models as a Key to Effective Teaching of Set Theory at Undergraduate Level

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Abstract

The paper is dedicated to the problem of optimization of teaching set theory at undergraduate level (as a part of Mathematical Logics course). The authors' original method of using mathematical models for teaching set theory is presented. Concrete recommendations for increasing effectiveness of teaching set theory are given. The paper contains original mathematical models which were proved to promote deep understanding of the key concepts of set theory in students, as well as their ability to apply those concepts to solving real-life problems.

All conclusions and recommendations are based on the results of modern research in education and on the authors' teaching experience. The effectiveness of presented technique was proven in practical teaching at Voorhees University, South Carolina, USA and at Denmark Technical College, South Carolina, USA.

Key Words: Teaching set theory, mathematical logics, discrete mathematics, methods for increasing effectiveness of teaching math in college, learner-centered education

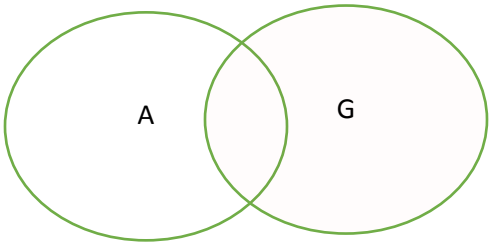
The practice of teaching mathematics shows that using mathematical models for teaching Mathematical Logic not only makes learning process more diverse and interesting to the learners but may also effectively stimulate logical and analytical thinking in students. The presented technique of teaching Set Theory is based on a learner-centered approach in education that is considered as one of the most effective instruments for improving the results of teaching

mathematics at undergraduate level [1]. This technique has the particular focus on the needs of the students who didn't have a good background in mathematics and need special help and support at college/university.

It's important to teach students how to distinguish between the possible cases of relative position of two sets (partial overlapping, no intersection, one set is the part of another set (subset of a set) [2]. To achieve this goal, firstly, some problems involving sets with numerical elements can be given (using Euler-Venn diagrams for illustration), and after enough practice is done, the teacher may proceed to real-life problems, similar to ones presented below.

Problem 1. Find (describe) the union and intersection of a set of all apples (A) and a set of all green objects (G).

Suggested answer. The sets A and G are *overlapping* (because some apples are green), so they can be graphed as follows.



The union AUG contains all objects that belong to either A or G (e.g. red apples, green notebooks, green apples). The intersection of our sets A∩G contains all elements that are common to both sets (green apples).

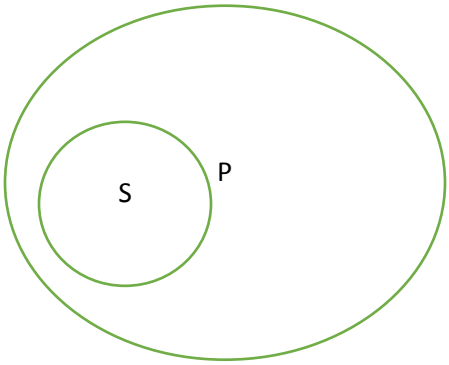
Problem 2. Find the union and intersection of a set of all cats (C) and a set of all dogs (D).

Suggested answer. An animal cannot be a cat and a dog at the same time, so the sets C and D *don't have any common elements (they are disjoint sets)*, which can be represented as a diagram below.

It's clear that C∩D=Ø (the intersection of our sets is an empty set), and the union CUD contains all objects that belong to either C or D (all cats and dogs).

Problem 3. Find the union and intersection of a set of all people (P) and a set of all students (S)

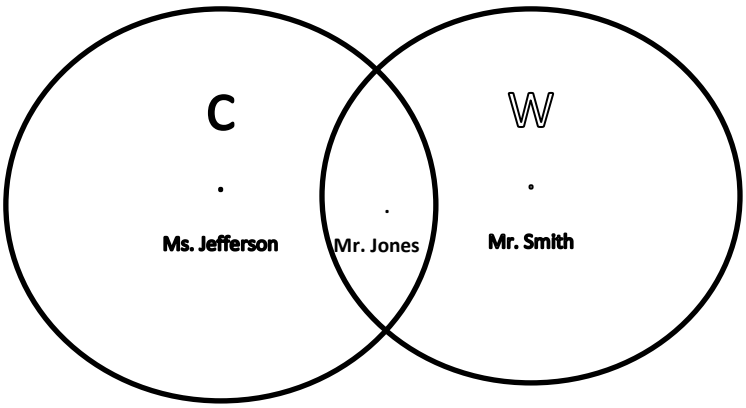
Suggested answer. The set of all students (S) is a *subset* of the set of all people (P) because all students are people. In other words, the set S in the part of the set P, the set of all students is *included* in the set of all people (S C P). With this said, we can graph our sets as following:



It's clear from the graph that SUP=P, and S∩P=S.

It's very important to help students develop skills of acquiring information form the Euler-Venn diagrams. The following models may stimulate students' ability to "read" and analyze the information presented on the diagrams.

Problem 4. Bank of America wants to hire a person with a Bachelor's degree in Computer Science who has work experience. Wells Fargo has an internship for people with a Bachelor's degree in Computer Science who don't have work experience. Given C is a set of all people with a Bachelor's degree Computer Science and W is a set of all people who have work experience find a right person for: a) Bank of America, b) Wells Fargo



Suggested answer. Let's analyze what information that is given in the diagram above.

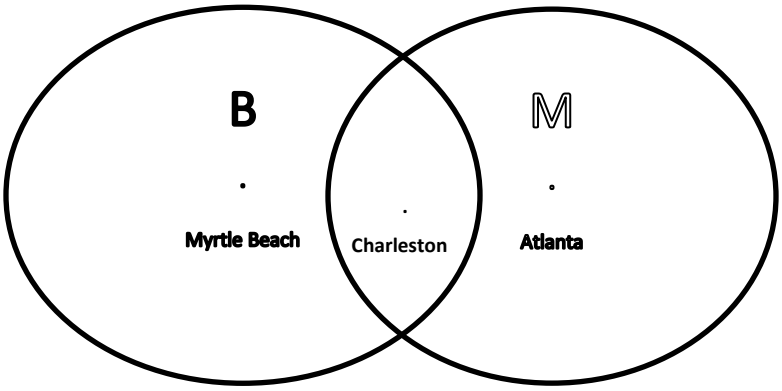
We see that Ms. Jefferson belongs to the set C only which means that she has a Bachelor's degree in Computer Science but she does NOT have work experience (because she doesn't belong to the set W).

The next person, Mr. Jones, belongs to BOTH sets C and W which means that he has a Bachelor's degree in Computer Science (because he belongs to C) and he has work experience (because he belongs to W).

And the last person, Mr. Smith, belongs to the set W only which means that he has work experience, but he does NOT have a Bachelor's degree in Computer Science (because he doesn't belong to the set C).

With this said the right person for the Bank of America is Mr. Jones and the right choice for Wells Fargo is Ms. Jefferson.

Problem 5. Patrick wants to spend his vacation at a city that has a beach and many museums. Given B is a set of all cities that have a beach and M is a set of all cities that have many museums help Patrick to make a right choice.

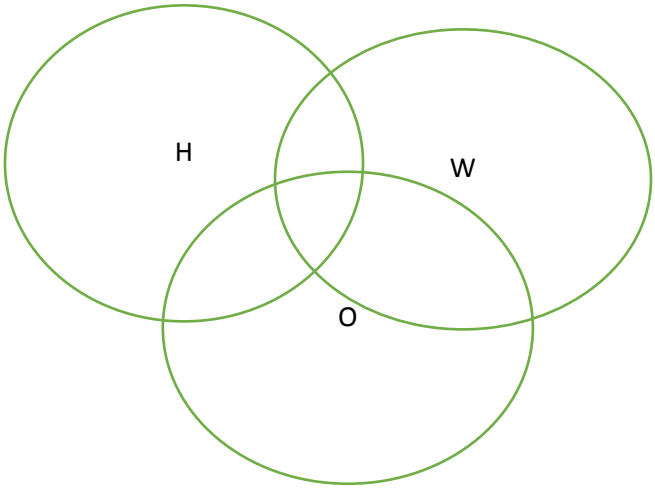


Suggested answer. According to the diagram, Myrtle Beach has a beach but it doesn't have many museums (since it belongs to the set B only), Atlanta has many museums but it doesn't have a beach (since Atlanta belongs to the set M only). Charleston belongs to the BOTH sets B and M (it's in the intersection of the sets), which means that it has both a beach and many museums, so Charleston is the right choice for Patrick.

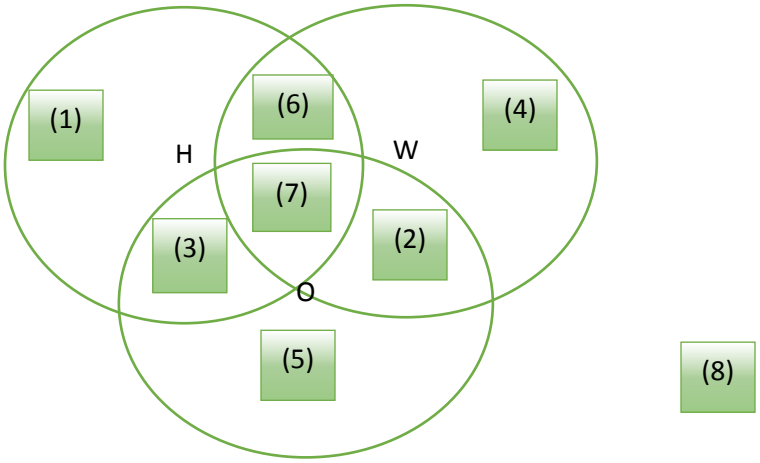
Problem 6. Given H is a set of all housekeepers, W is a set of all women and O is a set of all people who're 30 years old, place the following objects on the diagram:

1) A 20-years old housekeeper Mr. Smith
2) A 30-years old doctor Ms. Brown

3) A 30-years old housekeeper Mr. Bond
4) A 50-years old teacher Mrs. Allen
5) A 30-years old truck driver Mr. James
6) A 45-years old housekeeper Ms. Bennett
7) A 30-years old housekeeper Ms. Johnson
8) A 15-years old student Larry



Suggested answer:



Conclusion. Using mathematical models when teaching Set Theory may stimulate logical thinking and develop analytical and problem-solving skills in students. It also makes learning process more interesting and enjoyable to them. Even the students who tend to dislike mathematics, really enjoy learning through the presented technique and demonstrate significant improvement in understanding the key concepts of set theory and increase in their grades (by 34%), which is

especially important because strong math skills and an ability to think critically and logically are crucial for every learner.

References

1. Richard E. Hodel. An Introduction to Mathematical Logic, Dover Publications, 2016
2. Stephen Kleene. Mathematical Logic, Dover Publications, 2018
3. Friedberg, Solomon. Teaching Mathematics in Colleges and Universities: Case Studies for Today's Classroom/Solomon Friedberg. – Student ed. Conference Board of Mathematical Sciences, Rhode Island, 2001
4. Krantz, Steven. How to Teach Mathematics/Steven G. Krantz - 3rd edition. American Mathematical Society, Rhode Island, 2015, P. 26.