

The 3-dimensional and 2-dimensional Čerenkov effect with massive photons

Miroslav Pardy

Department of Physical Electronics and plasma physics

Masaryk University

Kotlářská 2, 611 37 Brno

Czech Republic

E-mail: pamir@physics.muni.cz

May 20, 2022

Abstract

The equations of massive electrodynamics are derived and the power spectrum formula for the 3-dimensional Čerenkov radiation of massive photons is found. It is argued that the massive Čerenkov effect can be observed in superconductive media, ionosphere plasma, waveguides and in particle laboratories. The same is valid for the Čerenkov radiation in the 2-dimensional medium including silicene. The 2D-Čerenkov effect with the massless photons was observed by the famous world laboratories (Adiv et al., 2022).

1 Introduction

The possibility that photon may be massive particle has been treated by many physicists. At the present time the great attention is devoted to discussion of the mass of neutrino and its oscillations, nevertheless theoretical problems with massive photons is of the same importance. The established fact is that the massive electrodynamics is a perfectly consistent classical and quantum field theory (Feldmann et al., 1963; Minkowski et al., 1971; Goldhaber et al. 1971). In all respect the quantum version has the same status as the standard QED. In this article we do not solve the radiative problems in sense of article by van Nieuwenhuizen (1973), our goal is to determine the Čerenkov effect (Čerenkov, 1934) of massive photons which is not in those article analyzed.

In particle physics and quantum field theory (Ryder, 1985; de Wit et al., 1986.; Commins et al. 1983) photon is defined as a massless particle with spin 1. Its spin is along or in opposite direction to its motion. The massive photon as a neutral massive particle is usually called vector boson. There are other well known examples of massive spin-1

particles. For instance neutral ϱ -meson, φ -meson and J/ψ particle, bosons $W^\pm$ and Z^0 in particle physics.

While massless photon is described by the Maxwell Lagrangian, the massive photon is described by the Proca Lagrangian from which the field equations follow. The massive electrodynamics can be considered as a generalization of massless electrodynamics. The well known area where the massive photon or boson plays substantial role is the theory of superconductivity (Ryder, 1985) plasma physics (Anderson, 1963) waveguides and so on. So, the physics of massive photon is meaningful and it means that also the Čerenkov effect with massive photons.

In order to be pedagogically clear, we treat in section 2 the massive spin 0 quantum field theory and then in section 3 the massive spin 1 field theory. In section 4 the massive Maxwell equations are derived. In that section power spectral formula is derived for the massive Čerenkov radiation for the situation with one charge moving in a medium. The 2D-Čerenkov effect is derived and discussed in the next section. The article includes the results of the author contribution (Pardy, 2015)

2 Massive spin 0 fields

We begin with the massive spin 0 fields as the most simple illustration how the source theory works (Schwinger, 1970). The action or spin 0 particles is according to source theory composed from the scalar source $K(x)$ and propagator Δ_+ in such a way that it gives the correct probability condition for the vacuum to vacuum amplitude. We show here that the action is

$$W(K) = \frac{1}{2} \int (dx)(dx') K(x) \Delta_+(x - x') K(x'), \quad (1)$$

and gives the right probability condition $|\langle 0_+ | 0_- \rangle|^2 \leq 1$, (Schwinger, 1970; Schwinger et al., 1976), where $(\hbar = 1)$

$$\langle 0_+ | 0_- \rangle^K = e^{iW(K)} \quad (2)$$

is the basic formula of the Schwinger source theory with $\langle 0_+ | 0_- \rangle$ being the vacuum to vacuum amplitude.

In order to prove that the quantity $\langle 0_+ | 0_- \rangle$ is really the vacuum to vacuum amplitude it is necessary to know the explicit form of the Green function $\Delta_+(x - x')$ which satisfies to equation

$$(-\partial^2 + m^2) \Delta_+(x - x') = \delta(x - x'). \quad (3)$$

From the last eq. follows that

$$\Delta_+(x - x') = \frac{1}{(-\partial^2 + m^2)} \int \frac{(dp)}{(2\pi)^4} e^{ip(x-x')}. \quad (4)$$

The formula (4) is not unambiguos and it is necessary to specify it by the ε -term, or,

$$\Delta_+(x - x') = \int \frac{(dp)}{(2\pi)^4} \frac{e^{ip(x-x')}}{p^2 + m^2 - i\epsilon}; \quad \epsilon \rightarrow 0_+. \quad (5)$$

Now, let us proof that $|\langle 0_+|0_- \rangle|^2$ is the probability of the persistence of vacuum. According to definition

$$\langle 0_+|0_- \rangle = \exp \left\{ \frac{i}{2} \int (dx)(dx') K(x) \Delta_+(x-x') K(x') \right\}, \quad (6)$$

we have

$$\begin{aligned} \langle 0_+|0_- \rangle &= \exp \left\{ \frac{i}{2} \int (dx)(dx') \int \frac{(dp)}{(2\pi)^4} K(x) \frac{e^{ip(x-x')}}{p^2 + m^2 - i\epsilon} K(x') \right\} = \\ &= \exp \left\{ \frac{i}{2} \int \frac{(dp)}{(2\pi)^4} \frac{K(p)K(-p)}{p^2 + m^2 - i\epsilon} \right\} = \exp \left\{ \frac{i}{2} \int \frac{(dp)}{(2\pi)^4} \frac{|K(p)|^2}{p^2 + m^2 - i\epsilon} \right\} \end{aligned} \quad (7)$$

as a consequence of eq. (5) and $K^*(p) = K(-p)$. Using the well known theorem

$$\frac{1}{x - i\epsilon} = P \left(\frac{1}{x} \right) + i\pi\delta(x); \quad \epsilon \rightarrow 0, \quad (8)$$

where P denotes the principal value of integral we get the following formula for the vacuum persistence:

$$|\langle 0_+|0_- \rangle|^2 = e^{-2\text{Im } W} = \exp \left\{ -2 \int \frac{(dp)}{(2\pi)^4} \pi |K(p)|^2 \delta(p^2 + m^2) \right\}. \quad (9)$$

Using

$$\delta(p^2 + m^2) = \frac{1}{2(\mathbf{p}^2 + m^2)^{1/2}} \left\{ \delta(p^0 - (\mathbf{p}^2 + m^2)^{1/2}) + \delta(p^0 + (\mathbf{p}^2 + m^2)^{1/2}) \right\}, \quad (10)$$

we get

$$2 \int \frac{(dp)}{(2\pi)^4} \pi |K(p)|^2 \delta(p^2 + m^2) = \int \frac{(d\mathbf{p})}{(2\pi)^3} \frac{1}{2p^0} |K(p^0, \mathbf{p})|^2, \quad (11)$$

and then,

$$|\langle 0_+|0_- \rangle|^2 = \exp \left\{ - \int d\omega_p |K(p)|^2 \right\}, \quad (12)$$

where

$$d\omega_p = \frac{(d\mathbf{p})}{(2\pi)^3} \frac{1}{2p^0} \quad p^0 = +(\mathbf{p}^2 + m^2)^{1/2}. \quad (13)$$

The expression (12) shows that in the presence of the scalar source $K(x)$ the probability for vacuum to remain a vacuum is equal or less than 1.

Now, let us show the derivation of the field equation from the action W for the scalar field φ , where

$$W = \frac{1}{2} \int K \Delta_+ K = \frac{1}{2} \int \varphi K = \frac{1}{2} \int \varphi (-\partial^2 + m^2) \varphi = \frac{1}{2} \int (\partial_\mu \varphi \partial^\mu \varphi + m^2 \varphi^2) =$$

$$2W - W = \int \varphi K - \frac{1}{2}[(\partial\varphi)^2 + m^2\varphi^2] = \int (dx) [K(x)\varphi(x) + \mathcal{L}(\varphi(x))] \quad (14)$$

with

$$\mathcal{L}(\varphi(x)) = -\frac{1}{2} [\partial_\mu \varphi \partial^\mu \varphi + m^2 \varphi^2]. \quad (15)$$

Let us put

$$\delta_\varphi W = 0, \quad (16)$$

or,

$$\int \delta\varphi K - [\partial_\mu \varphi \partial^\mu \delta\varphi + m^2 \varphi \delta\varphi] = 0. \quad (17)$$

After some modification we get

$$\int (dx) [K - (-\partial_\mu \partial^\mu \varphi + m^2 \varphi)] \delta\varphi = 0. \quad (18)$$

As variable φ is an arbitrary one the last integral is equal to zero only if

$$(-\partial^2 + m^2) \varphi(x) = K(x), \quad (19)$$

which is the Klein-Gordon equation with source $K(x)$ on the right side of equation. Now, let us derive the Proca equation for massive particles with spin 1 and generate the Maxwell equations for massive photons.

3 Massive fields with spin 1

We show the natural construction of the field of the particles with spin 1. The derivation of the action for this massive spin 1 fields is based on the modification of the derivation of spin 0 fields.

The relation

$$|\langle 0_+ | 0_- \rangle|^2 = \exp \{-2\text{Im } W\} \leq 1 \quad (20)$$

is postulated to be valid for all spin fields. Let us show here the construction of action and field equations concerning spin one.

If spin zero particles and fields are described by the scalar source, then a vector source denoted here as $J^\mu(x)$ can be considered as a candidate for the description of the spin 1 fields and particles. However, there exist some obstacles because source $J^\mu(x)$ has four components and spin one particles have only three spin possibilities. Nevertheless first, let us investigate by analogy with the spin zero fields the following form of the action for the unit spin fields:

$$W(J) = \frac{1}{2} \int (dx)(dx') J^\mu(x) \Delta_+(x - x') J_\mu(x'). \quad (21)$$

Then,

$$|\langle 0_+ | 0_- \rangle|^2 = e^{iW} e^{-iW^*} = \exp \left\{ - \int d\omega_p J^{*\mu}(p) J_\mu(p) \right\}. \quad (22)$$

However,

$$J^{*\mu}(p) J_\mu(p) = |\mathbf{J}(p)|^2 - |J^0(p)|^2 \leq 0, \quad \text{or, } > 0 \quad (23)$$

and it means that the quantity defined by eq. (21) cannot be considered as the probability of the persistence of vacuum.

The difficulty can be overcome by replacing the original form $J^{*\mu}(x) J_\mu(x)$ by the following invariant structure:

$$J^{*\mu}(p) \left[g_{\mu\nu} + \frac{1}{m^2} p_\mu p_\nu \right] J^\nu(p), \quad (24)$$

which can be with regard to its invariance, determined in the rest frame of the time-like vector p^μ , where $p^\mu = (m, 0, 0, 0)$ in the rest frame. Then, with $g_{\alpha\alpha} = (-1, 1, 1, 1)$ and $g_{\mu\nu} = 0$ for $\mu \neq \nu$ we have

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + \frac{1}{m^2} p_\mu p_\nu = \begin{cases} \delta_{kl}; & \mu = k; \quad \nu = l \\ 0; & \mu = 0; \quad \nu = 0 \\ 0; & \mu = k; \quad \nu = 0 \end{cases} \quad (25)$$

and

$$J^{*\mu}(p) \bar{g}_{\mu\nu} J^\nu(p) \equiv |\mathbf{J}|^2 \quad (26)$$

and now the quantity $|\langle 0_+ | 0_- \rangle|^2$ can be interpreted as the vacuum persistence probability.

At the same time $|\mathbf{J}|^2$ contains three independent source components, transforming among themselves under spatial rotation, as it is appropriate to unit spin.

After using eq. (24) it may be easy to get $W(J)$ in the space-time representation by the Fourier transformation, as it follows:

$$W(J) = \frac{1}{2} \int (dx)(dx') \left\{ J_\mu(x) \Delta_+(x - x') J^\mu(x') + \frac{1}{m^2} \partial_\mu J^\mu(x) \Delta_+(x - x') \partial'_\nu J^\nu(x') \right\}. \quad (27)$$

The field of spin one particles can be defined using the definition of the test source $\delta J^\mu(x)$ by the relation

$$\delta W(J) = \int (dx) \delta J^\mu(x) \varphi_\mu(x), \quad (28)$$

where φ_μ is the field of particles with spin 1. After performing variation of the formula (27) and comparison with eq. (28) we get the equation for field of spin 1 in the following form:

$$\varphi_\mu(x) = \int (dx') \Delta_+(x - x') J_\mu(x') - \frac{1}{m^2} \partial_\mu \int (dx') \Delta_+(x - x') \partial'_\nu J^\nu(x'). \quad (29)$$

The divergence of the vector field $\varphi_\mu(x)$ is given by the relation

$$\partial_\mu \varphi^\mu(x) = \int (dx') \Delta_+(x - x') \partial'_\mu J^\mu(x') - \frac{1}{m^2} \partial^2 \int (dx') \Delta_+(x - x') \partial'_\nu J^\nu(x') =$$

$$\frac{1}{m^2} \partial_\mu J^\mu(x), \quad (30)$$

as a cosequence of eq. (3), or,

$$-\partial^2 \Delta_+ = \delta(x - x') - m^2 \Delta_+. \quad (31)$$

Further, we have after applying operator $(-\partial^2 + m^2)$ on the equation (29) the following equations:

$$(-\partial^2 + m^2) \varphi_\mu(x) = J_\mu(x) - \frac{1}{m^2} \partial_\mu \partial_\nu J^\nu(x), \quad (32)$$

$$(-\partial^2 + m^2) \varphi_\mu(x) + \partial_\mu \partial_\nu \varphi^\nu(x) = J_\mu(x), \quad (33)$$

as a consequence of eq. (30).

It may be easy to cast the last equation into the following form

$$\partial^\nu G_{\mu\nu} + m^2 \varphi_\mu = J_\mu, \quad (34)$$

where

$$G_{\mu\nu}(x) = -G_{\nu\mu}(x) = \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu. \quad (35)$$

Identifying $G_{\mu\nu}$ with $F_{\mu\nu}$ of the electromagnetic field we get instead of eq. (33) and eq. (34) so called the Proca equation for the electromagnetic field with the massive photon.

$$(-\partial^2 + m^2) A_\mu(x) + \partial_\mu \partial_\nu A^\nu(x) = J_\mu(x), \quad (36)$$

$$\partial^\nu F_{\mu\nu} + m^2 A_\mu = J_\mu, \quad (37)$$

$$F_{\mu\nu}(x) = -F_{\nu\mu}(x) = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (38)$$

In case $m^2 \neq 0$, we can put $\partial_\mu A^\mu = 0$ in order to get:

$$(-\partial^2 + m^2) A_\mu(x) = 0, \quad \partial_\mu A^\mu = 0. \quad (39)$$

The solution of the system (39) is the plane wave

$$A_\mu = \varepsilon_\mu(\mathbf{k}) e^{ikx}, \quad k^2 = -m^2 \quad (40)$$

with $k\varepsilon(\mathbf{k}) = 0$, which is precisely the correct definition of a massive particle with spin 1. We will see in the next section how to generalize this procedure to the situation of the massive electrodynamics in dielectric and magnetic media and then to apply it to the determination of the massive Čerenkov radiation.

The equation (34) can be derived also from the action

$$W = \int (dx) (J^\mu(x) \varphi_\mu(x) + \mathcal{L}(\varphi(x))), \quad (41)$$

where

$$\mathcal{L} = -\frac{1}{2} \left(\frac{1}{2} (\partial^\mu \varphi^\nu - \partial^\nu \varphi^\mu) (\partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu) + m^2 \varphi^\mu \varphi_\mu \right), \quad (42)$$

where we have used the arrangement

$$\int (dx) \varphi^\mu (-\partial^2) \varphi_\mu = \int (dx) \partial^\nu \varphi^\mu \partial_\nu \varphi_\mu \quad (43)$$

and

$$\int (dx) \varphi^\mu \partial_\mu \partial^\nu \varphi_\nu = - \int (dx) \varphi^\nu \partial^\mu \partial_\mu \varphi_\nu = - \int (dx) \varphi_\mu \partial^\mu \partial^\nu \varphi_\nu. \quad (44)$$

Using the last equation (44) we get the Lagrange function in the following standard form:

$$\mathcal{L} = -\frac{1}{2} \left(\partial^\nu \varphi^\mu \partial_\nu \varphi_\mu - (\partial_\mu \varphi^\mu)^2 + m^2 \varphi^\mu \varphi_\mu \right). \quad (45)$$

If we use the A- and F-symbols, we receive from eq. (42) the Proca Lagrangian

$$\mathcal{L} = -\frac{1}{2} \left(\frac{1}{2} F^{\mu\nu} F_{\mu\nu} + m^2 A^\mu A_\mu \right), \quad (46)$$

or,

$$\mathcal{L} = -\frac{1}{2} \left(\partial^\nu A^\mu \partial_\nu A_\mu - (\partial_\mu A^\mu)^2 + m^2 A^\mu A_\mu \right). \quad (47)$$

By variation of the corresponding Lagrangians for the massive field with spin 1 we get evidently the massive Maxwell equations.

It is evident that the zero mass limit does not exist for $\partial_\mu J^\mu(x) \neq 0$. In such a way we are forced to redefine action $W(J)$. One of the possibilities is to put

$$\partial_\mu J^\mu(x) = mK(x) \quad (48)$$

and identify $K(x)$ in the limit $m \rightarrow 0$ with the source of massless spin zero particles. Since the zero mass particles with zero spin are experimentally unknown in any event, we take $K(x) = 0$ and we write

$$W_{[m=0]}(J) = \frac{1}{2} \int (dx)(dx') J_\mu(x) D_+(x-x') J^\mu(x'), \quad (49)$$

where

$$\partial_\mu J^\mu(x) = 0 \quad (50)$$

and

$$D_+(x-x') = \Delta_+(x-x'; m=0). \quad (51)$$

In case we want to work with electrodynamics in medium it is necessary to involve such parameters as velocity of light c magnetic permeability μ and the dielectric constant ε . Then the corresponding equations for electromagnetic potentials which are compatible with the Maxwell equations are as follows (Schwinger et al., 1976):

$$\left(\Delta - \frac{\mu\epsilon}{c^2} \frac{\partial^2}{\partial t^2}\right) A^\mu = \frac{\mu}{c} \left(g^{\mu\nu} + \frac{n^2 - 1}{n^2} \eta^\mu \eta^\nu\right) J_\nu, \quad (52)$$

where the corresponding Lorentz gauge is defined in the Schwinger et al. article in the following form

$$\partial_\mu A^\mu - (\mu\epsilon - 1)(\eta\partial)(\eta A) = 0, \quad (53)$$

where $\eta^\mu = (1, \mathbf{0})$ is the unit timelike vector in the rest frame of the medium. The four-potentials are $A^\mu(\phi, \mathbf{A})$ and the four-current $J^\mu(c\rho, \mathbf{J})$, n is the index of refraction of this medium.

The corresponding Green function $D_{+\mu\nu}$ in the x -representation is:

$$D_+^{\mu\nu}(x - x') = \frac{\mu}{c} \left(g^{\mu\nu} + \frac{n^2 - 1}{n^2} \eta^\mu \eta^\nu\right) D_+(x - x'). \quad (54)$$

$D_+(x - x')$ was derived by Schwinger et al. (1976) as follows:

$$D_+(x - x') = \int \frac{(dk)}{(2\pi)^4} \frac{e^{ik(x-x')}}{|\mathbf{k}^2| - n^2(k^0)^2 - i\epsilon}, \quad (55)$$

Or,

$$D_+(x - x') = \frac{i}{c} \frac{1}{4\pi^2} \int_0^\infty d\omega \frac{\sin \frac{n\omega}{c} |\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} e^{-i\omega|t-t'|}. \quad (56)$$

4 Massive photon in electrodynamics and the Čerenkov effect

The massive electrodynamics in medium can be constructed by generalization of massless electrodynamics to the case with massive photon. In our case it means that we replace only eq. (52) by the following one:

$$\left(\Delta - \frac{\mu\epsilon}{c^2} \frac{\partial^2}{\partial t^2} + \frac{m^2 c^2}{\hbar^2}\right) A^\mu = \frac{\mu}{c} \left(g^{\mu\nu} + \frac{n^2 - 1}{n^2} \eta^\mu \eta^\nu\right) J_\nu, \quad (57)$$

where m is mass of photon. The Lorentz gauge (53) is conserved also in the massive situation.

In superconductivity photon is a massive spin 1 particle as a consequence of a broken symmetry of the Landau-Ginzburg Lagrangian. The Meissner effect can be used as a experimental demonstration that photon in a superconductor is a massive particle. In particle physics the situation is analogous to the situation in superconductivity. The masses of particles are also generated by the broken symmetry or in other words by the Higgs mechanism. Massive particles with spin 1 form the analogue of the massive photon.

Kirzhnits and Linde proposed a qualitative analysis wherein they indicated that, as in the Ginzburg-Landau theory of superconductivity, the Meissner effect can also be realized in the Weinberg model. Later, it was shown that the Meissner effect is realizable

in renormalizable gauge fields and also in the Weinberg model (Kirzhnits et al., 1972; Yildiz, 1977) .

We concentrate in this article to the Čerenkov radiation with massive photons. The so called Čerenkov radiation was observed experimentally first by Čerenkov (1934) and theoretically explained by Tamm and Frank (1937) in classical electrodynamics as a shock wave resulting from a charged particle moving through a material faster than the velocity of light in the material. The source theory explanation was given by Schwinger et al. (1976) and the particle production by the Čerenkov mechanism was discussed by Pardy (1983a; 1983b). The Čerenkov effect at finite temperature in source theory was discussed by Pardy (1989; 1995) and the Čerenkov effect with radiative corrections, in electromagnetism and gravity was analyzed by Pardy (1994a; 1994b).

We will investigate how the spectrum of the Čerenkov radiation is modified if we suppose the massive photons are generated instead of massless photons. The derived results form an analogue of the situation with massless photons. According to Pardy (1989; 1995; 1994a; 1994b) and Dittrich (1978) with the analogy of the massless photon propagator $D(k)$ in the momentum representation

$$D(k) = \frac{1}{|\mathbf{k}|^2 - n^2(k^0)^2 - i\epsilon}, \quad (58)$$

the massive photon propagator is of the form (here we introduce $\hbar$ and c):

$$D(k, m^2) = \frac{1}{|\mathbf{k}|^2 - n^2(k^0)^2 + \frac{m^2 c^2}{\hbar^2} - i\epsilon}, \quad (59)$$

where this propagator is derived from an assumption that the photon energetic equation is

$$|\mathbf{k}|^2 - n^2(k^0)^2 = -\frac{m^2 c^2}{\hbar^2}, \quad (60)$$

where n is the parameter of the medium and m is mass of photon in this medium.

From eq. (60) the dispersion law for the massive photons follows:

$$\omega = \frac{c}{n} \sqrt{k^2 + \frac{m^2 c^2}{\hbar^2}}. \quad (61)$$

Let us remark here that such dispersion law is valid not only for the massive photon but also for electromagnetic field in waveguides and electromagnetic field in ionosphere. It means that the corresponding photons are also massive and the theory of massive photons is physically meaningful. It means that also the Čerenkov radiation of massive photons is physically meaningful and it is worthwhile to study it.

The validity of eq. (60) can be verified using very simple idea that for $n = 1$ the Einstein equation for mass and energy has to follow. Putting $\mathbf{p} = \hbar\mathbf{k}$, $\hbar k^0 = \hbar(\omega/c) = (E/c)$, we get the Einstein energetic equation

$$E^2 = \mathbf{p}^2 c^2 + m^2 c^4. \quad (62)$$

The propagator for the massive photon is then derived as

$$D_+(x - x', m^2) = \frac{i}{c} \frac{1}{4\pi^2} \int_0^\infty d\omega \frac{\sin\left[\frac{n^2\omega^2}{c^2} - \frac{m^2c^2}{\hbar^2}\right]^{1/2} |\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} e^{-i\omega|t-t'|}. \quad (63)$$

The function (63) differs from the the original function D_+ by the factor

$$\left(\frac{\omega^2 n^2}{c^2} - \frac{m^2 c^2}{\hbar^2}\right)^{1/2}. \quad (64)$$

From eq. (56) and (63) the potentials generated by the massless or massive photons respectively follow. In case of the massless photon, the potential is according to Schwinger defined by the formula:

$$V(\mathbf{x} - \mathbf{x}') = \int_{-\infty}^\infty d\tau D_+(\mathbf{x} - \mathbf{x}', \tau) = \int_{-\infty}^\infty d\tau \left\{ \frac{i}{c} \frac{1}{4\pi^2} \int_0^\infty d\omega \frac{\sin \frac{n\omega}{c} |\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} e^{-i\omega|\tau|} \right\}. \quad (65)$$

The τ -integral can be evaluated using the mathematical formula

$$\int_{-\infty}^\infty d\tau e^{-i\omega|\tau|} = \frac{2}{i\omega} \quad (66)$$

and the ω -integral can be evaluated using the formula

$$\int_0^\infty \frac{\sin ax}{x} dx = \frac{\pi}{2}, \quad \text{for } a > 0. \quad (67)$$

After using eqs. (66) and (67), we get

$$V(\mathbf{x} - \mathbf{x}') = \frac{1}{c} \frac{1}{4\pi} \frac{1}{|\mathbf{x} - \mathbf{x}'|}. \quad (68)$$

In case of the massive photon, the mathematical determination of potential is the analogical to the massless situation only with the difference we use the propagator (63) and the table integral (Gradshteyn et al., 1965):

$$\int_0^\infty \frac{dx}{x} \sin(p\sqrt{x^2 - u^2}) = \frac{\pi}{2} e^{-pu}. \quad (69)$$

Using this integral we get that the potential generated by the massive photons is

$$V(\mathbf{x} - \mathbf{x}', m^2) = \frac{1}{c} \frac{1}{4\pi} \frac{\exp\left\{-\frac{mc n}{\hbar} |\mathbf{x} - \mathbf{x}'|\right\}}{|\mathbf{x} - \mathbf{x}'|}. \quad (70)$$

If we compare the potentials concerning massive and massless photons, we can deduce that also Čerenkov radiation with massive photons can be generated. So, the determination of the Čerenkov effect with massive photons is physically meaningful.

In case of the massive electromagnetic field in the medium, the action W is given by the following formula:

$$W = \frac{1}{2c^2} \int (dx)(dx') J^\mu(x) D_{+\mu\nu}(x - x', m^2) J^\nu(x'), \quad (71)$$

where

$$D_+^{\mu\nu} = \frac{\mu}{c} [g^{\mu\nu} + (1 - n^{-2})\eta^\mu \eta^\nu] D_+(x - x', m^2), \quad (72)$$

where $\eta^\mu \equiv (1, \mathbf{0})$, $J^\mu \equiv (c\rho, \mathbf{J})$ is the conserved current, μ is the magnetic permeability of the medium, ϵ is the dielectric constant of the medium and $n = \sqrt{\epsilon\mu}$ is the index of refraction of the medium.

The probability of the persistence of vacuum follows from the vacuum amplitude (2) in the following form:

$$|\langle 0_+ | 0_- \rangle|^2 = e^{-\frac{2}{\hbar} \text{Im} W}, \quad (73)$$

where $\text{Im} W$ is the basis for the definition of the spectral function $P(\omega, t)$ as follows:

$$-\frac{2}{\hbar} \text{Im} W \stackrel{d}{=} - \int dt d\omega \frac{P(\omega, t)}{\hbar \omega}. \quad (74)$$

Now, if we insert eq. (72) into eq. (71), we get after extracting $P(\omega, t)$ the following general expression for this spectral function:

$$P(\omega, t) = -\frac{\omega}{4\pi^2} \frac{\mu}{n^2} \int d\mathbf{x} d\mathbf{x}' dt' \left[\frac{\sin\left[\frac{n^2\omega^2}{c^2} - \frac{m^2c^2}{\hbar^2}\right]^{1/2} |\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} \right] \times \\ \cos[\omega(t - t')] [\varrho(\mathbf{x}, t) \varrho(\mathbf{x}', t') - \frac{n^2}{c^2} \mathbf{J}(\mathbf{x}, t) \cdot \mathbf{J}(\mathbf{x}', t')]. \quad (75)$$

Now, let us apply the formula (75) in order to get the Čerenkov distribution of massive photons. The Čerenkov radiation is produced by charged particle of charge Q moving at a constant velocity $\mathbf{v}$. In such a way we can write for the charge density and for the current density:

$$\varrho = Q\delta(\mathbf{x} - \mathbf{v}t), \quad \mathbf{J} = Q\mathbf{v}\delta(\mathbf{x} - \mathbf{v}t). \quad (76)$$

After insertion of eq. (76) into eq. (75), we get ($v = |\mathbf{v}|$).

$$P(\omega, t) = \frac{Q^2}{4\pi^2} \frac{v\mu\omega}{c^2} \left(1 - \frac{1}{n^2\beta^2}\right) \int_{-\infty}^{\infty} \frac{d\tau}{\tau} \sin \left(\left[\frac{n^2\omega^2}{c^2} - \frac{m^2c^2}{\hbar^2} \right]^{1/2} v\tau \right) \cos \omega\tau, \quad (77)$$

where we have put $\tau = t' - t, \beta = v/c$.

For $P(\omega, t)$, the situation leads to evaluation of the τ -integral. For this integral we have:

$$\int_{-\infty}^{\infty} \frac{d\tau}{\tau} \sin \left(\left[\frac{n^2\omega^2}{c^2} - \frac{c^2}{m^2} \right]^{1/2} v\tau \right) \cos \omega\tau = \begin{cases} \pi, & 0 < m^2 < \frac{\omega^2}{c^2 v^2} (n^2\beta^2 - 1) \\ 0, & m^2 > \frac{\omega^2}{c^2 v^2} (n^2\beta^2 - 1). \end{cases} \quad (78)$$

From eq. (78) immediately follows that $m^2 > 0$ implies the Čerenkov threshold $n\beta > 1$. From eq. (77) and (78) we get the spectral formula of the Čerenkov radiation of massive photons in the form:

$$P(\omega, t) = \frac{Q^2}{4\pi} \frac{v\omega\mu}{c^2} \left(1 - \frac{1}{n^2\beta^2}\right) \quad (79)$$

for

$$\omega > \frac{mcv}{\hbar} \frac{1}{\sqrt{n^2\beta^2 - 1}} > 0, \quad (80)$$

and $P(\omega, t) = 0$ for

$$\omega < \frac{mcv}{\hbar} \frac{1}{\sqrt{n^2\beta^2 - 1}}. \quad (81)$$

Using the dispersion law (61) we can write the power spectrum $P(\omega)$ as a function dependent on k^2 . Then,

$$P(k^2) = \frac{Q^2}{4\pi} \frac{v\mu}{nc} \sqrt{k^2 + \frac{m^2c^2}{\hbar^2}} \left(1 - \frac{1}{n^2\beta^2}\right); \quad k^2 > \frac{m^2c^2}{\hbar^2} \frac{1}{n^2\beta^2 - 1} \quad (82)$$

and $P(\omega, t) = 0$ for $k^2 < (m^2c^2/\hbar^2)(n^2\beta^2 - 1)^{-1}$.

The most simple way how to get the angle Θ between vectors $\mathbf{k}$ and $\mathbf{p}$ is the use the conservation laws for an energy and momentum.

$$E - \hbar\omega = E', \quad (83)$$

$$\mathbf{p} - \hbar\mathbf{k} = \mathbf{p}', \quad (84)$$

where E and E' are energies of a moving particle before and after act of emission of a photon with energy $\hbar\omega$ and momentum $\hbar\mathbf{k}$, and $\mathbf{p}$ and $\mathbf{p}'$ are momenta of the particle before and after emission of the same photon.

If we raise the equations (83) and (84) to the second power and take the difference of these quadratic equations, we can extract the $\cos \Theta$ in the form:

$$\cos \Theta = \frac{1}{n\beta} \left(1 + \frac{m^2c^2}{\hbar^2k^2}\right)^{1/2} + \frac{\hbar k}{2p} \left(1 - \frac{1}{n^2}\right) - \frac{m^2c^2}{2n^2p\hbar k}, \quad (85)$$

which has the correct massless limit. The massless limit also gives the sense of the parameter n which is introduced in the massive situation. We also observe that while in the massless situation the angle of emission depends only on $n\beta$, in case of massive situation it depends also on the wave vector k . It means that the emission of the massive photons are emitted by the Čerenkov mechanism in all space directions. So, in experiment the Čerenkov production of massive photons can be strictly distinguished from the Čerenkov production of massless photons or from the hard production of spin 1 massive particles.

5 2D Čerenkov effect with Massive photon

5.1 2D Čerenkov effect with massless photon

In case of the the two-dimensional Čerenkov radiation by source theory formulation, the form of equations is the same with the difference that $\eta^\mu \equiv (1, \mathbf{0})$ has two space

components, or $\eta^\mu \equiv (1, 0, 0)$, and the Green function D_+ as the propagator must be determined by the two-dimensional procedure. In other words, the Fourier form of this propagator is with $(dk) = dk^0 d\mathbf{k} = dk^0 dk^1 dk^2 = dk^0 k dk d\theta$

$$D_+(x - x') = \int \frac{(dk)}{(2\pi)^3} \frac{1}{\mathbf{k}^2 - n^2(k^0)^2} e^{ik(x-x')}, \quad (86)$$

or, with $R = |\mathbf{x} - \mathbf{x}'|$

$$D_+(x - x') = \frac{1}{(2\pi)^3} \int_0^{2\pi} d\theta \int_0^\infty k dk \int_{-\infty}^\infty \frac{d\omega}{c} \frac{e^{ikR \cos \theta - i\omega(t-t')}}{k^2 - \frac{n^2 \omega^2}{c^2} - i\varepsilon}. \quad (87)$$

Using $\exp(ikR \cos \theta) = \cos(kR \cos \theta) + i \sin(kR \cos \theta)$ and $(z = kR)$

$$\cos(z \cos \theta) = J_0(z) + 2 \sum_{n=1}^\infty (-1)^n J_{2n}(z) \cos 2n\theta \quad (88)$$

and

$$\sin(z \cos \theta) = \sum_{n=1}^\infty (-1)^n J_{2n-1}(z) \cos(2n-1)\theta, \quad (89)$$

where $J_n(z)$ are the Bessel functions (Kuznetsov. 1962), we get after integration over θ :

$$D_+(x - x') = \frac{1}{(2\pi)^2} \int_0^\infty k dk \int_{-\infty}^\infty \frac{d\omega}{c} \frac{J_0(kR)}{k^2 - \frac{n^2 \omega^2}{c^2} - i\varepsilon} e^{-i\omega(t-t')}. \quad (90)$$

The ω -integral in (90) can be performed using the residuum theorem after integration in the complex half ω -plane.

The result of such integration is the propagator D_+ in the following form:

$$D_+(x - x') = \frac{i}{2\pi c} \int_0^\infty d\omega J_0\left(\frac{n\omega}{c} |\mathbf{x} - \mathbf{x}'|\right) e^{-i\omega|t-t'|}. \quad (91)$$

The spectral formula for the two-dimensional Čerenkov radiation is the analogue of the formula (77), where the charge density and current involves only two-dimensional velocities and integration is also only two-dimensional.

The difference is in the replacing mathematical formulas as follows:

$$\frac{\sin \frac{n\omega}{c} |\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} \longrightarrow J_0\left(\frac{n\omega}{c} |\mathbf{x} - \mathbf{x}'|\right). \quad (92)$$

So, instead of eq. (77), we get:

$$P(\omega, t) = \frac{e^2}{2\pi} \frac{\mu\omega v}{c^2} \left(1 - \frac{1}{n^2 \beta^2}\right) \int dt' J_0\left(\frac{n\omega}{c} |t - t'|\right) \cos[\omega(t - t')], \quad \beta = v/c, \quad (93)$$

where the t' -integration must be performed. Putting $\tau = t' - t$, we get the final formula:

$$P(\omega, t) = \frac{e^2}{2\pi} \frac{\mu\omega v}{c^2} \left(1 - \frac{1}{n^2 \beta^2}\right) \int_{-\infty}^\infty d\tau J_0(n\beta\omega\tau) \cos(\omega\tau), \quad \beta = v/c. \quad (94)$$

The integral in formula (94) is involved in the tables of integrals (Gradshteyn et al., 1963). Or,

$$J = \int_0^\infty dx J_0(ax) \cos(bx) = \frac{1}{\sqrt{a^2 - b^2}}, \quad 0 < b < a,$$

$$J = \infty, \quad a = b; \quad J = 0, \quad 0 < a < b, \quad (95)$$

In our case we have $a = n\beta\omega$ and $b = \omega$. So, the power spectrum in eq. (94) is as follows with $J_0(-z) = J_0(z)$:

$$P = \frac{e^2}{\pi} \frac{\mu v}{c^2} \left(1 - \frac{1}{n^2 \beta^2}\right) \frac{2}{\sqrt{n^2 \beta^2 - 1}}, \quad n\beta > 1, \quad \beta = v/c. \quad (96)$$

and

$$P = 0; \quad n\beta < 1, \quad (97)$$

which means that the physical meaning of the quantity P is really the Čerenkov radiation. And it is in our case the two-dimensional form of this radiation. The 2D-Čerenkov effect with the massless photons in the Mach cone was observed by the famous world laboratories (Adiv et al., 2022)

5.2 2D Čerenkov effect with Massive photon

In this case we write with regard to the formula (63) and (64) that the crucial formula for the derivation is the transition

$$\frac{\sin \left[\frac{n^2 \omega^2}{c^2} - \frac{m^2 c^2}{\hbar^2} \right]^{1/2} |\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} \longrightarrow J_0 \left(\left[\frac{n^2 \omega^2}{c^2} - \frac{M^2 c^2}{\hbar^2} \right]^{1/2} |\mathbf{x} - \mathbf{x}'| \right) \quad (98)$$

It means that after some operations we get instead the formula (34) the following radiative correction caused by the massive term:

$$P(\omega, t) = \frac{e^2}{4\pi^2} \frac{v\mu\omega}{c^2} \left(1 - \frac{1}{n^2 \beta^2}\right) \times$$

$$\int_{-\infty}^{\infty} \frac{d\tau}{\tau} J_0 \left(\left[\frac{n^2 \omega^2}{c^2} - \frac{m^2 c^2}{\hbar^2} \right]^{1/2} v\tau \right) \cos \omega \tau \quad (99)$$

In order to find the final contribution by mass, we are forced to determine the τ -integral in eq. (99). The integral in formula (99) is involved in the tables of integrals (Gradshteyn, 1963). Or,

$$J = \int_0^\infty dx J_0(ax) \cos(bx) = \frac{1}{\sqrt{a^2 - b^2}}, \quad 0 < b < a,$$

$$J = \infty, \quad a = b; \quad J = 0, \quad 0 < a < b, \quad (100)$$

In our case we have

$$a = \left[\frac{n^2 \omega^2}{c^2} - \frac{m^2 c^2}{\hbar^2} \right]^{1/2}, \quad b = \omega \quad (101)$$

Or,

$$\begin{aligned} & \int_{-\infty}^{\infty} \frac{d\tau}{\tau} J_0 \left(\left[\frac{n^2 \omega^2}{c^2} - \frac{c^2 m^2}{\hbar^2} \right]^{1/2} v\tau \right) \cos \omega\tau = \\ & = \begin{cases} \frac{1}{\sqrt{\omega^2(n^2\beta^2-1) - \frac{v^2 c^2}{\hbar^2} m^2}}, & 0 < m^2 < \frac{\hbar^2 \omega^2}{c^2 v^2} (n^2 \beta^2 - 1) \\ 0, & m^2 > \frac{\hbar^2 \omega^2}{c^2 v^2} (n^2 \beta^2 - 1) \end{cases} \end{aligned} \quad (102)$$

From eq. (102) immediately follows that $m^2 > 0$ implies the Čerenkov threshold $n\beta > 1$. From eq. (99) and (102) we get that the radiative corrections to the original spectral formula of the Čerenkov radiation are given by the formula (with $J_0(-z) = J_0(z)$):

$$P = \frac{e^2}{4\pi^2} \frac{v\omega\mu}{c^2} \left(1 - \frac{1}{n^2 \beta^2} \right) \frac{1}{\sqrt{\omega^2(n^2 \beta^2 - 1) - \frac{v^2 c^2}{\hbar^2} m^2}}. \quad (103)$$

The fundamental features of the 3D and 2D Čerenkov radiation are as follows:

- (1) The radiation arises only for particle velocity greater than the velocity of light in the dielectric medium.
- (2) It depends only on the charge and not on mass of the moving particles.
- (3) The radiation is produced in the visible interval of the light frequencies and partly in the ultraviolet part of the frequency spectrum. The radiation does not exist for very short waves, which follows from the dispersion theory of the index of refraction n , where $n < 1$.
- (4) The spectral dependency on the frequency is linear for the 3D homogeneous medium.
- (5) The radiation generated in the 3D medium at given point of the trajectory spreads on the surface of the Mach cone with the vertex at this point and with the axis identical with the direction of motion of the particle. The vertex angle of the cone is given by the relation $\cos \theta = c/nv$.
- (6) There is no Mach cone in the 2D dielectric medium. There is only the Mach angle in the 2D sheet. It follows from the fact that Čerenkov effect is the result of the collective motion of the 2D dielectric medium and it also follows from the quantum definition of the Čerenkov effect in the 2D structures. The conservation laws of momentum and energy for the Čerenkov effect are as follows:

$$\mathbf{p}_i = \mathbf{p}_f + \hbar \mathbf{k} \quad (104)$$

$$E_i = E_f + \hbar \omega, \quad (105)$$

where index i concerns the initial momentum and energy of an electron and index f concerns the final momentum and energy of an electron. Symbol $\mathbf{k}$ is the wave vector of emitted photon and $\hbar \omega$ is its energy. With regard to the situation that the motion of an electron is realized in the plane x-y, the 3D Mach cone cannot be realized (The existence

of Mach cone in our situation is the nonphysical escape of photons from 2D plane to the extra-dimension). So, the nonexistence of the Mach cone in the 2D structures is not mysterious. The 2D-Čerenkov effect with the massless photons in the Mach angle was observed by the famous world laboratories (Adiv et al., 2022)

6 Discussion

The distribution of massive photons generated by the Čerenkov radiation is derived here to our knowledge in the framework of the source theory for the first time and there is no conventional derivation of this effect in QED. As this effect was not discussed in physical literature, we fill up the gap by this article.

The velocity of the charged projectile which generates the massless Čerenkov radiation can be considered during the process of radiation constant because the energy loss due to radiative process is small. However, in case of massive Čerenkov effect the energy loss of the projectile may be large which means the projectile is strongly decelerated. It means the duration of the generation of massive photons is very short. The velocity can be considered constant only in case of very energetic and heavy charged projectile.

From the theoretical point of view, we used the massive electrodynamics which is only the generalization of the massless electrodynamics. So, our derivation of the Čerenkov radiation of massive photons can be considered also as a generalization of the situation with the massless photons.

The theory of the Čerenkov radiation of massive photons concerns the photons not only in superconductive medium but also plasma medium in electron gas, ionosphere medium or photons in waveguides. The possibility of the existence of the massive photons in neutron stars is discussed by Voskresensky et al. (1998). The bosons $W^\pm$ and Z^0 are also massive and it means that the generalization of our approach to the situation in the standard model is evidently feasible. Similarly, the generation of vectors mesons $\rho, \varphi, J/\psi$ by the Čerenkov mechanism may be possible. Probably, they can be generated in a such nuclear medium where they play role of mediators of nuclear forces.

The Čerenkov effect with massive photons can be in the experiment strongly distinguished from the classical effect because the emission of massive photons is distributed in all space directions.

We hope that with regard to the situation in physics of superconductivity, plasma physics, physics of ionosphere, waveguide physics, particle physics, where massive photons are present, sooner or later Čerenkov effect with massive photons will be observed and the theory presented in our article confirmed. The 2D-Čerenkov effect with the massless photons was observed in this year (Adiv et al., 2022) by the famous world laboratories. emission of light would appear.

”In nanofilms and nanowires made of Ag, Au, Cu there are so called surface plasmon polaritons, namely natural waves with a low phase velocity. These waves velocity could be by tens and hundreds times lower than the light velocity in vacuum. An electron with a velocity that is comparatively lower than a velocity of an electron that is able to emit in a uniform medium could emit Čerenkov light. Dispersion relations were calculated for the corresponding plasmons as well as the angles of emission of the plasmons of corresponding frequencies” (Zuev, 2009).

References

- Adiv, Y. et al..(2022). Observation of 2D Cherenkov radiation, arXiv: 2203.01698 [pdf], quant-ph, [physics.optics].
- Anderson, P. W. (1963). Plasmons, gauge invariance, and mass. *Physical Review* **130**(1), 439.
- Čerenkov, P. A. (1934). The visible radiation of pure liquids caused by γ -rays. *Comptes Rendus Hebdomadaires des Sciences de l'Académie des Sciences (USSR)* 2, 451.
- Commins, E. D. and Bucksbaum, P. H. (1983). *Weak Interactions of Leptons and Quarks*, Cambridge University Press, Cambridge.
- de Wit, B. and Smith, J. (1986). *Field Theory in Particle Physics*, Vol. I, Elsevier, Amsterdam.
- Dittrich, W. (1978). Source methods in quantum field theory. *Fortschritte der Physik* 26, 289.
- Feldman, G. and Mathews, P. T. (1963). Massive electrodynamics. *Physical Review* **130**, 1633.
- Goldhaber, A. S. and Nieto, M. N. (1971). Terrestrial and extraterrestrial limits on the photon mass. *Reviews of Modern Physics* **43**, 277.
- Gradshteyn, I. S. and Ryzhik, I. M. (1965). *Table of Integrals, Series and Products*, Academic Press, New York.
- Kirzhnits, A. D. and Linde, A. D. (1972). Macroscopic consequences of the Weinberg model. *Physics Letters* **42B**, 471.
- Minkowski, P. and Seiler, R. (1971). Massive vector meson in external fields. *Physical Review D: Particles and Fields* **4**,
- Pardy, M. (1983a). Particle production by the Čerenkov mechanism. *Physics Letters* **94A**, 30.
- Pardy, M. (1983b). Particle production by accelerated charges moving in condensed matter. *Journal of Physics G: Nuclear Physics* **9**, 853.
- Pardy, M. (1989). Finite-temperature Čerenkov radiation. *Physics Letters A* **134**(6), 357.
- Pardy, M. (1994a). The Čerenkov effect with radiative corrections. *Physics Letters B* **325**, 517.
- Pardy, M. (1994b). The gravitational Čerenkov radiation with radiative corrections. *Physics Letters B* **336**, 362.
- Pardy, M. (1995). The finite-temperature gravitational Čerenkov radiation. *International Journal of Theoretical Physics* **34**(6), 951.
- Pardy M. (2015). The two-dimensional Vavilov-Čerenkov radiation in LED, *Results in Physics* **5**, 6971
- Ryder, L. H. (1985). *Quantum Field Theory*, Cambridge University Press, Cambridge.
- Schwinger, J. (1970). *Particles, Sources and Fields*, Vol. I, Addison-Wesley, Reading, MA.
- Schwinger, J., Tsai, W. Y. and Erber, T. (1976). Classical and quantum theory of synergic Synchrotron-Čerenkov radiation. *Annals of Physics (New York)* **96**, 303.
- Tamm, I. E. and Frank, I. M. (1937). The coherent radiation of a fast electron in a medium. *Doklady Akademii Nauk SSSR* **14**, 107.

- van Nieuwenhuizen, P. (1973). Radiation of Massive Gravitons. *Physical Review D: Particles and Fields* **7**(8), 2300.
- Voskresensky, D. N., Kolomeitsev, E. E. and Kaempfer, B. (1998). Contribution of the massive photon decay channel to neutrino cooling of neutron stars. e-print: astro-ph/9802251.
- Yildiz, A. (1977). Meissner effect in gauge fields. *Physical Review D: Particles and Fields* **16**(12), 3450.
- Zuev, V. S. (2009). Vavilov-Čerenkov phenomenon in metal nanofilms, arXiv: 0907.1145, [Optics (physics.optics)].